J. ENVIRONMENTAL SYSTEMS, Vol. 19(2) 115-134, 1989-90

AN AQUACULTURE MODEL FOR WASTE HEAT UTILIZATION ASSESSMENT*

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ABSTRACT

Thermal effluents from power plants have been used to accelerate the growth of aquatic organisms. Aquaculture can be made more profitable by raising aquatic organisms under controlled temperature conditions to obtain maximum food conversion efficiency. In order to evaluate whether the benefits of increased productivity outweigh the costs of temperature control, it is necessary to simulate the operation of aquaculture facilities. This aquaculture model has two parts. First, a materials balance approach is used to estimate the growth of fish at various temperatures. Second, a heat balance provides a way to determine the mass flow of heated water which must be supplied to the pond to maintain the desired pond temperature under anticipated weather conditions.

INTRODUCTION

Every year, American power plants discharge about 11×10^{15} Btu of low-grade, "waste" heat. This heat is rejected to the environment as warm water at 60 to 110°F [1]. These temperatures are too low for most industrial processes, but they are ideal for many living organisms. Fish, livestock, and plants grow faster at optimum temperatures, and require less nutrients. Biological waste treatment

* **Editor's Note:** This is the second in a series of eight articles on the utilization of waste heat from power plants. The first article (pp. 95-114, herein) presented a method for site-specific assessment of technology options and a summary of results. Future articles describe models for simulating the greenhouse, livestock, crop drying, and wastewater treatment components of an integrated waste heat utilization complex.

doi: 10.2190/F93C-YWGA-407C-7TGR
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is accelerated, so a greater volume of wastes can be handled. Air flow requirements for crop drying can be reduced if the temperature of the air is elevated.

Further efficiency improvements may be obtained by linking together several operations into a single integrated complex. This mimics the natural cycling of nutrients among plants and animals, thereby minimizing both waste disposal and feed costs. Consider the arrangement shown in Figure 1. The waste-laden effluent of the aquaculture facilities passes through a series of waste treatment ponds used for water hyacinth and algae production. The water hyacinths are harvested mechanically and fermented into ethanol, while the algae are filtered biologically by clams in the clam and crayfish pond. The renovated water is aerated and returned to the aquaculture facility.

Livestock shelters for broiler chickens and swine litters provide ample manure for the anaerobic digesters. Municipal sewage and refuse can be added as necessary to achieve the proper moisture content and chemical composition. The anaerobic digestion process yields methane gas, which can be burned to provide backup heating whenever waste heat supplies are inadequate. The liquid by-product supernatant is treated in the algae pond, while the solid sludge portion becomes fertilizer for the greenhouses. This complex produces fish, shellfish, livestock, vegetables, flowers, ethanol, and methane for wholesale markets, and also provides waste treatment and crop drying services.

By varying the proportion of the complex which is devoted to each particular technology, we can adapt this arrangement to a specific site. For example, a complex in Minnesota could emphasize cold-water fish such as trout, while a complex in Arkansas might favor warm-water shellfish such as prawns. Crop drying will be more profitable in a rural area, while urban centers may pay a premium for waste treatment services.

In designing such a system, we are faced with a bewildering array of power plant operating data, local market prices, anticipated weather conditions, biological production functions, and interconnections among diverse production facilities. Perhaps the only way to analyze such a complicated system is to simulate its performance under numerous sets of conditions, and then use optimization techniques to select the best configuration for each specific site. (For a more detailed explanation, see [2] or [3].)

In this article, we describe a model for simulating the operation of aquaculture facilities. This model can be combined with others to aid in the design of integrated waste heat utilization complexes. It is based on materials and heat balances which are presented following a brief review of aquaculture options appropriate for the United States. The model described here provides a means for calculating aquaculture production, feed consumption and thermal requirements. It has been incorporated into a larger decision-model for testing the feasibility of waste heat utilization options. Numerical results will be presented in a later paper in this series.

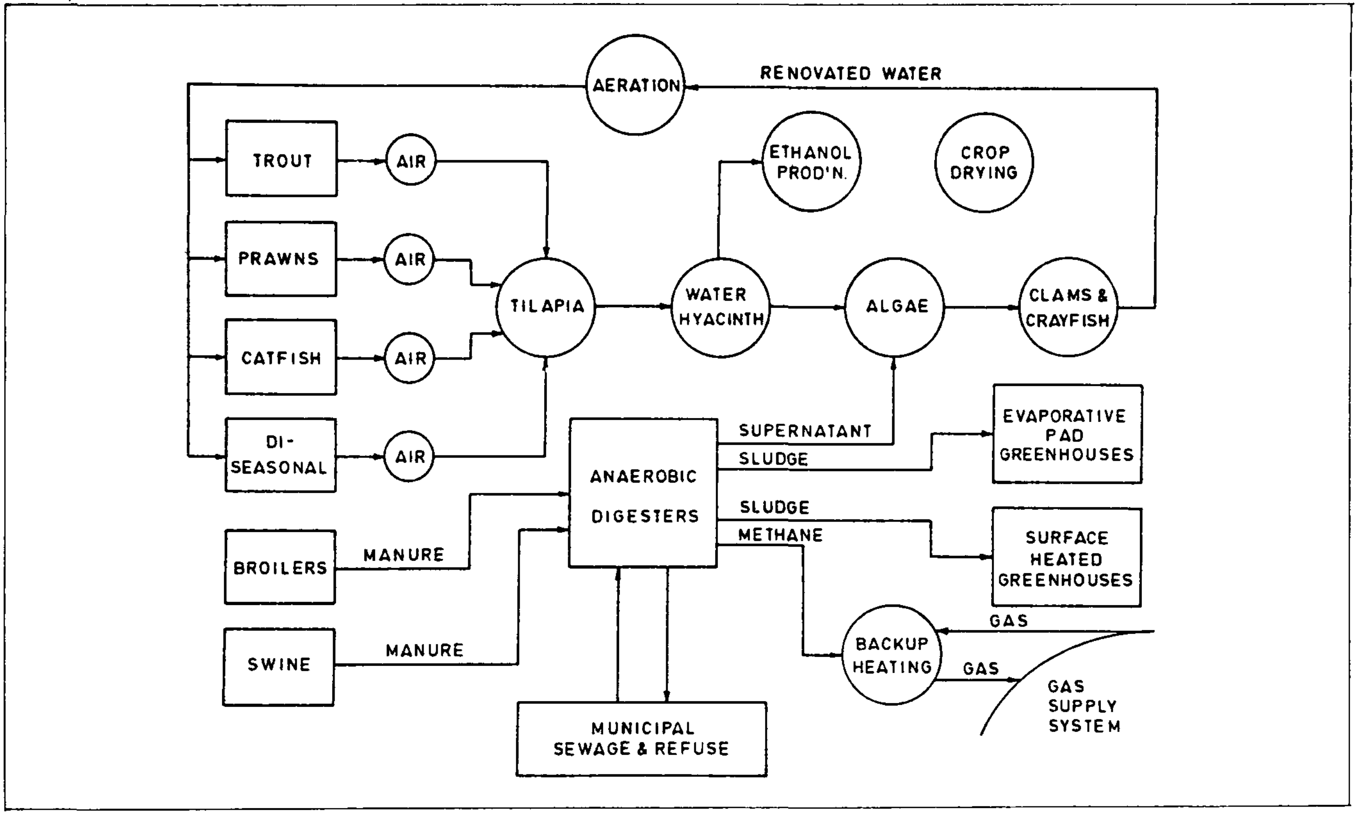

Figure 1. Spectrum of integrated waste heat utilization options.

AQUACULTURE

The practice of raising aquatic organisms for food is known as "aquaculture." This may be as simple as supplemental feeding of fish in a natural pond, or as complex as artificial control of all environmental factors in a computer-monitored raceway. The basic methods of fish farming have been known for centuries, but until recently the natural supplies of fish and shellfish seemed adequate. Modern culture techniques will enable aquaculture to become a major source of protein, in response to a rapidly growing demand for seafood and limited natural yields [4].

Water temperature is crucial in the farming of aquatic organisms, since oxygen consumption, food consumption, growth rate, and conversion efficiency of food to flesh are more highly temperature-dependent in cold-blooded than in warm-blooded species. In catfish, for example, a 9°F reduction from optimum temperature reduces the growth rate by 50 percent and the food conversion efficiency by 20 percent [5]. These rather substantial beneficial effects have led to numerous thermal aquaculture projects. The most comprehensive survey to date identified over fifty such efforts [5]. A small sample of these appears in Table 1.

Only freshwater organisms will be considered in this article because most potential waste heat sources use fresh water cooling. To narrow the focus even further, we consider only the most promising freshwater organisms. However, the reader should be aware that marine oysters, clams, lobsters, shrimp, flatfish, and finfish have been cultured with some success.

The aquaculture of some species is highly developed. Nearly all of the thirty million pounds of trout marketed each year in the United States are raised on commercial trout farms located in thirty-eight states [6]. Catfish farms produce over ninety million pounds annually [7]. Therefore, catfish and trout are excellent candidates for thermal aquaculture; their culture requirements are well understood, and established markets exist for both.

Freshwater prawns (which are similar to shrimp) are more difficult to raise, but can be sold at very high prices [5]. They require warm temperatures, which could be readily attainable using waste heat. They may be grown independently, or, alternate with trout on a seasonal basis. Lower heat requirements and reduced biological stress are possible by growing warmwater species (e.g., prawns) in the summer and cold water species (e.g., trout) in the winter. This "diseasonal approach" has been advanced by Guerra, Godfriaux, and Sheahan, among others [8].

While prawns are regarded as fragile, tilapia have a reputation for being "nearly indestructible." Tilapia thrive on very inexpensive feeds, and thus have been promoted as a "protein source of the future" [9]. Their only weakness is that they are tropical fish, although waste heat could help to overcome this obstacle.

Table 1. Some Heated Water Aquaculture Projects in the United States with Freshwater Organisms [5]

Organization	*Location*	*Stage*
Freshwater Prawns		
Public Service Electric & Gas	Trenton, NJ	Research
Texas Electric Company	Monahans, TX	Research
Sierra Pacific Power Co.	Xerington, NE	Research
Catfish		
Aquarium Farms, Inc.	Freemont, NE	Commercial
Fish Breeders of Idaho	Buhl, ID	Commercial
Cultural Catfish, Inc.	Colorado City, TX	Commercial
Eels		
Public Service Electric & Gas	Trenton, NJ	Research
Tilapia		
Aquarium Farms, Inc.	Freemont, NE	Commercial
Fish Breeders of Idaho	Buhl, ID	Commercial
Weisbart and Weisbart, Inc.	Alamosa, CO	Commercial
Yellow Perch		
Idaho State University	Pocatello, ID	Research
Vermont Yankee Nuclear Power Corp.	Rutland, VT	Feasibility
Trout		
Public Service Electric & Gas	Trenton, NJ	Research
Maine Salmon Farms	Wiscasset, ME	Commercial

A large number of other aquatic organisms were considered and ultimately excluded from this study for various reasons. For example, carp suffer from very low market prices, while perch and striped bass have both proved very difficult to culture [5]. Likewise, eels are highly susceptible to disease, pollution, and parasites [10].

Thus, there are five viable aquaculture options: trout, catfish, prawns, tilapia, and diseasonal trout/prawns. These are described in greater detail in the following sections.

Trout

Rainbow trout, which are common in North American lakes and streams, consume insects, snails, amphipods, and fish, and can survive temperatures of 32 to 80°F. Eggs are deposited, fertilized, and buried in gravelly areas [11]. Spawning may be controlled by regulating water temperature [12]. Maximum food conversion efficiency and growth rate are observed at 63°F; market size of twelve ounces is reached in 250 days at a constant 59°F [13].

The Public Service Electric and Gas Company has conducted extensive research on trout in two 100 foot × 12 foot raceways, each 6 feet deep. Rainbow trout (*Salmo gairdneri*) were stocked at 4 lb/ft^3 as 6.5 inch fingerlings. The Mercer Generating Station facility harvested 165,590 trout, weighing 67,830 lb, at a density of 7 lb/ft^3, in April 1982. These extraordinary densities were made possible by supersaturating the raceway water with oxygen. Liquid oxygen was vaporized, mixed with raceway water at a pressure of 85-90 psi, and thus maintained dissolved oxygen levels at 12 mg/L in the water entering at 2500 gal/min (gpm). Demand feeders dispensed food pellets whenever trout nudged pressure-sensitive levers [14].

Catfish

Channel catfish (*Ictalurus punctatus*) inhabit both streams and lakes, feeding on aquatic insects, fish, and shellfish. After spawning in late spring, eggs are laid in a single mass which is guarded by the male. Fingerlings may be grown to 1 lb market size during a 180 day period in which water temperatures exceed 60°F [15].

Food conversion efficiency reaches a maximum of 1.5:1 at 84°F, and declines to 2.5:1 at either 70 or 98°F [16]. Production rates of 0.5 lb/ft^3 have been obtained in 100 ft raceways with a 1.5 hour turnover time; flow rates of 500 gpm were needed to keep the dissolved oxygen level above 4 mg/L [17]. Intensive culture techniques can boost production rates ten-fold to the 5 to 10 lb/ft^3 rates achieved with a seventeen minute turnover time at the Tennessee Valley Authority's Gallatin Project [5]. Catfish have also been raised successfully in floating cages placed in the discharge canal of a power plant [18].

Prawns

These crustaceans may be found in streams, lakes or brackish estuaries, feeding on live or dead fish, invertebrates, and vegetable matter. After mating, females migrate to brackish estuaries, where each deposits over 70,000 eggs in water with salinity of 12 to 16 ppt and pH of 7.5 to 8.0. These eggs hatch into larvae which pass thorugh a series of free-swimming and benthic stages, feeding on zooplankton and other larval invertebrates. Malaysian prawns (*Macrobrachium rosenbergii*) reach maturity in six to nine months [19]. Considerable mortality is observed outside temperatures between 75 and 87°F, with the higher portion of the range preferred for rapid growth [20].

Juvenile prawns weighing 0.005 ounces can reach 0.3 ounces in 112 days with 18 percent mortality when stocked at $0.3/ft^3$, but reach only 0.15 ounces with 50 percent mortality when stocked at $18/ft^3$ [21]. Overstocking leads to cannibalism and reduced growth, despite the presence of adequate food and oxygen supplies. If netting is draped into the water, the territory available to each prawn is increased, and mortality declines [19].

Tilapia

These hardy fish can withstand temperatures as high as 100 to 112°F, or dissolved oxygen levels as low as 1 mg/L, but must be kept at temperatures above 43 to 54°F, depending on the species [22]. Most tilapia are mouth-breeders, and high reproductive rates can lead to over-population and stunted growth. This can be controlled by producing monosex cultures through hybridization, or by feeding hormones to young tilapia to bring about sex reversal.

Although wild tilapia eat plankton, algae, and plants, cultured tilapia will accept odd foods such as rice bran, manure, corn meal, kichen scraps, rotten fruit, coffee pulp, flour, and catfish chow as supplements [4]. These feeding habits have prompted many to propose waste treatment systems using tilapia. One facility was constructed so that effluent from catfish raceways flowed into tilapia raceways; the tilapia raised overall system productivity by consuming uneaten food and solid wastes [9]. Weisbart and Weisbart, Inc., has achieved a food conversion ratio of 1.5:1 using pelleted feed in a system which utilized 80°F geothermal water [23].

MATERIALS BALANCE

The aquaculture simulation model is based upon the construction of a materials balance on the fish and a heat balance on the pond. First, we consider the materials balance. The production of energy in the form of fish flesh is the difference between the fish's metabolizable energy intake and the heat loss from the body of the fish [24]:

$$P = Q - H \tag{1}$$

where: P = production of energy as fish tissue, Btu/hr
Q = metabolizable energy intake, Btu/hr
H = heat lost to the environment, Btu/hr.

Note that the metabolizable energy intake does not include energy lost in the form of wastes; that is,

$$Q = (\text{food intake}) - (\text{feces} + \text{urine} + \text{gill excretions}). \tag{2}$$

The amount of food which a fish must eat to achieve a given metabolizable energy intake is:

$$F = \frac{Q}{M} \tag{3}$$

where: F = amount of food eaten, lb/hr
M = metabolizable feed energy, Btu/lb.

Production is measured indirectly, at the time of harvest, by weighing the amount of fish produced. Most empirical data on operational performance of fish are reported in the form of weight gain. Therefore, it is often more meaningful to express production in terms of pounds gained rather than Btu's. We can use the relationship:

$$P = CG \tag{4}$$

where: C = energy content of the fish, Btu/lb
G = rate of weight gain by the fish, lb/hr.

Combining equations 1, 3, and 4, equation 5 is obtained:

$$CG = FM - H. \tag{5}$$

Solving for G:

$$G = \frac{FM - H}{C} . \tag{6}$$

This formula for G is derived from theoretical considerations. In order to calculate the food requirements, we need to equate this to an empirical formula for G and solve for F.

Curves relating fish weight gain to temperature in terms of percentage of the maximum growth rate, G_{max}, are quite common in the literature [25]. These are of the form:

$$G = g(T)G_{max} \tag{7}$$

where g(T) represents the function relating fish weight gain and temperature. We can substitute equation (7) for G into equation (6) and solve this expression for F:

$$g(T)G_{max} = \frac{FM - H}{C} \tag{8}$$

or,

$$F = \frac{H + g(T)CG_{max}}{M} \tag{9}$$

A common form for g(T) expressions is [26]:

$$g(T) = V^{x}e^{x(1-V)} \tag{10}$$

where

$$V = \frac{T'_{max} - T'}{T'_{max} - T'_{opt}} \tag{11}$$

and: x = constant
T' = water temperature, °C
T'_{max} = upper lethal temperature, °C
T'_{opt} = optimum growth temperature, °C .

Table 2. Values of Q_{10} for Van't Hoff's Law [29]

	Temperature Range, °C					
	0-5	*5-10*	*10-15*	*15-20*	*20-25*	*25-30*
Q_{10}	10.9	3.5	2.9	2.5	2.3	2.2

Heat loss may be found using Van't Hoff's Law [27]:

$$H = H_o Q_{10}^{0.1T'} \tag{12}$$

where: H_o = metabolic rate at 0°C, Btu/hr
Q_{10} = coefficient based on effective heat transmissivity, dimensionless.

Van't Hoff's Law follows from the approximate doubling of reaction rates in simple chemical systems for each 10°C rise in temperature [28]. It is most applicable to poikilotherms (cold-blooded animals), such as fish and shellfish. Values for Q_{10} as a function of temperature are given in Table 2.

Assuming constant temperature, weight gain can be calculated using [30]:

$$\frac{dW}{dt} = K_o W^\alpha S \tag{13}$$

with

$$S = 1 - \left(\frac{W}{W_{max}}\right)^\mu \tag{14}$$

where: W = weight of the fish, lb
t = time, hr
K_o = constant, dimensionless
α = constant, dimensionless
μ = constant, dimensionless
S = growth-limiting factor, dimensionless
W_{max} = maximum attainable weight, lb.

This is because growth depends roughly on the surface area of the digestive tract, and so is approximately proportional to $W^{0.67}$ (which follows from an area to volume ratio). Empirical data suggest that a $W^{0.75}$ proportionality is more accurate [27]. Other factors, such as the size of the fish tank, may limit growth. The S factor expresses the observation that growth slows as the weight of the creature approaches some W_{max}. This factor may or may not be significant, depending on how densely the fish are stocked.

If we determine K_o for fish at optimum temperature, we have:

$$G_{max} = KW^\alpha S \tag{15}$$

where: $K = K_o$ at optimum temperature.

Recall Van't Hoff's Law (equation (12)):

$$H = H_0 Q_{10}^{0.1T'} \, . \tag{12}$$

The overall metabolism is also proportional to W^{α}. If we assume that S limits growth but not metabolism, and if we evaluate K_0 for fish at some minimum temperature:

$$H_0 = JW^{\alpha} \tag{16}$$

where J is the value of K_0 at this temperature.

Recall the equation we derived for F (equation (9)):

$$F = \frac{H + g(T)CG_{max}}{M} \, . \tag{9}$$

We can now combine equation (12) with equation (16) to form:

$$H = JW^{\alpha}Q_{10}^{0.1T'} \, . \tag{17}$$

Equations (15) and (17) are substituted into equation (9) to give:

$$F = \frac{JW^{\alpha}Q_{10}^{0.1T'} + g(T)CKW^{\alpha}S}{M} \, . \tag{18}$$

The final step is to factor out the W^{α}:

$$F = \frac{W^{\alpha}(JQ_{10}^{0.1T'} + g(T)CKS)}{M} \, . \tag{19}$$

Thus, for a given temperature, weight of fish, and energy content of feed, we can calculate the amount of food required.

Once we have F, we can calculate the gain by using the results of equations (17) and (19) in equation (6). We note that:

$$G = \frac{dW}{dt} \, . \tag{20}$$

Therefore, expressing this relationship over discrete time intervals,

$$W_{n+1} = W_n + G(t_{n+1} - t_n) \tag{21}$$

where:

$(t_{n+1} - t_n)$ = growth iteration time interval, hours.

This model may be applied to the species described in this article using the data presented in Table 3. For a given operating temperature (T') and an initial weight (W_0), the weight of the organism can be determined iteratively using equation (21). The value of G in equation (21) is determined using equation (6). In equation (6), F is evaluated from equation (19), and H, from equation (17). The various inputs which are required are available as the organism specific data in Tables 2 and 3, or as design parameters (T' and W_0). This model may be extended to other species if sufficient data are available.

Table 3. Aquaculture Data for Growth Model [2]

Property	*Trout*	*Catfish*	*Prawns*	*Tilapia*
M	3960	3735	3654	3735
J	2.1111	1.11	1.237	1.4875
C	2158	2016	2058	2094
K	0.0054	0.0059	0.0039	0.0079
W_{max}	2.0	5.8	0.22	2.0
α	0.75	0.75	0.75	0.75
μ	0.25	0.25	0.25	0.25
X	4	4	3	4
T_{max}	27	36	35	38
T_{opt}	17	29	30	29

HEAT BALANCE

A typical pond heat budget is presented by Thackston and Parker [31]:

$$H_t = H_c + H_e + H_b + H_a + H_s + H_g, \tag{22}$$

where:
H_t = net heat gained or lost by pond, Btu/hr
H_o = heat gained or lost by conduction to air, Btu/hr
H_e = heat lost by evaporation, Btu/hr
H_b = long wave back radiation of the body of water to space, Btu/hr
H_a = absorbed long wave radiation, Btu/hr
H_s = absorbed solar radiation, Btu/hr
H_g = heat gained or lost by conduction to the ground, Btu/hr

Heat Loss by Conduction

The conductive heat losses are simply [31]:

$$H_c = (0.00543)vp(T_a - T_w)A \tag{23}$$

or,

$$H_c = \frac{(0.16247)v(T_a - T_w)A}{\exp\left(\dfrac{32.15E}{1545(T_a + 460)}\right)} \tag{24}$$

where: v = wind speed, mph
T_a = air temperature, °F
T_w = water temperature, °F
E = elevation of the site, ft
A = area of pond or raceway, ft^2
p = atmospheric pressure, inches of Hg .

Note that an empirical relationship has been used for the atmospheric pressure p in equation (23) to arrive at equation (24). This relationship is [31]:

$$p = \frac{29.92}{\exp\left(\dfrac{32.15E}{1545\,(T_a + 460)}\right)} \tag{25}$$

Heat Loss by Evaporation

The evaporative loss calculation is adapted from Root [32]:

$$H_e = 200h_{hl}(p_w - p_a)A \tag{26}$$

where: h_{hl} = heat loss coefficient, Btu-hr^{-1}-ft^{-2}-°F^{-1}
$= 0.32v + 0.42$
p_w = water vapor pressure of the air at the water surface, psi
p_a = water vapor pressure of the surrounding air, psi.

Empirical formulae are also available for the vapor pressures and for the wet bulb temperature [31]:

$$p_w = 0.491 \exp\left(17.62 - \left(\frac{9501}{T_w + 460}\right)\right) \tag{27}$$

$$p_a = 0.491 \exp\left(17.62 - \left(\frac{9501}{T_{wb} + 460}\right)\right) \tag{28}$$

$$T_{wb} = (0.655 + 0.36R)T_a \tag{29}$$

where: R = relative humidity, decimal
T_a = air temperature, °F
T_w = water temperature, °F
T_{wb} = wet bulb temperature, °F .

Heat Loss by Radiation

Radiative losses are found by [31]:

$$H_b = 0.97\sigma(T_w + 460)^4 A \tag{30}$$

where: 0.97 = emissivity of the surface, dimensionless
σ = Stefan-Boltzman Constant = 1.714×10^{-9} Btu/hr^{-1}-ft^{-2}-°F^{-4}.

However, there are also radiative gains due to absorption of radiation given off by the surroundings [33]:

$$H_a = -e_{at}\sigma(T_a + 460)^4 A \ , \tag{31}$$

where: e_{at} = sky emittance (approximately 0.82), dimensionless.

Heat Gain by Solar Radiation

Additional heat is gained through direct radiation from the sun [34]:

$$H_s = -gAI \ , \tag{32}$$

where: g = fraction of horizontal radiation gained (approximately 0.80)
I = horizontal insolation, Btu-ft^{-2}-hr^{-1}.

Heat Loss to the Ground

Finally, we have the heat losses by conduction to the ground. The lining of the pond will be at essentially the same temperature as the pond. Therefore, the heat loss will depend primarily on the heat transfer coefficient of the soil, and the difference between the temperature of the water and the effective temperature of the soil. Along the sides, this effective temperature will be the average of the air temperature and the surface water temperature. Actually, this effective temperature depends on the depth of frost penetration, but this estimate will do as an approximation.

Beneath the bottom of the pond, the effective temperature of the distant soil is the ground water temperature. Since there is a considerable time lag between the temperature of the ground water and that of the air, and yet the ground water must, on average, be in thermal equilibrium with the atmosphere, we can use the mean annual temperature as an estimate of the ground water temperature.

The heat lost to the ground is [35]:

$$H_g = 0.1A(T_w - T_{gw}) + 0.1P_L d(T_w - T_{eff}) \ , \tag{33}$$

where: T_{gw} = ground water temperature, °F
T_{eff} = effective ground temperature, °F
0.1 = heat transfer coefficient of the ground, Btu-hr^{-1}-ft^{-2}-°F^{-1}
P_L = perimeter of the pond, ft
d = depth of the pond, ft.

The net heat gained or lost by the pond, H_t, is determined using equation (22). The evaluation of this equation in turn requires site-specific weather data (v, T_a, T_{wb}, R, I, T_{gw}, and T_{eff}), known or assumed design parameters (T_w, A, P_L, E, and d), and known constants (h_{hl}, σ, e_{at}, and g).

FLOW REQUIREMENTS

The most straightforward way to transfer heat from thermal effluent to water in a pond, without mixing the streams of water, is to pass the effluent through pipes laid on the bottom of the pond. The construction and maintenance of such a system is very simple, and piping is readily available. The system uses the minimum material possible, since the water transporting vessel and the heat transfer surface are combined.

Several authors, including Vaughan, Holmes, and Bell [36], discuss submerged pipe heat exchangers. They point out that the thermal conductivity of the pipe material plays a very small role in determining the overall heat transfer. Rather, the forced convection caused by the velocity of the fluid inside the pipe, and the natural convection outside the pipe, are the determining factors.

Incropera and Rog experimented with several pipe diameters and spacings [37]. They concluded that one-inch pipes spaced six inches apart marked the best trade-off among cost, efficiency of heat transfer, and pressure losses from friction.

The following empirical equation, derived from Incropera and Rog's data [37], describes the dependence of the heat transfer coefficient on the mass flow rate of the water, to within a few percent:

$$U = 2.466(m)^{0.308}, \tag{34}$$

where: m = mass flow rate of the water in the pipe, lb/hr
U = heat transfer coefficient of the pipe, Btu-hr^{-1}-ft^{-1}-°F^{-1}.

Note that the value of U obtained through this expression is to be multiplied by the length of one-inch pipe measured in feet.

In practice, the flow rates required to heat a large pond or raceway are too large for a single loop of one-inch pipe. Therefore, we divide each pond into sections of fifty square feet. Since the pipes are spaced six inches apart, a single section (zone) would have pipes totaling one hundred feet in length. The pond is in thermal equilibrium, so the heat loss from the pond is balanced by the heat gain from the pipe zones. The number of zones needed can be found by dividing the pond area by the fifty square feet served by each pipe zone. Each pipe zone must replenish a fraction of the overall heat loss:

$$H_p = H_t\left(\frac{50}{A}\right), \tag{35}$$

where: H_p = heat released by a single zone of pipes, Btu/hr.

We would like to know how much warm water to supply, given the heat loss calculated using equations (22) and (35). A heat balance is obtained by equating the heat lost from the pipe with the heat lost by the water in the pipe and the heat transmitted through the pipe [38].

$$-H_p = mc(T_2 - T_1) = UL(LMTD)\,. \tag{36}$$

Note that:

$$(LMTD) = \frac{(T_d - T_1) - (T_d - T_2)}{\ln\left(\frac{(T_d - T_1)}{(T_d - T_2)}\right)} \tag{37}$$

$$= \frac{T_2 - T_1}{\ln\left(\frac{(T_d - T_1)}{(T_d - T_2)}\right)} \tag{38}$$

where: (LMTD) = log mean temperature difference, °F
c = specific heat of water (approximately 1.0 Btu-lb^{-1}-°F^{-1})
T_d = pond temperature desired, °F
T_1 = temperature of water entering the pipe, °F
T_2 = temperature of water exiting the pipe, °F
L = length of the pipe section, ft.

Neglecting H_p for the moment, we can rearrange equation (36):

$$\frac{UL}{mc} = \frac{(T_2 - T_1)}{(LMTD)}\,. \tag{39}$$

We use the expression for (LMTD) found in equation (38), and observe that the two $(T_2 - T_1)$ factors cancel:

$$\frac{UL}{mc} = \frac{T_2 - T_1}{\left(\frac{T_2 - T_1}{\ln\left(\frac{T_d - T_1}{T_d - T_2}\right)}\right)} \tag{40}$$

$$= \ln\left(\frac{T_d - T_1}{T_d - T_2}\right) = -\ln\left(\frac{T_d - T_2}{T_d - T_1}\right). \tag{41}$$

We exponentiate both sides and rearrange [38]:

$$\exp\left(\frac{-UL}{mc}\right) = \frac{T_d - T_2}{T_d - T_1} = \frac{T_d - T_2 + T_1 - T_1}{T_d - T_1}\,. \tag{42}$$

This gives us an expression for the exit temperature:

$$T_2 = T_1 + (T_d - T_1)\left(1 - \exp\left(\frac{-UL}{mc}\right)\right). \tag{43}$$

Unfortunately, we cannot solve explicitly for m or T_2 using equations (34) and (43). We must use an iterative procedure:

1. Assume $T_2 = T_d$ to initialize m (only in step 2).

2. $m = \frac{H_p}{c(T_1 - T_2)}$ (from equation (36)).
3. Use equation (34) to calculate U.
4. Use equation (43) to calculate T_2.
5. $m = \frac{H_p}{c(T_1 - T_2)}$ (from equation (36)).
6. Repeat steps 3 through 5 if

$$\left| \frac{m_{new} - m_{old}}{m_{new}} \right| > 0.01$$

This iterative procedure is used to estimate the mass flow of water through a single pipe zone. For the entire pond, the total required mass flow, m_t, is:

$$m_t = m \frac{A}{50} . \tag{44}$$

In this manner, we determine the mass flow rate of water at a given temperature (T_1) required to maintain the pond at a specified temperature (T_d). The required inputs include H_t from section 4, certain design parameters (A, T_1, and L), and the constant c. The pond temperature, in turn, is related to the growth rate of the fish using the equations developed previously.

CONCLUSIONS

The beneficial use of thermal effluents has an intuitive appeal. Rather than increasing ecosystem stress in the form of thermal pollution, we are able to derive tangible benefits in the form of increased food production. However, quantifying these benefits has proved elusive.

This aquaculture model provides a means for predicting the feed consumption and thermal requirements of an aquaculture facility, along with food production. The materials balance enables us to calculate growth and ultimate yield, based upon principles of metabolism. The heat balance of the pond (or raceway) translates ambient conditions into the amount of heat which must be replenished. The flow of thermal effluent which is needed to maintain optimum temperatures is found through an iterative procedure.

We have identified trout, catfish, prawns, and tilapia as being particularly well suited for thermal aquaculture projects, based upon their performance in pilot studies and their marketability. Waste disposal and feed costs can be minimized by placing these aquaculture facilities within an integrated waste heat utilization complex. Waste heat aquaculture represents a major opportunity for improving the overall efficiency of energy utilization, waste treatment, and food production.

NOMENCLATURE

The following is a summary list of the nomenclature used throughout this article:

A = Area of pond or raceway, ft^2
C = Energy content of fish, Btu/lb
E = Site elevation, ft
F = Food consumption, lb/hr
G = Rate of fish weight gain, lb/hr
G_{max} = Maximum rate of fish weight gain, lb/hr
H = Heat loss to environment, Btu/hr
H_o = Metabolic rate at 0°C, Btu/hr
H_a = Absorbed long-wave radiation, Btu/hr
H_b = Long-wave back radiation, Btu/hr
H_c = Heat exchange by conduction to air, Btu/hr
H_e = Heat loss by evaporation, Btu/hr
H_g = Heat exchange to the ground, Btu/hr
H_p = Heat released by a single zone of pipes, Btu/hr
H_s = Absorbed solar radiation, Btu/hr
H_t = Net heat lost by pond, Btu/hr
I = Horizontal insolation, $Btu\text{-}ft^{-2}\text{-}hr^{-1}$
J = Value of K_o at minimum temperature
K = Value of K_o at optimum temperature
K_o = Dimensionless constant
L = Length of pipe section, ft
LMTD = Log mean temperature difference
M = Metabolizable feed energy, Btu/lb
P = Production of energy as fish tissue, Btu/hr
P_L = Pond perimeter, ft
Q = Metabolizable energy intake, Btu/hr
Q_{10} = Coefficient based on effective heat transmissivity, dimensionless
R = Relative humidity, dimensionless decimal
S = Dimensionless growth limiting factor
T' = Water temperature, °C
T_a = Air temperature, °F
T_d = Desired pond temperature, °F
T_{eff} = Effective ground temperature, °F
T'_{max} = Upper lethal temperature, °C
T'_{opt} = Optimum growth temperature, °C
T_{gw} = Ground water temperature, °F
T_w = Water temperature, °F
T_{wb} = Wet bulb temperature, °F
T_1 = Temperature of water entering pond, °F
T_2 = Temperature of water exiting pond, °F
U = Pipe heat transfer coefficient, $Btu\text{-}hr^{-1}\text{-}ft^{-1}\text{-}°F^{-1}$
V = Dimensionless temperature factor
W = Weight of fish, lb
W_{max} = Maximum attainable weight, lb
c = Specific heat of water, $Btu\text{-}lb^{-1}\text{-}°F^{-1}$
d = Pond depth, ft

e_{at} = Sky emittance, dimensionless
g = Fraction of horizontal radiation gained, dimensionless
$g(T)$ = Function relating fish weight gain and temperature
h_{hl} = Heat loss coefficient, $Btu\text{-}hr^{-1}\text{-}ft^{-2}\text{-}°F^{-1}$
m = Mass flow rate of water in pipe, lb/hr
m_t = Total mass flow rate of water required for the entire pond, lb/hr
p = Atmospheric pressure, in. of Hg
p_a = Vapor pressure of the air, psi
p_w = Vapor pressure at the water surface, psi
t = Time, hr
v = Wind speed, mph
x = Dimensionless constant
α = Constant, dimensionless
μ = Constant, dimensionless
σ = Stefan-Boltzman Constant = $1.714 \times 10^{-9} Btu\text{-}hr^{-1}\text{-}ft^{-2}\text{-}°F^{-4}$

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