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DAILY ABSORBED SOLAR RADIATION AT AIR-WATER INTERFACE

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ABSTRACT

A single analytical expression is developed which describes the daily total absorbed solar radiation at an air-water interface. This function is valid for latitudes between 23.45° and 58.80°. The development assumes that the incoming solar radiation vector is constant. This implies clear sky or constant cloud cover conditions. Reflectance at the water surface is accounted for as well as the daily variation in solar declination. Accuracy of the developed equation is very good when compared to actual values. This function can be very useful in developing analytical models of temperature distributions in water systems. It can be used to provide limiting or average values of temperature variations.

INTRODUCTION

Accurate prediction of temperature profiles in water bodies has been the subject of many papers in the literature [1–5]. The largest single component to the heat flux at the air-water interface is the incoming solar radiation absorbed at the surface [6]. Models which attempt to develop analytical solutions have been hindered by the problems associated with describing incoming solar radiation actually absorbed with a single function of time. The main problem arises from the fact that the reflectivity at the surface is a function of solar zenith angle. Present calculation procedures require separate equations to calculate solar zenith angle, solar declination angle, and hour angle.

Dake and Harleman described the incoming radiation as both a constant and a quadratic with time [2]. No theoretical justification was given. They were able

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to obtain analytical solutions to vertical temperature profiles in lakes for a four month period using a constant radiation flux.

Yotsukura, Jackman, and Faust approximated the net heat flux at the air water interface by expanding each term in a Taylor series and neglecting higher than first order terms [7]. Their incoming radiation function was handled as a piecewise constant so that the linearization was only strictly valid for constant incoming solar radiation.

Presently, there is no single function which describes the daily absorbed solar radiation as a function of time. The purpose of this study is to develop a single equation for this purpose. This equation is valid under conditions where cloud cover is constant on a daily basis and is also valid for latitudes between 23.45° and 58.80°. It is within this latitude region that this function has a single maximum during a yearly cycle. This function could prove valuable in developing analytical solutions for the daily temperature profiles in water bodies.

DERIVATION OF GOVERNING EQUATION

This section describes the derivation of an equation which describes daily solar radiation which actually penetrates through the water surface. This equation is a function of the latitude ϕ and day of year D_y.

The equation which describes the position of the sun at any latitude and time is [8]:

$$\cos Z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h \qquad (1)$$

where Z = zenith angle, δ = declination angle, and h = hour angle. The solar declination angle δ is approximated by:

$$\delta = \frac{23.45\pi}{180} \sin\left\{\frac{360}{365}\left[\frac{\pi}{180}(D_y - 80)\right]\right\} \qquad (2)$$

where D_y = annual day number and all angles are measured in radians.

$$h = \frac{\pi}{12} t - \pi \qquad (3)$$

where t is measured in hours starting at sunrise and ending at sunset. This corresponds to an average of twelve hours of sunlight.

In order to determine the solar radiation which penetrates the water surface, it is necessary to take into account reflectance at the water surface. Viskanta and Toor present a figure which shows the reflectance ρ plotted versus $\sin^2 Z$. The value of ρ is between 0 and 1 and is the fraction of incoming solar radiation reflected at the surface [9]. An excellent approximation of this curve is

$$\rho = \frac{0.018}{\cos^2 Z} \qquad (4)$$

Therefore, the actual radiation absorbed at the water surface is:

$$R_a = R(1 - \rho), \tag{5}$$

where R_a = absorbed radiation at the water surface and R = incoming radiation vector. Assuming R equals one for normalization purposes,

$$R_a = (1 - \rho) \cos Z. \tag{6}$$

Combining equations (4) and (6),

$$R_a = \cos Z - 0.018 \sec Z.$$

Using equations (1) and (3),

$$R_a = a + b \cos\left(\frac{\pi}{12}t - \pi\right) - \frac{0.018}{a + b \cos\left(\frac{\pi}{12}t - \pi\right)} \tag{8}$$

$$a = \sin \phi \sin \delta \tag{9}$$

$$b = \cos \phi \cos \delta \tag{10}$$

For a given position, a and b are constants for a given day.

R_a can now be integrated over the course of the daylight hours for one day. A plot of R_a versus time is symmetric about solar noon for clear sky conditions. Equation (11) gives a general description of the daily absorbed radiation (B):

$$B = 2 \int_{t_o}^{12} R_a dt \tag{11}$$

where t_o = sunrise.

Substituting equation (8) into equation (11),

$$B = 24a - 2at_o - \frac{24}{\pi} b \sin\left(\frac{\pi}{12}t_o - \pi\right) + \frac{\frac{24}{\pi}(0.036)}{\sqrt{|a^2 - b^2|}} \tan^{-1} \left[\frac{\sqrt{|a^2 - b^2|}}{a + b} \tan \frac{\left(\frac{\pi}{12} t_o - \pi\right)}{2}\right] \tag{12}$$

Note that t_o is calculated by

$$R_a(t_o) = 0 = (1 - \rho) \cos Z. \tag{13}$$

Substituting equations (1), (3), and (4), one obtains

$$t_o = 24 - \frac{12}{\pi} \left[\pi + \cos^{-1} \left(\frac{0.1342}{\cos\phi \cos\delta} - \tan \phi \tan \delta\right)\right] \tag{14}$$

At this point, a solution for the total daily absorbed radiation for any given day B requires solution of equations (9), (10), (12), and (14). It would be desirable from a modeling viewpoint to obtain one equation which would calculate B as a function of ϕ and D_y. For $23.45° < \phi < 58.8°$, B is a single maximum function which can be very closely approximated by a cosine function. So, for various values of ϕ between 23.45° and 58.80°, equations (9), (10), (12), and (14) were solved to obtain values of B versus D_y. For the purpose of developing analytical models of thermal distribution in water it is desirable to fit this information to a function of the form

$$B = C - A \cos \left[\frac{2\pi}{365} (D_y - 355) \right] \tag{15}$$

C and A are functions of ϕ. Figure 1 shows values of C and A plotted versus latitude ϕ and the relationship is approximately linear. Therefore, a "least-squares" fit should yield values for A and C which can be substituted into equation (15).

C and A were determined for each ϕ in the following fashion. C represents the average value of the B computed and A is the amplitude of the variation throughout the year.

$$A = 3.4625\,\phi + 0.5870 \tag{16}$$

$$C = -3.6749\,\phi + 7.8389 \tag{17}$$

Substituting equations (16) and (17) into equation (15), we obtain the desired result.

$$B = (-3.6749\,\phi + 7.8389) - (3.4625\,\phi + 0.5870) \cos \left[\frac{2\pi}{365} (D_y - 355) \right] \tag{18}$$

Equation (18) can now be used for any ϕ between 0.4093 radians and 1.0263 radians. This single equation will calculate the daily total absorbed radiation at an air-water interface. Equation (18) is based on a unit amount of incoming solar radiation to the water surface. Therefore to use this equation, it is necessary to determine the actual amount of incoming solar radiation to the surface. This can be accomplished by one measurement of the incoming solar radiation such as a direct measurement of the solar radiation flux or a measurement of the normal component to the water surface and the solar zenith angle.

RESULTS

To determine the accuracy of equation (18), B was calculated using equation (18) and equations (9), (10), (12), and (14). The latitude chosen was 0.7044 radians (40.4°) which is the latitude for Laramie, Wyoming. The largest error is 3.62 per cent which occurred on $D_y = 92$. The root-mean-square deviation for the entire year was 0.101. The value of B varied from 2.100 to 8.407. Therefore,

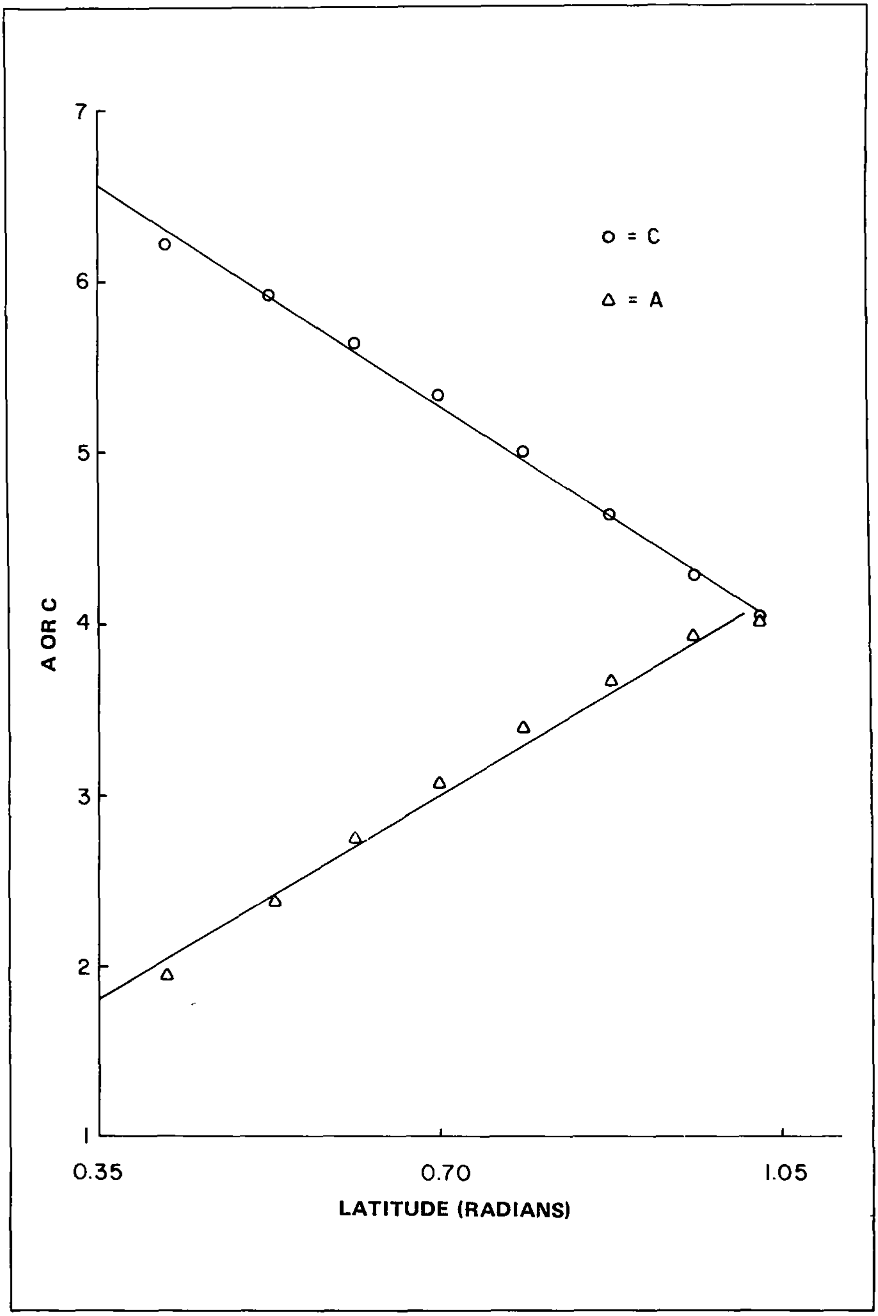

Figure 1. A or C (equation (15)) versus latitude.

equation (18) proves a very useful and accurate approximation to the daily absorbed solar radiation at a water surface.

CONCLUSIONS

An analytical expression has been developed which describes the daily total absorbed solar radiation at an air-water interface under clear sky or constant cloud cover conditions. This function takes into account the reflection of solar radiation at the interface. The expression is valid for latitudes between 0.4093 radians (23.45°) and 1.0263 radians (58.80°). The expression also assumed that the incoming solar radiation vector is a constant. This function was shown to yield very good approximations to the actual value and can serve as a useful tool in modeling water temperature distributions in water systems.

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