

A SIMPLE MODEL OF SELF-REGULATION IN LARGE NATURAL HIERARCHICAL SYSTEMS*

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ABSTRACT

The characteristics of large hierarchical systems existing in nature are discussed from the standpoint of self-regulation problems. By way of example, some biophysical, ecological and economic systems are considered. A cooperative mechanism of self-regulation which enables the system to function ideally is proposed. A mathematical model for large hierarchical system involving a basic medium and hierarchical transport network responding to variations in the basic medium parameters are formulated. The governing equations that describe evolution of this system in terms of two continuous fields are presented. The meaning of these fields are discussed for different natural systems.

INTRODUCTION

Large hierarchical systems occurring in nature are characterized by such a great information flow that none of the system elements can possess the whole amount of information required to govern the system. The aim of the present article is to focus attention on the fact that in such large hierarchal systems there can be a cooperative mechanism of regulation which involves individual responses of each element to the corresponding hierarchical piece of information and which leads to ideal system functioning, due to self-processing of information.

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As a typical example of such systems we may regard living tissue where blood flowing through the vascular network of arterial and venous beds supplies cellular tissue with oxygen, nutritious products, etc. At the same time blood withdraws carbon dioxide and products resulting from life activities of the cellular tissue. Both the arterial and venous beds are of tree form, containing a large number of hierarchy levels and having similar structures. The system response to disturbances in life activity occurs, for instance, through vessel response to variations of the carbon dioxide concentration in blood, which gives rise to expansion or contraction of arteries (for a review, see, e.g., [1, 2]). A similar situation takes place also in respiratory systems where oxygen going through the hierarchical system of bronchial tubes reaches small vessels (capillaries).

The organizational structure of large firms is a clear example of an economic hierarchical system. Managers of all functions and all levels make up a management network [3]. Roughly speaking, the management network controls the money flow toward the organization's "bottom," comprising workers as well as the flow of products in the opposite direction. The term "control" here means governing the money and product redistribution inside the firm by dint of information flow through the management network. In performing technological processes, wages paid to workers are in a sense transformed into the firm's products.

The existence of a tremendous variety of goods in the market, in contrast to a relatively small number of raw materials, suggests that there must be large hierarchical systems in the market. In this context we note that the flow of goods, after reaching the consumers, gives rise to money flow in the opposite direction.

Ecological systems are also complex in structure, can involve a larger number of "predator-prey" levels, and are grounded on some medium (for example, plankton) [4]. Dynamics of ecosystems are governed by biomass and energy flows in trophic networks.

In the present article we consider a model of large hierarchical systems, one that generalizes the main characteristics of natural hierarchical systems mentioned above. This system is grounded on a distributed basic living medium sustained by a nutrition supply system and a draining system that withdraws products resulting from life activities. For these purposes, there is a complex transport network whose architectonics is organized in such a manner that flow of transport agents through the basic medium are the same at each point, all other factors being equal. To meet this condition the transport network may involve supplying and draining beds of tree form. In principle, the two beds can coincide with each other in real space. Interaction between the transport agent and the basic medium causes interchange of the supplied and withdrawn living products. The transport supply system agent flow through the basic medium should keep the concentrations of these products inside a certain region called the vital region, which is the aim of regulation.

Since the motion of the transport agent dissipates energy, it is necessary that a certain external force be applied to the system, affecting the overall flow of the

transport agent. In living tissue, blood pressure plays the role of this external force [1, 2]. In ecological systems, the transport agent flow is caused by gradient of general potential along the transport network. It should be noted that exergy can play the role of this potential [5]. In economic systems the total utility function [6] can be regarded as filling the role of the external force causing products and money flow.

In contrast to artificial systems, natural ones are able to adapt to variations in the environment. Under ordinary conditions the behavior of a natural system as a whole and the individual behavior of its different elements are likely to follow the strategy of minimum expenditure rate. This agrees with the minimum entropy production principle stated in nonequilibrium thermodynamics [7]. However, for such complex and nonequilibrium systems as biological and ecological ones, specification of particular expressions for the entropy production rate is far from being solved and the minimum entropy production principle can be applied to these systems at the phenomenological level only.

Keeping in mind the aforesaid, we shall state below a simple mathematical model that captures the main properties of these hierarchical active systems and exhibits ideal self-regulation.

FORMAL MODEL

Let us consider a system consisting of the distributed basic medium \mathcal{M} and the transport hierarchical network \mathcal{N} (Figure 1). The transport network involves dendriform supplying and draining beds (the left- and right-side networks in Figure 1). The medium \mathcal{M} is a d -dimensional homogeneous continuum. The state of the medium \mathcal{M} is described by the field θ evolving according to the equation

$$\frac{\partial \theta}{\partial t} = q - \eta \theta + D \nabla^2 \theta \quad (1)$$

Here q is the generation rate of the state variable θ due to life activity, D is the diffusivity, ∇^2 is the d -dimensional Laplace operator, and η is the volumetric rate of the transport agent flow. We note that the generation rate q , in principle, can depend on the spatial coordinates \vec{r} and the time t as well as the variables θ and η . The term $-\eta \theta$ in Eq. (1) describes the dissipation rate of the variable θ due to exchange of life activity products between the medium \mathcal{M} and transport agent.

In order to complete Eq. (1) we need to describe evolution of the field η which is controlled by the transport agent flow in the network \mathcal{N} and, thus, to describe architectonics and properties of the hierarchical transport network. Geometry of both the beds is assumed to be the same, so we specify it for the supplying bed only. The stem of this bed splits into $g = 2^d$ branches of the first level Figure 1. Each of the first level branches, in turn, splits into g branches of the second level and so on. The branches of the last level (N) are directly connected with basic

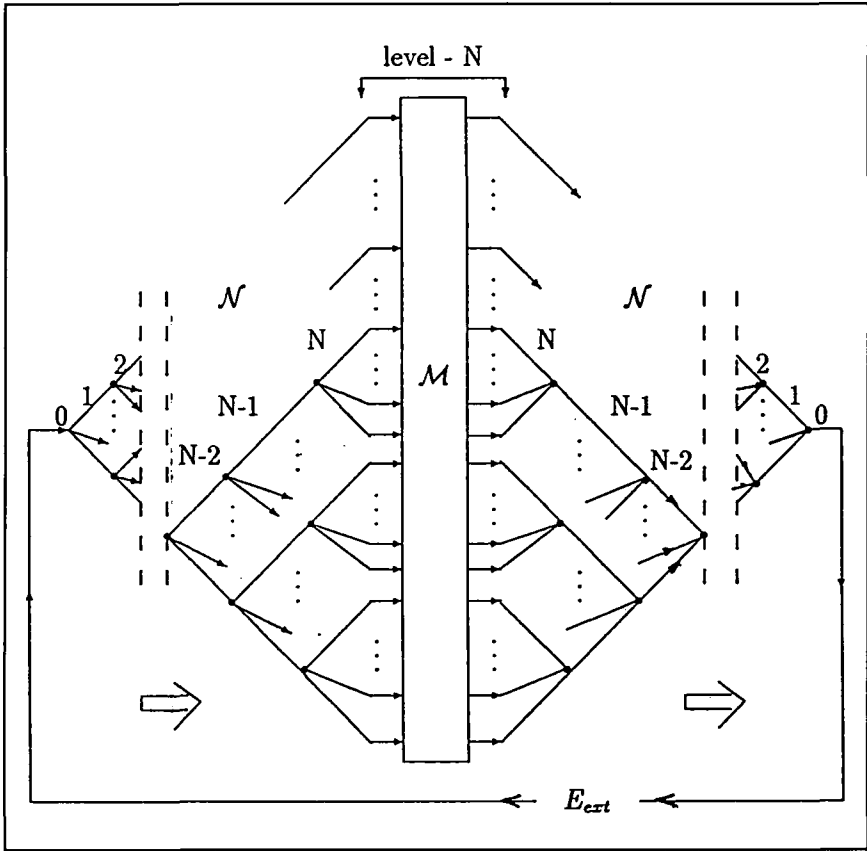


Figure 1. The structure of hierarchical system.

medium \mathcal{M} . The bed is organized in such way that each branch i of a given level n supplies a certain domain \mathcal{M}_n^i as a whole (called a fundamental domain of level n). All the fundamental domains of a given level n form the basic medium \mathcal{M} . The fundamental domains of the last level will be also referred to as elementary domains. Each elementary domain is bound up with one of the last level branches. The last level number N is assumed to be much larger than unity: $N \gg 1$, and its length l_N may be regarded as infinitely small. Whence it follows, in particular, that inside an elementary domain the variable θ may be treated as a constant.

Going through the supplying bed, the transport agent flow continuously splits into smaller streams, and, passing through the last level branches, delivers products needed for life activity to the basic medium. At the same time, the

transport agent is saturated with the products of life activity. Thereby the variable θ is also assigned to the transport agent flowing through the draining bed.

The pattern of transport agent flow on the network \mathcal{A} obeys, first, the conservation law at branching points. In particular, for a given branching point B for example, of the draining bed we can write

$$\sum_B J_{in} = J_{out} \tag{2}$$

where J_{in} and J_{out} are the transport agent flow on branches going in and out of the point B , and the sum runs over all the branches leading to this point. Second, the total dissipation rate due to transport agent flow through the network \mathcal{A}

$$D\{J_i\} = \sum_i \frac{1}{2} \Lambda_i J_i^2 - J_0 E_{ext} \tag{3}$$

attains its minimum subject to the transport agent flow conservation. Here Λ_i is a kinetic coefficient corresponding to branch i , E_{ext} is the external force that gives rise to the total flow of transport agent and, thus, flow of the products needed for the basic medium life, J_0 is the flow through the stem, and the sum runs over all the branches of the network \mathcal{A} . The supplying and draining beds are assumed to be identical. Therefore the flow pattern on the supplying and draining beds as well as the coefficient $\{\Lambda_i\}$ must be mirror images of each other, with reversed flow direction.

The agent flow through the draining bed gives rise to flow of life activities products. In particular, agent flow J_i on branch i causes the flow of life activity products $J_i \theta_i$, where θ_i is the state variable assigned to branch i . Distribution of the state variable θ over the draining bed is determined by agent flow pattern $\{J_i\}$ on it and the conservation of life activity product flow. In particular for a given branching point B we can write

$$\sum_B [J_i \theta_i]_{in} = [J_i \theta_i]_{out} \tag{4}$$

The variable θ distribution over the draining bed and the basic medium are related by the condition that the total dissipation rate of the variable θ in a elementary domain \mathcal{M}_N^i of the basic medium must be equal to the life activity product flow through the corresponding branch of the last level, i.e.,

$$J_i \theta_i = \int_{\mathcal{M}_N^i} d\vec{r} \nabla \theta \eta \tag{5}$$

Furthermore, by definition, the agent flow pattern and the field η are connected by the expression

$$J_i = \int_{\mathcal{M}_N^i} d\vec{r} \eta \quad (6)$$

In order to complete the present model we should specify the response of the transport network to variations in the field θ . Due to the supplying and draining beds being mirror images of each other, we may describe this response for the draining bed only. We assume that for each branch, for example, the branch i , time variations in transport coefficient Λ_i are directly controlled by the variable θ_i assigned to this branch, and obey the equation

$$\tau \frac{d\Lambda_i}{dt} + \Lambda_i = \Lambda_n^0 (1 - \theta_i) \quad (7)$$

for $\Lambda_i > 0$. The quantity Λ_i may vary in the region $\Lambda_i \geq 0$. Here τ is the delay time of the network response and Λ_n^0 is the formal transport coefficient for fixed $\theta_i = 0$ whose value is assumed to be equal for all branches. The value Λ_n^0 depends on n as

$$\Lambda_n^0 = \Lambda^{02\text{nd}} \rho(n) \quad (8)$$

where $\rho(n)$ is a smooth function such that $\sum_{n=0}^{\infty} \rho(n)$ is convergent and Λ_0 is a constant. The smoothness of the function $\rho(n)$ follows from the requirement that branches of all the levels take part in control of the transport agent flow redistribution.

RESULTS AND DISCUSSION

Under the adopted assumptions, Eq. (2-7) can be reduced to the equation governing evolution of the field η in the basic medium, which is of the form

$$\tau \frac{\partial \eta}{\partial t} = \eta_0 - \eta(1 - \theta) \quad (9)$$

where η_0 is the constant volumetric flow rate of transport agent for fixed $\theta = 0$. As follows from Eqs. (1) and (9), the variable θ cannot leave the interval (0,1) for a long time for any generation rate q , and there is a local relationship between the volumetric flow rate η and the state variable θ . The two properties allow us to regard the problem stated above as a model for an active hierarchical system with ideal self-regulation. The term ideal self-regulation here means that, first, the system cannot leave its vital region for a long time; second, variations in the state variable θ at a certain point of the basic medium give rise to variations in the agent flow rate η at the same point only. In other words, disturbances in the field θ inside a certain domain do not cause system responses at points lying beyond this domain.

We now discuss the main features of this model that are responsible for the ideal self-regulation. Minimizing function (3) subject to Eq. (2) for all the branching points, we find that the distribution of the transport agent flow over the network \mathcal{N} is determined by a certain potential E and, for a branch i , the transport agent flow $J_i = \Delta E_i / \Lambda_i$, where ΔE_i is the potential difference across the branch i . So the less Λ_i , the greater the flow J_i on the branch i . Let us assume that, for example, in the domain Q (Figure 2) the variable θ exceeds its normal value to the balance in life activities being disturbed. In order to smother the increase in the variable θ , the system should increase the transport agent flow rate η in the domain Q . The system responds by decreasing the kinetic coefficients $\{\Lambda_i\}_\rho$ along the whole path \wp on the network \mathcal{N} that leads from the stem of the supplying bed to the domain Q and, then, from this domain to the draining bed stem (Figure 2). Information required of this system behavior is delivered by the distribution of the variables $\{\theta_i\}_\rho$ over the path \wp on the draining bed. In fact, increases of the variable θ in the domain Q must give rise to the corresponding increases of all the variables $\{\theta_i\}_\rho$, as follows from equations (4) and (5). By virtue of Eq. (7), increases in θ_i lead to decreases in coefficient Λ_i . This relation between the field θ and the kinetic coefficients $\{\Lambda_i\}$ of the network \mathcal{N} is the essence of the self-regulation process in the active hierarchical system under consideration.

Variations in the field θ located in domain Q can, in principle, cause alteration of the transport agent flow rate at the exterior points due to the flow redistribution over the network \mathcal{N} . However, in contrast to the points of the domain Q , at the exterior points, for example, at points of a domain Q' (Figure 2), decreases of different kinetic coefficients belonging to the collection $\{\Lambda_i\}_\wp$ give rise to variations

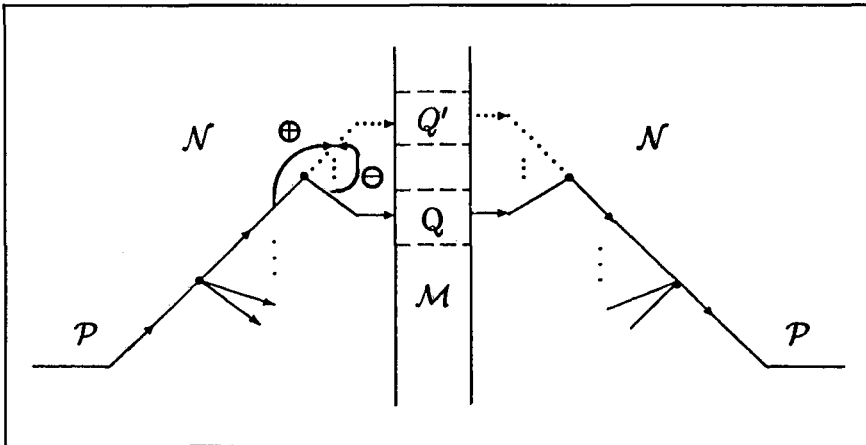


Figure 2. Schematic representation of the cooperative mechanism of ideal self-regulation.

in the field η different in sign. Figure 2 schematically shows the sign of this effect for different branches of the path \wp . In the given model, due to the specific forms of Eq. (8) and the right-hand side of Eq. (7), the net effect is reduced to zero. For other forms of these functions the second term in Eq. (9) will be of nonvocal form; i.e., time variations in the field η at a given point will also be determined by values of the variable θ at other points. In living natural tissue, for example, in order to reduce this effect of the nonideality, arterial and venous beds contain a system of anastomoses, i.e., vessels joining arteries or veins of the same level [2]. In such ways, the architectonics of large natural systems are organized in such a manner as to make their functioning as ideal as possible.

It should be pointed out that the distribution of the variable θ over the draining actually provides the system with the information on the distribution of θ over the basic medium in a form that enables the system to respond immediately to counteract disturbances in life activities. Indeed, according to Eq. (7), a given branch i of the draining bed responds solely to variations in the variable θ_i being in effect the mean value of the field θ in the domain where the total transport agent flow is directly controlled by this branch (Figure 2).

For different natural systems the variables θ and η are distinctive in meaning. For example, for living tissue the variable θ can be treated as concentration of carbon dioxide, or the tissue temperature; the variable η corresponds to the blood flow rate, the blood pressure plays the role of the external potential E_{ext} , and $d = 3$. A similar situation takes place in respiratory systems, where the variable θ is the oxygen concentration. In economic systems the quantity η is the flow rate of goods and the variable θ is the price. In this case the external potential E_{ext} is likely can be treated as the total utility function of the production process in a certain industry. In models for organization and functioning of firms the basic medium is the firm "bottom," comprising workers, and the variables η and θ are quantities proportional to the wages of workers and the amount of products, respectively. For economical systems the basic medium dimension d can be large, and differ among industries. Concerning ecological systems the variables η and θ seem may be regarded as the rate of biomass flow and the energy content of biomass.

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