Allocation of Resources for Neighborhood Improvement

RICHARD I. POLLACK

Dept. of Operations Research and Systems Analysis
Polytechnic Institute of Brooklyn

ABSTRACT

City planners have long made decisions on urban improvement projects primarily on an intuitive basis. A model is presented which serves to provide a structural framework within which these decisions can be made, and their effects on the quality of life in the urban area are examined.

Standards by which to measure the improvement in the quality of life in a neighborhood are developed. Indices are selected to represent such improvements arithmetically. Alternative urban improvement projects are examined for their costs and effects on the standards set for quality of life in the neighborhood. Mathematical programming techniques are used to allocate funds to these projects such that the improvement in the quality of life is maximized within a given budget.

An illustrative example of this model is formed for a fixed planning horizon. Extension of the model to apply to a variable planning horizon is discussed.

Introduction

The goal of the city planner is to improve the quality of life in the urban area. He can implement a large variety of programs designed to meet the various objectives of the city. The model discussed herein finds the optimal allocation of financial resources to urban improvement projects with respect to improvement in the quality of life.

The improvement in the quality of life in a neighborhood is defined mathematically by quantifying the objectives of the neighborhood and measuring the contribution of each program towards them. A measure of the relative importance of each objective to the residents is introduced into the model to adjust expenditures to their needs.

The techniques of mathematical programming are employed to find the optimal allocation of funds to the projects. Sensitivity analysis yields extensive information concerning the change in the objectives caused by manipulating the parameters of the model.

A dynamic model is postulated to extend the principles developed here to apply to long range situations. This model examines and adjusts expenditures based on their effect on the quality of life in the neighborhood at specified intervals of time.

The Model

A model may be constructed which relates the programs under consideration to the primary objective, the improvement in the quality of life (Q). This measure must be represented in some way by the effectiveness of the programs over a particular period of time, in terms of the elimination of the neighborhood's problems. Some measure of the effectiveness of a program with respect to alleviation of a problem must be determined and a method to combine all the individual improvements into one index to represent Q must be found.

Obviously, an index must be proposed to measure the severity of a problem at a given time. In some cases such indices are obvious and the levels are easily measurable, for example, overcrowding in a school can be measured by subtracting the design capacity from the actual student load. Variables such as the number of neighborhood residents with incomes at the poverty level and the overcrowding in apartments are also easily measurable, these data are available from the Department of Social Services and the Census Bureau, respectively, and acceptable levels for both can be set. In other cases the level variables, that is, the numerical indices of the severity of the problems, are measurable, but data are not conveniently available. In these cases acceptable levels may or may not be known, for example, the average noise level in decibels, the density of industrial traffic on residential streets, and the average travel time to the central business district. Finally, some factors which enter into Q may be extremely difficult to quantify, for example, the visual enhancement of a neighborhood, or the comfort of travel. In these cases some ingenuity must be used to bring these into the model. It is heartening to note that several notable operations researchers maintain that one can develop an index to measure anything that one can develop an intuitive feeling for.[1]

These indices must be combined in some way to be used in the calculation of Q. Whatever the fundamental relationship it is necessary that the units be consistent, we may not add the output of a job training program to the output of a slum clearance program. Several methods have been proposed to

solve this problem. The most rigorous method utilizes the concepts of utility theory where each level variable is mapped onto the utility scale via a transformation function. This method, while theoretically the most accurate, is perhaps the least likely to give meaningful results in practical applications due to the tremendous effort required to identify a single transformation function. The least rigorous method is to specify a simple yes or no depending on whether the program affects the objective or not. The method used in the following analysis is to form a decimal fraction representing the portion of the objective accomplished.

Each problem will have a certain level of urgency associated with it. This is equivalent to the relative importance of solving this problem completely as compared to the other problems. It is possible to quantify these levels by using a rigorous form of questionnaire such as the one due to Churchman and Ackoff.[1] In this method the decision maker, who may be a city planner or a representative sample of neighborhood residents, is asked to produce a simple ordinal ranking of all possible objectives. A value of 1, a relative weight, is assigned to the lowest ranked objective and the decision maker is asked to provide weights relative to this for all the other objectives. Then questions of the form: "Is complete solution of a particular problem preferable to solution of a particular combination of other problems?" are asked for all possible combinations of objectives. This yields a set of inequalities of preference which can be used to adjust the weights assigned previously.

Thus, Q is a function of the initial and optimal values of the level variables, the relative weights, and an intermediate solution level for each objective. Remaining to be identified is only the functional relationship. Intuitively, addition is very attractive. This would yield Q of the form:

$$\sum_{i=1}^{n} a_i X_i$$

where a_i is the relative importance of objective i and X_i is the percentage of that objective which is accomplished. This functional relationship is satisfying because each independent objective which is accomplished completely contributes its own weight to Q. It handles marginal solutions as a linear ratio, hence the elimination of half of a problem contributes half the relative weight to Q. It allows a positive Q if at least one factor is nonzero. Furthermore, a linear objective function allows the use of the theory of Linear Programming and its preprogrammed algorithms. The result is:

$$\max Q = \sum_{i=1}^{n} a_i X_i$$

Linear Programming theory allows the use of linear constraints, such as the

budget and minimal solution level constraints which are discussed further below.

Methodology

For the time independent, fixed planning period model one must:

1. Identify objectives and programs.
2. Select indices for the level variables.
3. Identify starting and solution levels.
4. Use the Churchman—Ackoff procedure to find relative weights.
5. Identify cost per unit output of various programs.

Each program proposal is examined for the following information:

1. What is the change in each objective for each dollar invested in this program?
2. Does this program interact with any other program? Linear Programming assumes independence here, so any interactions will require modifications in the solution procedure to account for nonlinearities. Note that this refers to effects one program has on another, not the effects of one program on the objectives of another. It is anticipated that this will not normally present any difficulty.
3. What is the time span of the program?
4. What are the fixed and variable costs? If we are planning only for a fixed period of time we can combine them, however, these results are not realistic for the following reason. The cost per unit is not constant, but decreasing due to the effect of a larger output sharing the fixed cost. This effect may serve to make continuation of a program past the set period less costly than continuation or initiation of a different program.

The objective function can be expressed as follows:

$$\max Q = \sum_{i=1}^{n} a_i \frac{\Delta x_i}{x_i^s - x_i^*}$$

where

Δx_i = a positive change in level of variable i

x_i^* = solution level of variable i

x_i^s = starting level of variable i

Care must be taken to assure that a step towards solution is in the positive direction. If a program detracts from an objective it may cause Δx_i to be negative. The levels are selected such that $x^s - x_i^*$ is positive. If this is false, the two can be interchanged in order to reverse the sign.

If we let k_{ij} = the unit output of program j to objective i per dollar
and
N_j = the total invested in program j
and
b_i = the starting level minus the solution level $(x_i^s - x_i^*)$
then

$$\sum_j N_j k_{ij} = \Delta x_i$$

thus

$$\max Q = \sum_i \sum_j \frac{N_j k_{ij} a_i}{b_i}$$

is an equivalent objective function subject to the budget constraint

$$\sum_j N_j \leqslant N$$

where N is the total budget.

If minimal and/or maximal standards are required they are included as
follows:

$$\sum_j N_j k_{ij} \geqslant C_{i \ min}$$

$$\sum_j N_j k_{ij} \leqslant C_{i \ max} = b_i$$

of course, non-negativity constraints hold.

If a_i is constant, and the programs are independent with negligible fixed
cost, the above is the formulation of the model. If these assumptions do not
hold, nonlinearities are introduced into the model and different methods
must be called on to yield an optimal solution. If the constraints form a
convex region standard techniques are available, otherwise an algorithm must
be constructed to search the function at the expense of additional computer
time.

Results

This linear program allocates available funds to the program alternatives in
order to maximize the improvement in the quality of life. It is likely that a
class of optimal solutions will arise. Due to limitations in the data it is not
possible to clearly distinguish an optimal allocation, however we can carefully
study the effects of changing the parameters of the model through
post-optimality analysis.

Post-optimality analysis yields the effects of a change in a parameter on
the objective function. In this way it is possible to study the sensitivity of the

solution to the input data. One might choose to select a probable range for each input parameter and rerun the model with the parameters at their opposite extremes in order to identify sensitive solutions. Further, binding constraints will be identified and the effects of relaxing such requirements, or tightening nonbinding constraints can be examined.

The model gives the planner an *indication* of a solution rather than an exact one, and the ability to simulate the effects of various perterbations of the system. The main function of the model is to structure decisions which have previously been made in an unstructured environment, primarily on an intuitive level. While the model's usefulness may be limited by the accuracy of the input data, it should be recognized that the results of the linear programming allocation and the sensitivity analysis are at least as good as those a planner could provide heuristically, and probably much better.

Stochastic Properties

As previously noted this model is time independent, assuming all programs act for a fixed time period and no further. In reality this will not be the case. Some programs will take longer than others, some will be "one shot" improvements and others will continue indefinitely. All will have different transient and steady-state outputs, many will have significant fixed costs in addition to their variable costs. The model cannot be realistic without taking these into consideration.

Suppose time delays are included in the achievement of objectives and the model is run many times using the output of one stage as the input to the next. Q is set equal to zero at the beginning, and all subsequent values of Q are added. Relative weights are recalculated at each stage, if necessary. In this way we can reexamine our investments at each stage with respect to progress already made, and eliminate the requirement of a fixed planning horizon.

This proposal results in a great deal more data to be analyzed. It allows us to look at the neighborhood in both the long and short run. We find that different initial allocations will be optimal at different stages. This type of model accurately reflects the fixed and variable portions of the costs because the distinction can be made between them which is not possible in the time-independent formulation. Associated with this dynamic model must be either projected data for each variable for the length of time the model is to be run or a submodel which predicts such data, to include independent changes in the level variables. Without these considerations we would fall victim to the pitfall of near-sighted planning, short range effects which usually reverse in the long run. These extensions enable the model to represent the dynamic characteristics of Q as a function of planning decisions.

In its most mundane application the model will allow the optimization of

the quality of life at different stages, or at combinations of stages. If the length of the time spans involved are selected such that there is confidence in the projected data, we may then optimize a function of the values of Q at different stages, for example, the sum or even the product of the individual qualities at times $t = 1, 2, 3, \ldots n$, or perhaps some weighted average of the Q's.

If the complex dynamic characteristics and the forecasting submodel are included the model may be run for long periods of time by merely specifying the initial conditions. Intermediate variables, exogeneous in the former case, are endogeneous to the dynamic model. Jay W. Forrester has pioneered the field of urban dynamic models, but as yet little or no work has been done in the dynamic allocation of resources with respect to the optimization of specific objectives in a neighborhood. Also, while Forrester was interested in the structure of the urban system as a whole, here we are interested in only a small subset, the neighborhood.

The use of post-optimality analysis is broadened here because the effects of a change at any stage will be calculated by modifying parameters and using post-optimality procedures at the given stage, and rerunning the model from this point.

The computer programming effort required for this model is bound to be sizable. We expect this to be so due to the complexity of the dynamic characteristics of the optimization to take place.

An Example

For the purpose of illustrating the concepts presented here, a limited model will be constructed using data taken from the Red Hook area of Brooklyn, N. Y. Estimates of data such as might be expected in Red Hook are made where such data are unavailable.

From the many factors comprising the quality of life several are selected to be used in the objective function of the model. It is beyond the scope of this effort to treat all of the possibilities due to the dimensions of the subsequent analysis. These factors are examined for the appropriate indices and their starting and solution levels in Red Hook. The Churchman-Ackoff procedure is used to find the relative weights to be used in the model. It is hoped that these few objectives will provide a representation of many of the problems to be encountered in the complete analysis.

The factors in Q are:

1. level of industrial traffic through residential streets
2. average noise level in residential areas
3. number of inadequately housed families
4. overcrowding in the schools

The indices are:

1. vehicles per hour
2. decibles
3. number of families
4. number of students,

respectively.

In cases where quantification is difficult, one must attempt to analyze one's intuition for the criteria with which he regards the variable in his own mind. While this may not seem rigorous it is at least as good in every case as a planner would do. Furthermore, the interpretation of the results takes difficulties in estimation into account.

As a further example of this technique, suppose we wish to measure the comfort of travel to the central business district during rush hours. How can an analyst quantify something as intangible as comfort? He merely analyzes his own feelings about when he is comfortable. It is a function of several variables including the probability of standing in public transit, the density of people per car, the probability of traveling a congested route, vehicle temperature on public transit, and the time spent in these "uncomfortable" conditions. Relative weights for these could be determined via the Church-man-Ackoff procedure and a functional relationship decided upon, perhaps based on several interviews in which the passenger describes his comfort under various conditions. Alternatively, in this case a "psychophysiological" approach could be taken where travel conditions are simulated while heartbeat, blood pressure, and other physiological indicators are monitored on several subjects. These data can be transformed into a rating of comfort. The result of either method is a quantification of commuter comfort.

The optimal levels of the objectives will not often be difficult to estimate once the indices have been determined. For the objectives listed above the optimal levels are:

1.   0 vehicles
2. 10 decibels
3.   0 families
4.   0 students

These levels have been assigned arbitrarily for the purpose of illustration. The corresponding starting levels are:

1. 100 vehicles per hour
2. 50 decibels
3. 14000 families
4. 170 students

One can now determine relative weights by applying the procedure due to Churchman and Ackoff.

1. Rank outcomes in order of preference, labeling O_1 as the most preferred ... to O_n as least preferred. Assign a relative weight of 1 to O_n.
2. Have the decision maker assign values relative to O_n.
3. Have the decision-maker look at the following table. Go down each column until the choice on the left-hand side is preferred or equal to the right; then go to the next column.
4. Adjust the numbers from 2. to suit the inequalities from 3.

The order of preference is:

O_1 inadequately housed families
O_2 overcrowding in the schools
O_3 average noise level
O_4 level of industrial traffic

If O_4 is given a value of 1.0, then:

$O_4 = 1.0$
$O_3 = 1.5$
$O_2 = 4$
$O_1 = 6$

The table is as follows:

O_1 or $O_2 + O_3 + O_4$ $\qquad$ O_2 or $O_3 + O_4$
O_1 or $O_2 + O_3$

The inequalities resulting from the table are:

$O_1 > O_2 + O_3 + O_4$
$O_2 > O_3 + O_4$

The original values are now adjusted

$6 > 4 + 1.5 + 1$ $\quad$ change O_1 to 7
$4 > 1.5 + 1.0$ $\quad$ consistent

Therefore the relative weights are:

objective	a_i
level of industrial traffic	1.0
noise	1.5
overcrowding in schools	4.0
inadequately housed families	7.0

Several hypothetical programs are considered. In each case the contribution of the program to each objective per unit cost must be evaluated. Since contracts and proposals usually include this information this requirement places little additional burden on the user of the model. These programs are chosen to illustrate the capability of the model to deal with widely disparate and closely related factors.

The programs to be evaluated are:

1. Widening and repaving of streets around the perimeter of Red Hook and rerouting of all industrial traffic. The proposed route avoids residential areas and will, therefore, have the effect of eliminating all industrial traffic from residential streets. It is thought that the industry will welcome the new route because it will speed their traffic. The road requires no new right of way. Community groups favor it due to the decrease in unsightly and noisy traffic from their streets.

2. Community job action training to train heads of inadequately housed families to improve their income level. In families without a male head, women will be trained and day care centers provided. As these families move out this housing must be eliminated or more people will move in, keeping the number of inadequately housed families constant. This program also affects average income in the community, but this is not one of the objectives in the example.

3. Elimination of overcrowding in the schools may be accomplished two ways, either by construction of a new school or by the addition of mobile teaching units to the existing school. If both choices are specified the model will select the least expensive. In the case of equal cost alternatives the Churchman-Ackoff procedure could be used to rate them on desirability which could be used as a multiplier similar to the relative weight. The stochastic model discussed earlier would also take into consideration future demand which, if increasing, would probably favor a new, larger building; if decreasing, would probably favor the mobile units; if constant, the desirability would be the most significant factor and the most desirable building would be selected. For the purposes of this illustration only the mobile units will be evaluated.

4. Renewal of 100 brownstones and other buildings with the intent of creating a historic area similar to Brooklyn Heights. The Heights and the surrounding communities (Cobble Hill, Boerum Hill, and Carroll Gardens) have undergone a rennaisance in recent years due to their proximity to the central business district and the flavor of their architecture. Red Hook shares this architectural color and an attempt can be made to attract new, and wealthier, people to this area for the same reasons. These

improvements would provide both housing and an improvement in the image of Red Hook due to the sense of community created by the restoration.

The numerical aspects of these programs pertaining to the model are as follows (these are estimates for illustrative purposes):

1. Twenty-five blocks along the perimeter of the area would be involved at an average cost of $20,000 per block, for a total of $500,000. This would eliminate all 100 vehicles per hour and the segment of noise pollution attributed to industrial vehicles, estimated at 25 decibels. This breaks down to $5000 per vehicle per hour and $20,000 per decibel.
2. Community job action produces one trained person who upgrades his housing for each $3000.
3. A mobile unit with a capacity of 25 students costs $20,000.
4. Buildings are renovated one at a time at a cost of $200,000 each. Each building contains ten units.

Therefore, the k_{ij} are as follows:

<div align="center">objectives</div>

j \ i	1	2	3	4
1	$\dfrac{4 \text{ vph}}{\$20,000}$	$\dfrac{1 \text{ decibel}}{\$20,000}$	0	0
2	0	0	$\dfrac{1 \text{ family}}{\$3,000}$	0
3	0	0	0	$\dfrac{25 \text{ students}}{\$20,000}$
4	0	0	$\dfrac{10 \text{ units}}{\$200,000}$	0

programs

Expenditures on the various programs must be made in the discrete intervals represented above. For example, the above table indicates that each building contributes ten units towards the third objective at a total cost of $200,000. Due to the structural integrity of the building we may not choose to spend $100,000 for five units. The mathematical programming algorithm must account for this practical consideration, or the model adapted for an integer programming solution. The arrays of the parameters may be written:

$$a_i = 1, 1.5, 7, 4$$
$$b_i = 100, 40, 14000, 170$$
$$c_{i \, min} = 20, 10, 1000, 75$$
$$N = \$5,000,000$$

and the linear program is

$$\max Q = \sum_i \sum_j \frac{N_j k_{ij} a_i}{b_i}$$

such that

$$\sum_j N_j k_{ij} \leqslant b_i \qquad \text{for all } i$$

$$\sum_j N_j k_{ij} \geqslant c_{i \text{ min}}$$

$$\sum_j N_j \leqslant N$$

$$N_j > 0 \qquad \text{for all } j$$

For this formulation $500,000 will be invested in program 1, $4,320,000 in program 2, and $140,000 in program 3. Due to its high cost and low contribution to Q, program 4 is bypassed entirely. This solution corresponds to the elimination of all 100 vehicles per hour of industrial traffic on residential streets, a reduction in the noise level of 25 decibels, and new classroom space for 175 students. Additional funds will be allocated to program 2 until the upper constraint, b_i, is met. At this point the model will begin allocating funds to program 4.

Conclusion

This model is potentially a valuable aid to planners in achieving the ultimate goal of improving the quality of life. The model provides a framework in which the planner can clearly specify his objectives based on the desires of the people most affected by his plans. It allows the planner to simulate the effects of alternative investment plans, and indicates the direction in which an optimum may be found.

The model uses the bare minimum of data necessary to perform the allocation. The information required for the quantification of the quality of life in the neighborhood is the only additional data needed beyond that which the planner must use for an intuitive decision. It should be clear that this requirement is a small burden when compared with the potential benefit.

Application of the model necessitates careful evaluation of the input parameters. Accuracy of the results is a direct function of the care taken in specifying the input. Also, the model makes the basic assumption of independence between programs; if this is not so the formulation must be modified by the addition of a constraint equation representing the

interaction. To obtain realistic results applicable to an urban area we must examine the effects of the allocations at discrete points in time over a long planning horizon. The basic principle for this extension is the iterative application of the fixed planning period model using the output of each phase as the input to the next, and using exogeneous forecasted data to account for changes in the level variables which are independent of the allocations made by the model. There are added difficulties in estimating the input parameters for this version of the model due to the inherent uncertainty in a forecast. None of these problems appear to be insurmountable, however.

It is of the greatest importance that the model be evaluated in comparison with the planning function as it currently exists. Despite the limitations of the model its effect is to improve the planning function by providing structure to a formerly unstructured process.

Acknowledgement

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