J. ENVIRONMENTAL SYSTEMS, Vol. 24(2) 169-190, 1995-96

BEVERAGE CONTAINER DESIGN AND SEPARATION

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ABSTRACT

Beverage containers make up a significant portion (at about 25%) of the packing materials that constitute 33 percent of the municipal solid waste stream. Despite the containers' similar shapes and sizes, only about 30 percent of glass, expanded polystyrene (EPS) and tetrapak containers are recovered. With this research, the mechanical separation capability of container materials from municipal waste is investigated. Breakage theory is reviewed and applied to predict the weight fraction of separated particles by screen size. The results show that the empirical parameters of maximum particle size y, breakage ratio r, and slope factor q can be established to predict the particle size distribution within plus or minus 10 percent. Factorial screening tests reveal that container size, material, size-material, and size-shape interactions influence the achievable separation. The next research phase will modify the breakage theory to incorporate these additional factors and should investigate the influence of size, shape, and materials on separation behavior, particularly for flexible containers.

INTRODUCTION

Beverage containers and packaging materials make up approximately 33 percent of the municipal waste stream. Despite similar shapes and sizes, only about 30 percent of glass, tetrapak, and styrofoam containers are recovered and recycled. The remaining two-thirds are disposed of in landfills or incinerators unless the containers can be mechanically separated from the waste stream in

doi: 10.2190/0RVQ-JXKV-GFJ0-DQ3J
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material recovery facilities (MRFs). However, characteristics that determine the behavior of common glass, expanded polystyrene (EPS), and tetrapak containers in mechanical separation processes are poorly understood. In order to predict and optimize container recovery rates, these container characteristics must be investigated. The results can then also be used to optimize the design of beverage containers so they can be more effectively separated. In order to predict the behavior of beverage containers in mechanical separation, the theory of breakage is reviewed and applied.

REVIEW OF BREAKAGE THEORY

The breakage theory and the results from its use for municipal solid waste are reviewed here to generate hypotheses the experimental design for the separation tests.

Breakage theory, as used for municipal solid waste (MSW) size reduction [1, 2] consists of three elements:

1. the initial distribution of particle sizes in the feed material, denoted as a vector F with elements f_i to denote the weight fraction of particles in each size range where $\Sigma f_i = 1$;
2. the selection function, S, a vector of elements s_i, denotes the fractions of each particle size range i that are selected for breakage, and, hence, undergo size reduction; the complimentary function $1\text{-}s_i$ is the fraction of each size range that is not broken; and
3. the breakage function, B_{xy} that denotes the fraction of broken particles in the product size ranges x, smaller than the input particle size range y.

The initial, or feed, particle size distribution F for raw municipal refuse (or individual components of the waste stream) is represented by the sigmoid curves of cumulative weight fraction under particle size x as a function of the log x [3]. The feed waste used by Trezek [1] and by [2] fit this type of distribution. In contrast, the beverage containers tested in this research consist of uniform objects in two size ranges: 102.5 mm for 1,000 mL containers, and 65 mm for 250 mL containers. As a result, the container recovery tests use a special case of the general sigmoid F distribution, which appears as a single step function from 0 to 100 percent on the cumulative particle size distribution at the characteristic size of the containers. This characteristic input size is the larger of the smaller two dimensions of the container.

The selection function S in its simplest form is a constant Π for all size ranges. Trezek found the constant Π selection function to provide good results for the breakage of municipal refuse in single, double and triple shredding steps [1]. Vesilind et al., however, determined that the Π selection function gave large errors when predicting the breakage of single materials e.g., polystyrene, wood, paper, cardboard, except for glass [2]. They determined variable selection functions S that provided good results for different materials (see Table 1). Trezek tested more complex selection functions for repeated breakage (as in hammermills or multiple shredding) and found them to be inferior to the simple Π selection function for mixed MSW [1].

Notably, all particles of the two brittle materials (glass and polystyrene) are completely selected in the largest size range, while flexible cardboard and mixed waste are not totally selected in their largest ranges. Glass particles are totally selected above 7.1 mm size, while only 84 percent of cardboard particles above 124.2 mm are selected. This table shows that the selection function is specific to the material, and, depends on the input particle size and the brittleness. The glass and EPS in our test containers are larger than the largest mean sizes in Table 1 as 14.3 mm for glass and 50.8 mm for EPS. As a result, the selection functions are likely to be 1.0 for glass and polystyrene containers in the test. Tetrapak containers sizes (102.5 mm and 65 mm) are between the largest and second mean sizes for cardboard in Table 1 and, hence, are assigned selection functions prorated by mean size of between 0.35 and 0.84.

Breakage functions are used to describe the distribution of particles over the range of (screen) sizes after breakage. Several breakage functions have been suggested from the breakage of brittle materials (e.g., Rosin-Rammler [4]). The following breakage functions were actually tested for solid waste [1].

Gaudin-Meloy and modified Gaudin-Meloy equations predict the fraction of the output waste smaller than size x as a function of the input particle size y for single fracture breakage:

$$\text{Gaudin-Meloy} \qquad B_{xy} = 1 - \left(1 - \frac{x}{y}\right)^r \tag{1}$$

where r is an empirical parameter that reflects the number of product particles resulting from the breakage of an input particle, or, the number of breaks in a selected input particle. A modified version of Gaudin-Meloy [5] adds a second parameter, q, to increase the coarse size curve-fitting capacity:

$$\text{Modified Gaudin-Meloy} \qquad Y = \left[1 - \left(1 - \frac{x}{y}\right)^r\right]^q \tag{2}$$

where q is a slope factor for the particle size distribution (PSD).

A different function by Broadbent-Callcott was also tested, consisting in its modified form of a quotient as a function x/y and of the parameter n as an exponent of the exponential of the negative particle size ratio of product particle size x to the characteristic particle size y_o of the feed.

Table 1. Selection Functions for Waste Materials

Expandable Polystyrene (EPS)		Cardboard		Glass		Municipal Refuse	
Geom Mean Particle Size (mm)	Selection Function S_i	Geom Mean Particle Size (mm)	Selection Function S_i	Geom Mean Particle Size (mm)	Selection Function S_i	Geom Mean Particle Size (mm)	Selection Function Si
50.8	1.0	124.2	0.84	14.3	1.0	20	0.81
28.6	0.71	50.8	0.35	7.1	1.0	10	0.37
14.3	0	28.6	0.0	3.6	0.76	5	0.0
7.1	0.14	14.3	0.0	1.8	0.34	2.5	0.0
				0.6	0.19		
				0.3	0.0		

Source: Vesilind et al., 1986.

$$\text{Modified Broadbent-Callcot} \qquad B_{xy} = \frac{\left[1 - \exp\left(-\frac{x}{y}\right)^{n}\right]}{[1 - \exp(-1)]} \tag{3}$$

With n = 1, this equation is equal to the original Broadbent-Callcott equation [1].

Trezek applied the Π breakage theory with varying values for the Π selection function the Gaudin-Meloy and original and modified versions of Broadbent-Callcott [1]. For primary shredding, the Gaudin-Meloy function with Π equal to 0.93 and r equal to 7 provided the best fit of the primary shredded product from raw municipal waste. For secondary and tertiary shredding, the modified Broadbent-Callcott equation gave the best fit with Π equal to 0.814 and n equal to 0.845 for secondary shredding and Π equal to 0.44 and n equal to 1.0 for tertiary shredding. All results fit the product particle size distributions very well, with squared residual values between 0.0011 to 0.0049.

Vesilind et al. applied the original Broadbent-Callcott breakage function to individual materials in the waste stream in an attempt to determine the Π selection functions [2]. Although no results were presented, they observed that the Broadbent-Callcott function did not represent the actual particle size distributions very well, except for glass. Thus, some calibration of the breakage function parameters may be necessary to fit the product particle size distribution.

The combined breakage theory uses a matrix equation to predict product size distribution vector P as a function of 1) the feed particle size distribution as a vector F, 2) the selection function as a diagonal matrix S, and 3) the breakage function as a triangular matrix B. Thus, with the error as a vector E, the product size distribution P is

$$P = B * S * F + (1 - S) * F + E \tag{4}$$

or, in full matrix form

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & & \\ \vdots & \vdots & \vdots & & \\ b_{n1} & b_{n2} & . & . & b_{nn} \end{bmatrix} \begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & 0 & \\ 0 & 0 & s_3 & \\ \vdots & \vdots & \vdots & \\ 0 & & 0 & 0\ s_n \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_n \end{bmatrix} + \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ \vdots & \vdots & \vdots & \\ 0 & & 0 & 0\ 1 \end{bmatrix} \cdot \begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & 0 & \\ 0 & 0 & s_3 & \\ \vdots & \vdots & \vdots & \\ 0 & & 0 & 0\ s_n \end{bmatrix} \right\} \cdot \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_n \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_n \end{bmatrix} \tag{5}$$

to represent the n equations for each particle size range in the form

$$p_1 = b_{11} * s_1 * f_1 + (1 - s_1) * f_1 + e_1$$
$$p_2 = b_{21} * s_1 * f_1 + b_{22} * s_2 * f_2 + (1 - s_2) * f_2 + e_2$$

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$$p_n = b_{n1} * S_1 - f_1 + b_{n2} * S_2 * f_2 + \ldots + e_n \tag{6}$$

where the first i-2 terms on the right hand side represent the contributions of broken particles from larger particles, the second to last term represents the unbroken particles which stay in the same particle size range, and the last term is the error term for each particle size range. With the proper feed particle size distribution, selection function and breakage function, this matrix allows the prediction of the output product particle size distribution.

Previous literature has demonstrated that the current form of the Π breakage theory using the Broadbent-Callcott equation is unsuitable for predicting the product particle size distribution of specific materials, such as wood, certain plastics and cardboard [2]. The breakage theory depends on material feed size as the primary factor in predicting the product particle size distribution [1]. Although the effect of the specific material involved is often considered indirectly through the inclusion of empirical coefficients (r,q), little or no consideration has been given to the possible effects that a container's material, shape or aperture (open-mouthed or enclosed) can have on the breakage behavior. Items with curved surfaces may be more difficult for shredding edges to successfully grasp and break while enclosed items may have greater structural integrity and thus better resist breakage. An empirical approach was therefore adopted to: 1) test breakage theory for beverage containers with observed breakage results and modify the breakage parameters to fit the predicted to the observed PSDs; and 2) test the effects of the physical characteristics of size, shape, aperture, and material on the breakage and separation behavior of beverage containers.

The first research statement, therefore, is that the breakage theory in its matrix form is capable of predicting the breakage and separation behavior of simple, uniform container objects based on the simple feed particle size distribution, the selection function chosen from Vesilind et al. [2] for the material and feed size, and the Gaudin-Meloy breakage function as shown to be most accurate for primary shredding in Trezek [1]. The comparison of predicted and measured particle size distributions will test this hypothesis. A calibration of empirical parameters will be conducted to optimize the fit if necessary.

The theory uses input particle size y, breakage ratio r, and slope factor q as the three independent variables ("parameters") to predict product size. Although the breakage theory shows good results for mixed waste through the adaptation of empirical parameters, breakage theory use for containers of different shape, closure, and material brittleness seems to be simplistic. Indeed, Vesilind et al. results show some of the difficulties, although the fit of measured and predicted particle size distributions were not shown for the optimal selection functions [2]. Nonetheless, experimental testing of the breakage theory seems reasonable, because it is the only theory to date used to predict the product particle size. The second research hypothesis states that, in addition to size, other factors are important for predicting the product particle size. Separation experiments are designed to test for the influence of the suite of variables that were judged to influence shredding and separation behavior and to be relevant for container design:

1. container size (1,000 mL or 250 mL),
2. geometry of the container (round or rectangular cross section),
3. closure (with or without top or lid), and
4. brittleness of the material (glass, expanded polystyrene or tetrapak materials).

This extended suite was tested through factorially designed experiments. In the next section, the experimental design and procedure are discussed.

EXPERIMENTAL METHODS

Three beverage container types—1) glass jars, 2) expanded polystyrene (EPS) containers (coffee cups), and 3) tetrapak aseptic packages (juice boxes)—were selected to represent recoverable materials that may be collected and processed in material recovery facilities (MRFs). The characteristics of the three container types represented different types of material, making them ideal for these experiments. Two sizes (250 mL and 1,000 mL), shapes (cylindrical and rectangular), and apertures (open or closed) were tested for glass and tetrapak containers. EPS containers were tested for size only because of the difficulty of manufacturing rectangular EPS cups and because aperture did not affect breakage.

The product size distribution for each individual test was then characterized by the modal (highest) interval density of shredded material. Because the screen size intervals vary, the highest mass percentage of shredded material per any interval screen size range (%/mm) was chosen to represent the maximum concentration of shredded material that could be expected from that particular container configuration. Within each container type, these modal interval density values were found to be normally distributed as they form a straight line passing through their own centroidal point (mean value of the variant, 50% probability) when plotted on normal probability paper [6]. Although only four values were available for the EPS containers, given the apparent normal distributions of the other two container types, the EPS modal values are also assumed to be normally distributed.

Product particle size distributions were predicted with breakage theory for each of the two container sizes and three container materials. Selection values of 1.0 for glass and EPS, and of 1.0 (1,000 mL) and 0.695 (250 mL) for tetrapak containers were derived by interpolation of cardboard results as reported in Vesilind et al. [2], see Table 1.

The Gaudin-Meloy breakage function with an r value of 7.0 was used based on Trezek's findings for primary shredding of raw MSW [1]. In subsequent trials, r values were varied and the modified Gaudin-Meloy breakage function with empirically determined values of q was tested. Table 2 summarizes the (a) feed particle distribution F, (b) the selection function S, and (c) the Gaudin-Meloy breakage function coefficients r and q for the original, modified and final prediction trials.

Table 2. Experimental Deisgn — Breakage Theory Parameters

	Glass		EPS		TP	
	1,000 mL	250 mL	1,000 mL	250 mL	1,000 mL	250 mL
(a) Original Trial						
Input Distribution F f_1 =	1.0	1.0	1.0	1.0	1.0	1.0
	(@ 102.5 mm)	(65 mm)	(@ 102.5 mm)	(65 mm)	(@ 102.5 mm)	(65 mm)
Selection Function S S_1 =						
	1.0	1.0	1.0	1.0	0.691	0.444
Breakage Ratio r					-interpolated	-cardboard
	7.0	7.0	7.0	7.0	7.0	7.0
(b) Modified Trial						
Input Distribution F	1.0	1.0	1.0	1.0	1.0	1.0
Selection Function S	1.0	1.0	1.0	1.0	0.691	0.444
Breakage Ratio r	5.3	3.9	2.4	1.5	1.3	0.93
(c) Final Trial						
Input Distribution F	1.0	1.0	1.0	1.0	1.0	1.0
Selection Function S	1.0	1.0	1.0	1.0	0.691	0.444
Breakage Ratio r	5.3	3.9	2.4	1.5	1.3	0.93
Slope Ratio q	1.5	1.5	1.5	1.5	1.5	1.5
	2.5	2.5	2.5	2.5	2.5	2.5

The shredding and screening experiment was designed as a 2^3 factorial to empirically test the effects of three independent variables: 1) size, 2) shape, and 3) aperture; and variable interactions size-shape (1 × 2), size-geometry (1 × 3), shape-aperture (2 × 3), and size-shape-aperture (1 × 2 × 3).

The shredding and separation tests were run with four container configurations for each glass and tetrapak container and with two different sizes of EPS containers. The results were averaged within each container configuration. The results are shown as the means and standard deviations particle size distribution (PSD) parameters as Rosin-Rammler characteristic particle size X_0 (screen size at which Y = 63.2% of particle pass) and slope n (slope of log {(ln[1/(1-Y)]} vs. log X). The averaged particle size distribution curves for each container type and size are shown in Figure 1. The resulting modal densities and the corresponding screen sizes are also shown in Table 3. As expected, the glass product particles are the smallest and are tightly distributed a Rosin-Rammler (RR) characteristic particle size X_0 of 35 to 38 mm (1,000 mL) and around 33 mm (250 mL). The high values for the RR slope n of 6.7 to 7.4 and the high modal densities of 2.7 to 2.9 percent/mm further indicate that glass product particles fall within a narrow size range between 15 to 25 mm. EPS containers break into larger, particles with the characteristic particle size equal to about 60 mm. The particle size distribution, however, is flatter as indicated by lower RR slope n of 1.96 and lower modal density of 1.59 percent/mm at 25 to 50 mm screens. Tetrapak product particle sizes are the largest, with X_0 average sizes of 111 mm (1,000 mL) and 87 mm (250 mL). Rosin-Rammler slopes n are lowest at 1.22 (1,000 mL) to 1.97 (250 mL). The modal PSD densities are low at 1.08 percent/mm (1,000 mL) to 1.92 percent/mm (250 mL) and occur at the 80 to 125 mm screen size (see Table 3). The results reflect the differences in brittleness of the material, while within container variation may be explained by size, shape, and aperture (see below).

For the purposes of the ANOVA, the configurations were considered the treatments while the replicates were considered as blocks. Some of the container configurations were run with fewer than three replicates. The eight missing modal interval densities were estimated by the values that minimized the sum of the squares of the errors in the ANOVA. This common procedure [6] results in an approximate ANOVA with the degrees of freedom for the error term reduced by the number of estimated values. However, this ANOVA is considered in conjunction with the factorial analysis results.

RESULTS AND ANALYSIS

Breakage Theory Predictions

In the original trial predictions, the predicted characteristic particle sizes X_0 using the selection function and breakage ratio values from Trezek [1] and

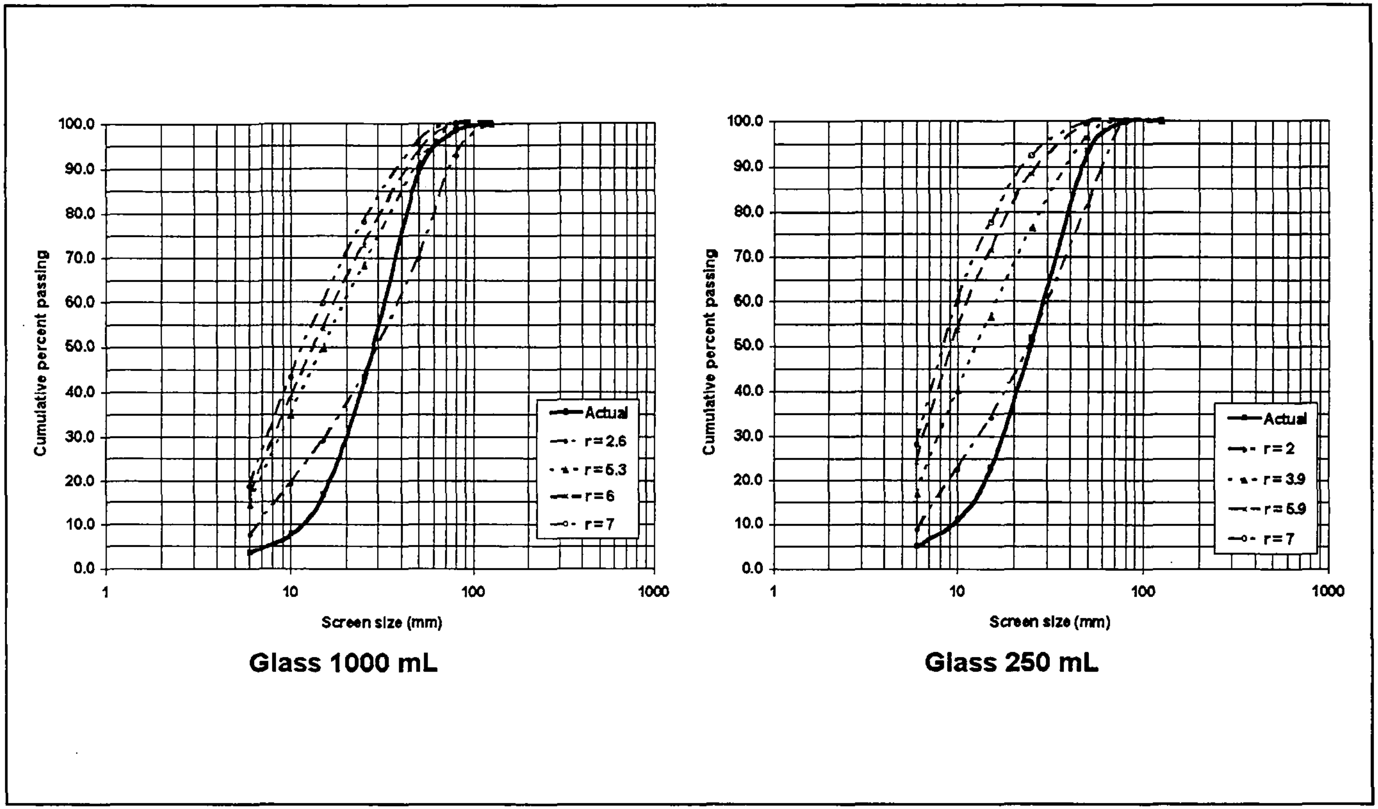


Figure 1. Measured mean particle size distributions versus predicted breakage size distributions for various r-values.

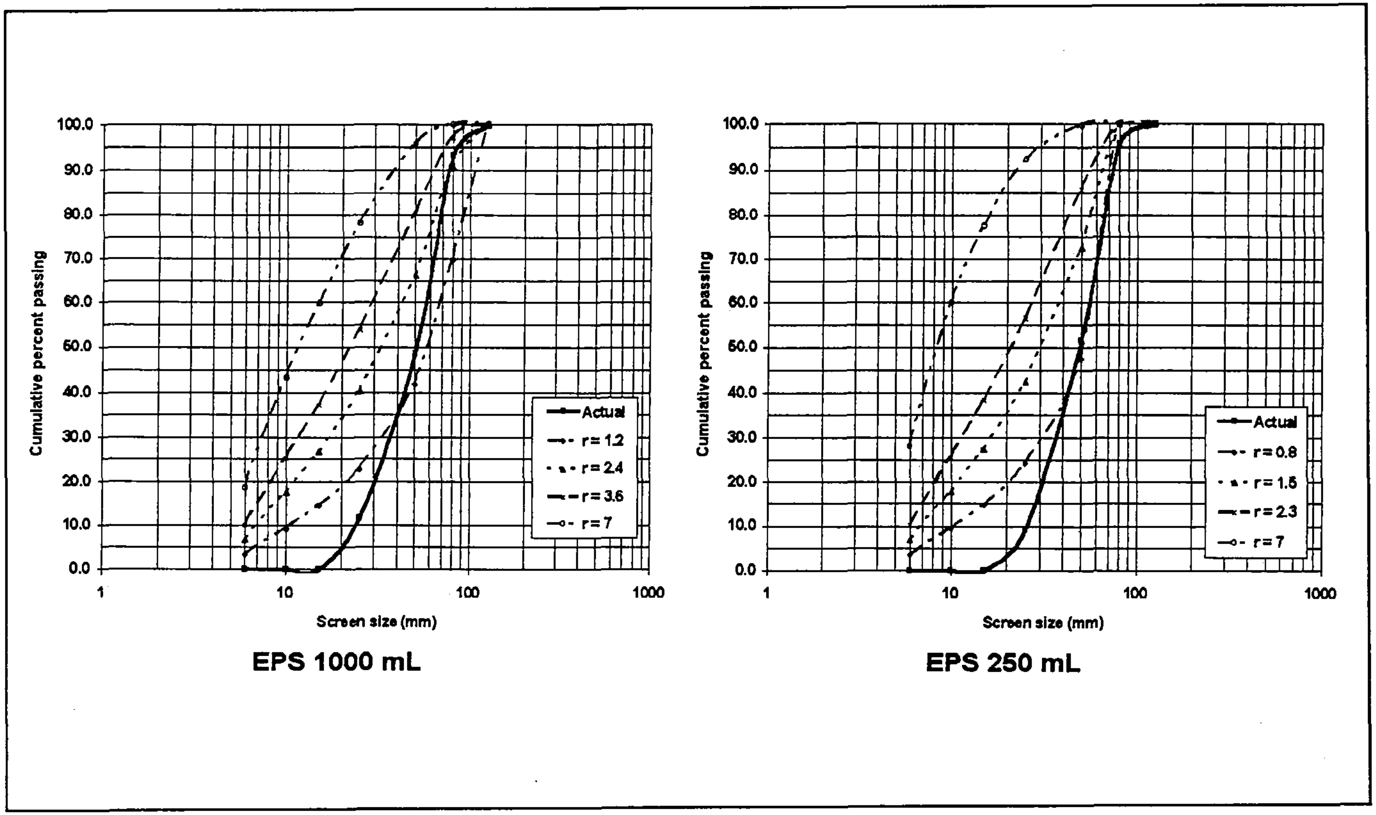


Figure 1. (Cont'd.)

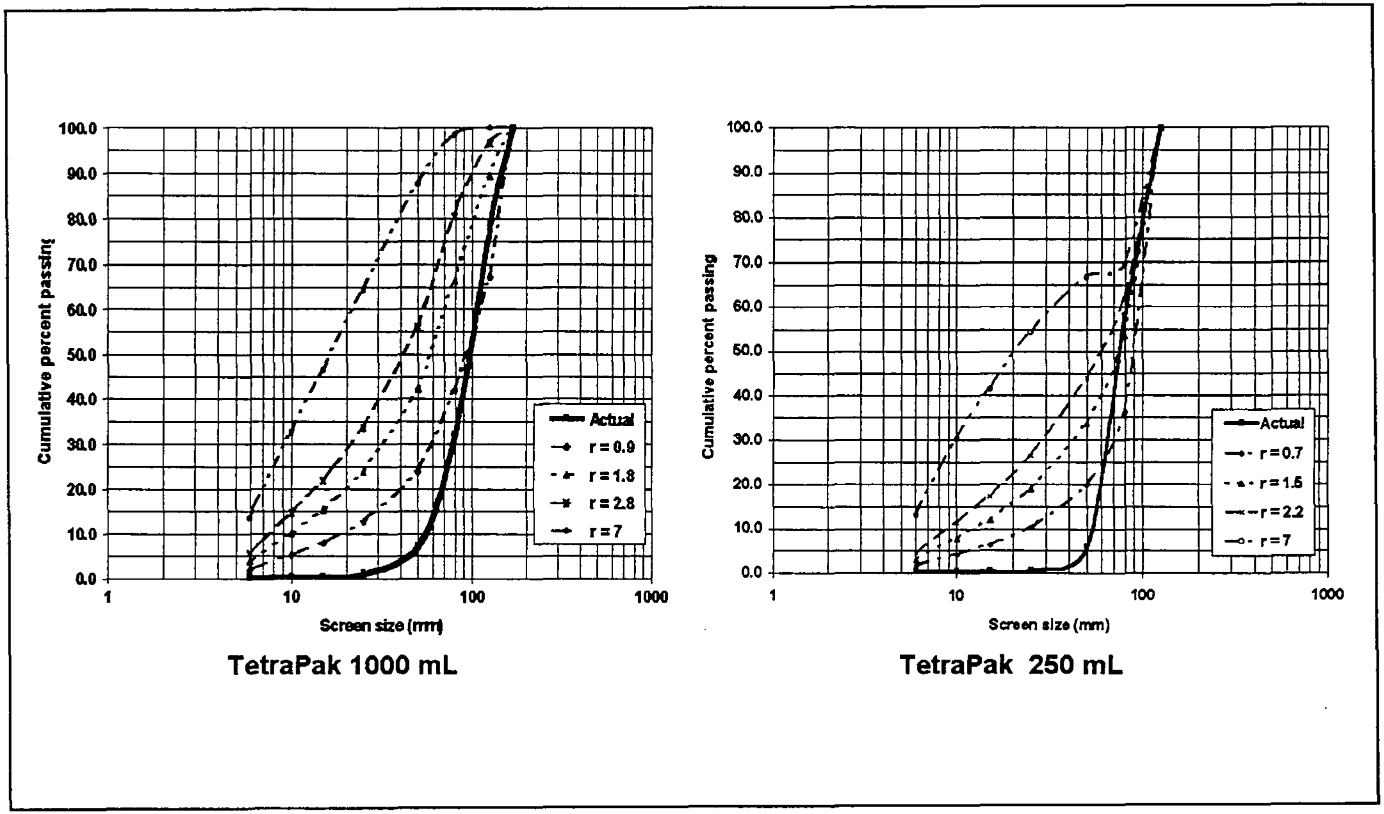


Figure 1. (Cont'd.)

Table 3. Separation Test Results

					Rosin-Rammler Characteristic Particle Size		Modal Density	
Container	Size	Shape	Aperture	Sample Size n =	X_0 (mm)	Slope n (–)	Mean (%/mm)	Screen Size of Modal
Glass	1,000 mL	Rectangular	Enclosed	2	37	6.55	2.6	15
	1,000 mL	Rectangular	Open	2	35	6.97	2.73	15
	1,000 mL	Cylindrical	Enclosed	2	38	7.18	2.62	15
	1,000 mL	Cylindrical	Open	1	37	7.47	3.14	15
Average					36.75	7.04	2.71	
Standard Deviation					1.26	0.39	0.34	
Glass	250 mL	Rectangular	Enclosed	2	33	6.70	2.84	15
	250 mL	Rectangular	Open	2	33	6.70	2.49	15
	250 mL	Cylindrical	Enclosed	2	33	6.70	3.09	15
	250 mL	Cylindrical	Open	2	33	6.70	3.06	15
Average					33	6.70	2.87	
Standard Deviation					0.00	0.28		
EPS	1,000 mL	Cylindrical	Open	2	60	1.70	1.69	25
	250 mL	Cylindrical	Open	2	59	2.21	1.49	50
Average				59.5	1.96	1.59		
Standard Deviation				0.71	0.37	0.14		
Tetrapak	1,000 mL	Rectangular	Enclosed	2	110	1.24	1.12	80
	1,000 mL	Rectangular	Open	3	118	1.36	1.08	80
	1,000 mL	Cylindrical	Enclosed	1	102	1.23	1.15	80
	1,000 mL	Cylindrical	Open	2	115	1.06	0.96	80
Average					111.25	1.22	1.08	
Standard Deviation					6.99	0.13	0.08	
Tetrapak	250 mL	Rectangular	Enclosed	3	99	1.71	1.44	50
	250 mL	Rectangular	Open	3	98	1.54	1.56	80
	250 mL	Cylindrical	Enclosed	1	72	2.54	2.59	80
	250 mL	Cylindrical	Open	1	78	2.08	2.09	80
Average					86.75	1.97	1.92	
Standard Deviation					13.79	0.44	0.53	

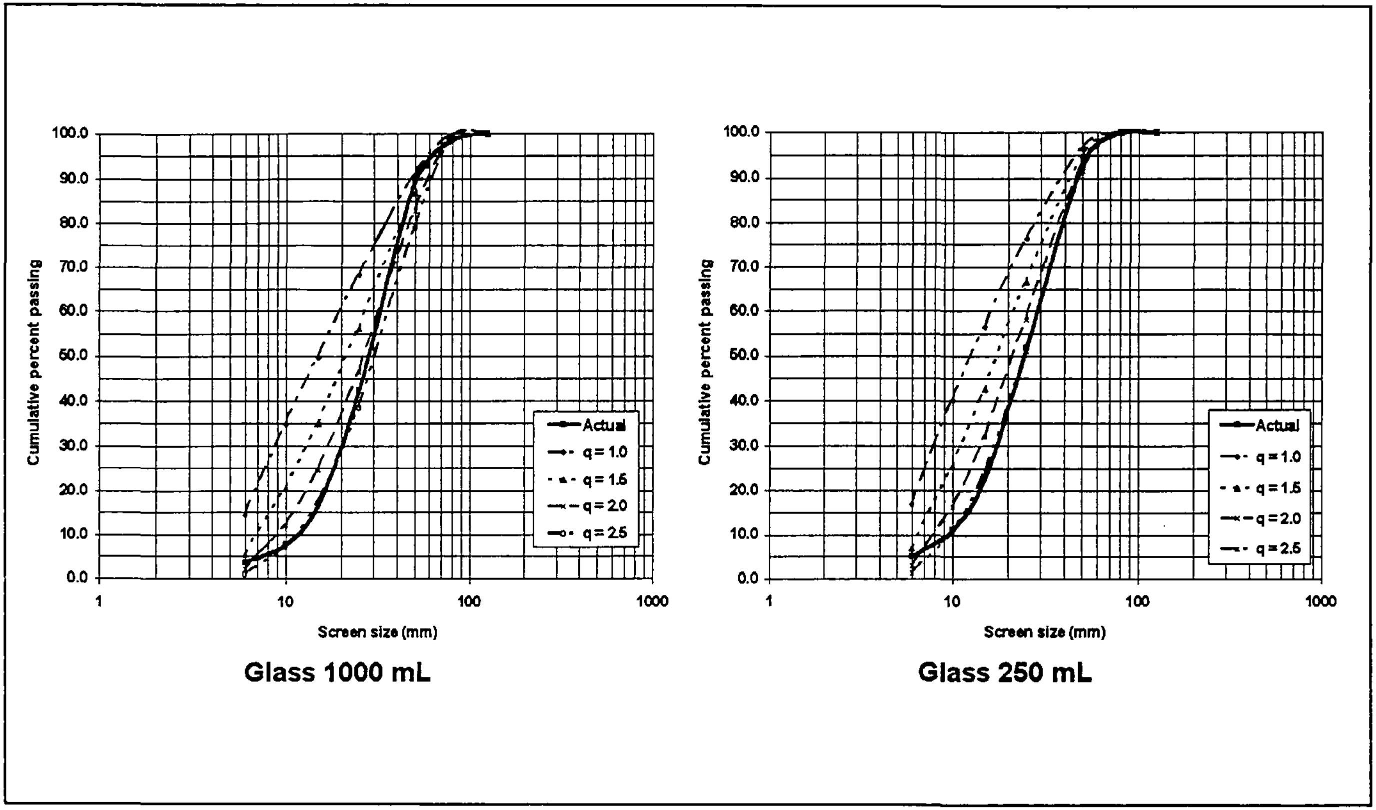
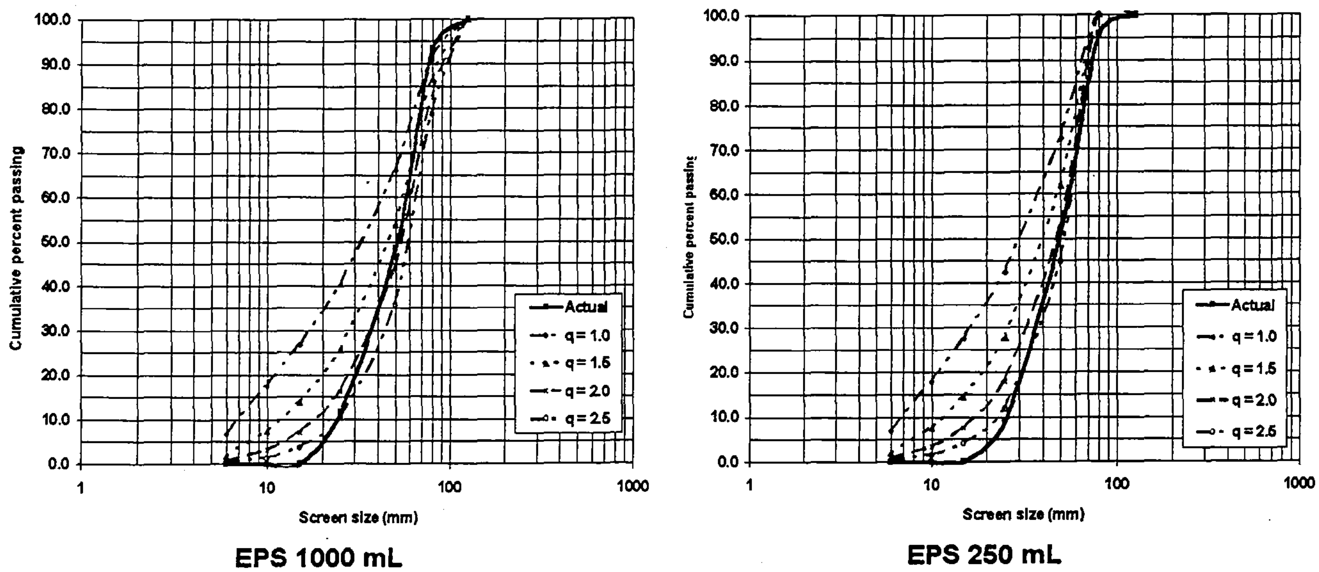


Figure 2. Measured mean particle size distributions versus predicted breakage size distributions for various q-values.



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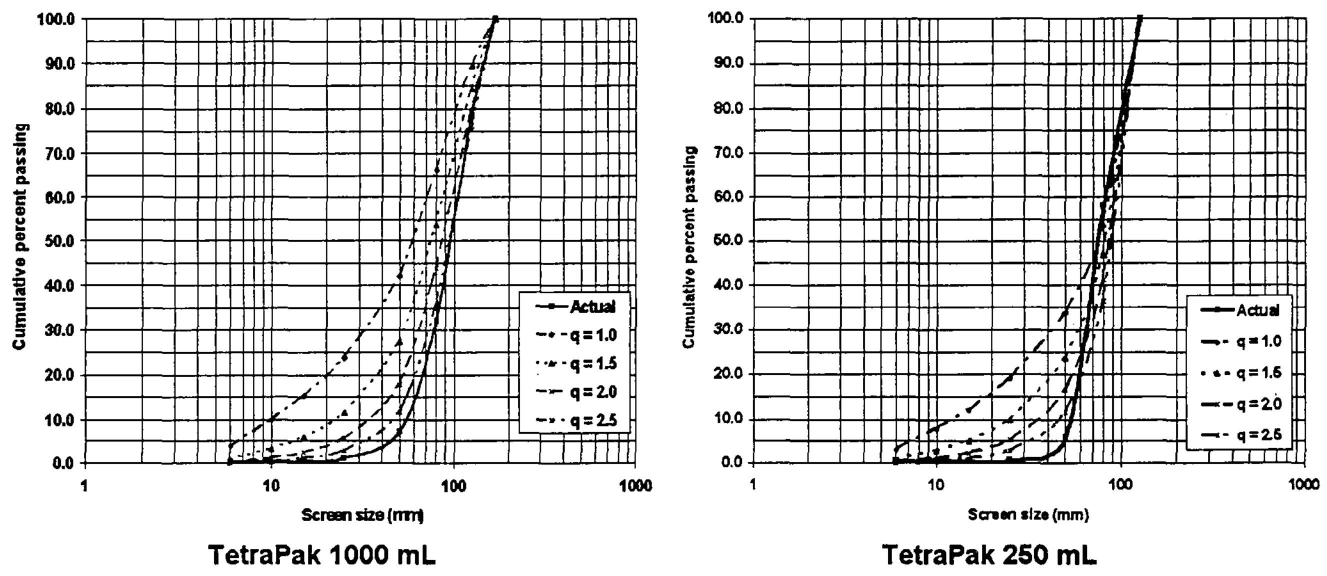

Figure 2. (Cont'd.)

Vesilind et al. [2] (see Table 2) are significantly smaller than the measured values (see Figure 1). For glass, the breakage theory underestimates the characteristic size by 15 to 18 mm (at 18 mm vs. 33 to 38 mm measured), for EPS, the difference is 42 mm (at 18 mm predicted vs. 60 mm measured) and for tetrapak containers, the difference increases to 92 mm.

In a subsequent, modified trial prediction, the product sizes were inspected and the values for the exponent r were calibrated in the breakage model to test for better fit. The average ratio of product particle size to feed particle size were determined from the separation test results to estimate new r values. The adjusted r values were 5.3 (1,000 mL) and 3.9 (250 mL) for glass, 2.4 (1,000 mL) and 1.5 (250 mL) for EPS, and 1.3 (1,000 mL) and 0.93 (250 mL) for tetrapak (Table 2). These modified r values were used to recalculate predicted product particle sizes (see Figure 1). Figure 1 shows the comparison of the results with measured particle size distributions. In addition, r values were rated to plus 100 percent and minus 50 percent of the calculated, modified values.

Adjustments were made to breakage ratio, r, values to improve the fit of the breakage function to measured PSD's. These curves are also shown. Notable differences remain between predicted and measured characteristic particle sizes X_0 (read in Figure 1 at the 63.2% level) and for smaller particles.

The third trial of predictive breakage theory used the modified Gaudin-Meloy breakage function and varied the slope coefficient q from 1.5 to 2.5 (see Table 2). The resulting changes in slope provide much better fit for glass and EPS particle size distributions with differences in characteristic particle size X_0 of less than 5 mm for the best fitting curves (q = 2.0 to 2.5). Tetrapak predictions are better, but discrepancies remain in the larger particle sizes (about 60 to 80 mm). These differences stem from the use of the input distribution F (102.5 mm or 65 mm) as y, the largest particle size. The largest shredded tetrapak particles measured up to 165 mm and exceeded the largest input particles. As a result, when the largest measured particle size $y' = 165$ mm was substituted into the breakage functions, the predicted fit (PSD's) the measured PSD's very well. Slight discrepancies remain at particle sizes below 50 mm.

Applying the Gaudin-Meloy breakage function with empirical values for the selection function S and for the breakage ratio r yielded poorly fitting predictions for all three container types. The modified Gaudin-Meloy breakage function with empirically determined slope factor q and maximum particle size y yielded accurately fitting particle size distribution curves to predict characteristic particle size X_0 and modal density of glass, EPS and tetrapak containers to within plus or minus 10 percent accuracy.

The maximum particle size, breakage ratio r, and slope factor q are possibly affected by container size, shape, geometry, and material. The influence of these factors is tested by factorial analysis of the test results.

FACTORIAL TEST SERIES

Glass Containers

The results of the glass container runs analyzed by factorial analysis produced the following model.

$$\eta = 2.719 - 0.151x_1 - 0.056x_2 + 0.067x_3 + 0.149x_1x_2 - 0.028x_1x_3 - 0.013x_2x_3 - 0.093x_1x_2x_3 \tag{7}$$

η represents the model outcome, here the modal density; the constant is the average value of all configurations (or runs), which is subsequently altered by adding or subtracting the x-terms. The actual effect of each factor, when going from the −1 to +1 level, is double the magnitude of the coefficient for that factor listed in the equation. The standard error of each effect is ±0.0999, except for the constant, which is ±0.05.

The significant effects consist of the size (x_1) and the size/shape (x_1,x_2) interaction at the 95 percent confidence level. Increasing the glass container size from 250 mL to 1000 mL will decrease the modal interval density by 0.301 percent/mm. The combination of increasing the size and switching from cylindrical to rectangular glass containers increases the modal interval density by 0.299 percent/mm. The effect of size for glass containers changes the constant by only 0.151 percent/mm to either extreme size level. This is a change of approximately 6 percent.

The compact form of the model appears therefore as

$$\eta = 2.719 - 0.151\,x_1 + 0.149\,x_1x_2 \tag{8}$$

where the other terms are omitted because they are insignificant.

ANOVA was also applied to the glass data. Based on a one-sided *F*-test at the 0.05 and 0.01 level of significance, the size factor and size-shape interaction were significant. Other factors were insignificant (see Table 4a). This is consistent with the factorial results.

Expanded Polystyrene Container Tests

Cylindrical, open EPS containers of 1,000 mL and 250 mL size were tested in replicate. A two-sample *t*-test was performed to evaluate if the mean modal densities of the two EPS containers were different. Based on this comparison, the means are different at the 0.05 significance level, but not at the 0.01 level. By inspection of the PSD curves, however, the difference in resulting distributions of 1,000 mL and 250 mL particles is small. Thus, while statistically significant, the differences may not be practically important.

Table 4. Statistical Analysis of Breakage Tests

a) Glass Container — ANOVA

Source of Variation	Deg. of Free, *d.f.*	SS	MS (SS/*d.f.*)	F_{calc} = MS/MS Error	Sig @ 0.05	Sig @ 0.01
Replicates	1	0.0346	0.0346	0.876	No	No
Size	1	0.334	0.334	8.456	Yes	No
Shape	1	0.061	0.061	1.544	No	No
Size Shape	1	0.28	0.28	8.304	Yes	No
Geometry	1	0.059	0.059	1.494	No	No
Size Geom.	1	0.019	0.019	0.481	No	No
Shape Geom.	1	0.001	0.001	0.0253	No	No
3-factor	1	0.121	0.121	3.063	No	No
Error	6	0.237	0.0395			

b) EPS Container — t-Test

t-calc.	t-tab @ 0.05	Sig @ 0.05	t-tab @ 0.01	Sig @ 0.01
4.430	4.303	Yes	9.925	No

Source: Tam, 1994.

c) Tetrapak Container — ANOVA

Source of Variation	Deg. of Free, *d.f.*	SS	MS (SS/*d.f.*)	F_{calc} = MS/MS Error	Sig @ 0.05	Sig @ 0.01
Replicates	2	0.5026	0.251	1.549	No	No
Size	2	4.379	2.190	13.519	Yes	Yes
Shape	2	1.810	0.905	5.586	Yes	No
Size Shape	4	1.459	0.365	2.253	No	No
Geometry	2	0.243	0.122	0.753	No	No
Size Geom.	4	0.001	0.00025	0.00154	No	No
Shape Geom.	4	0.252	0.063	0.389	No	No
3-factor	6	0.066	0.011	0.0679	No	No
Error	6	0.972	0162			

Tetrapak Containers

The average modal density from both sizes (1,000 mL and 250 mL) is constant at 1.5 percent/mm. This average is derived from the testing of four large and four small container configurations and thus represents a midpoint. The factorial analysis for the tetrapak containers produced the following linear model:

$$\eta = 1.499 - 0.420x_1 - 0.200x_2 + 0.076x_3\,0.222x_1x_2 \\ - 0.018x_1x_3\,0.095x_2x_3 + 0.059x_1x_2x_3 \tag{9}$$

The standard error of each effect is ±0.1957, except the far constant which is ±0.0979. These results show that container size is the most significant effect. The effect of shape is marginally significant. Increasing the size of the container from 250 mL to 1000 mL will decrease the interval density by (2·0.42, or) or 0.841 percent/mm. Therefore, the 0.841 percent/mm change represents a deviation of 0.420 percent/mm to either size level from the midpoint. This would represent a midpoint-to-extreme change of approximately 28 percent.

Given this evaluation, the model appears as

$$\eta_{TP} = 1.499 - 0.420x_1 - 0.200x_2 \tag{10}$$

with the remaining terms eliminated because they are insignificant.

The one-sided ANOVA *F*-tests revealed that size was significant at both the 0.05 and 0.01 significance levels. Shape was significant only at the 0.05 level. These findings are consistent with results of the factorial analysis.

A final test of material brittleness was conducted. The modal densities were analyzed in relation to material brittleness. Glass containers produce the highest modal densities at 2.7 percent/mm, followed by EPS with 1.6 percent/mm and tetrapaks at 1.2 percent/mm. The differences are significant between glass and EPS and tetrapak are significant. The differences show increasing particle size densities with increasing propensity to fracture. Glass is highest, followed by EPS and tetrapak. However, container size reverse this trend as 250 mL tetrapaks produce higher densities.

The comparison of the shredding behavior confirms that the different container materials do produce different modal densities. It appears that the modal density increases as the brittleness of the material increases and as size decreases.

The factorial and ANOVA results show that for glass, size-shape interactions, for EPS size, and for tetrapak size and shape affect the modal densities of the product PSDs at the 95 percent confidence level. Only tetrapak size is, however, significant at the 99 percent level. Moreover, the differences in modal densities for EPS and tetrapaks are practically small. In contrast, however, the container material brittleness increases model density over 100 percent from 1.2 percent/mm for tetrapaks, to 2.8 percent/mm for glass particles.

As a result, size and shape are marginally significant for brittle materials, but size is a highly significant factor for pliable container materials. Aperture has no consistent effect.

DISCUSSION AND CONCLUSION

Separation experiments with glass, expanded polystyrene (EPS), and tetrapak containers of 1,000 mL and 250 mL sizes were used to test the predictive capability of the breakage theory and to determine whether container characteristics (size, shape, aperture, and material) affect mechanical separation of beverage containers.

The modified Gaudin-Meloy breakage function with empirically derived selection function S, breakage ratio r, and slope factor q provided good fit with measured product particle size distributions (PSDs) and modal particle densities (as %/mm screen size). Thus, the breakage theory can be adapted to model container behavior if the proper values for maximum particle size y, breakage ratio r, and slope factor q can be empirically determined. These factors will probably vary with the type of shredder and the size, shape, geometry, and material of the container. The composition of the mixed refuse or the co-mingled recyclable stream may also affect container breakage behavior. Brittle materials appear to be more accurately predictable with breakage theory because they are less affected by size, shape, and other characteristics.

Glass and EPS product particle size distributions appear to be only marginally affected by size and shape because they fracture as brittle materials. However, tetrapak product particle size is significantly affected by input particle size. The results show that container material, size, and size-material and size-shape interactions affect the separation behavior, particularly of soft ductile materials. Small containers of more brittle material will separate into more concentrated fractions on smaller screen sizes. This research is an initial step toward predicting municipal solid waste components' separation behavior. Container (and other particles) characteristics need to be systematically tested, both as single materials and co-mingled with the solid waste stream and other recyclables. The breakage behavior of specific materials needs to be modified and further investigated if product design is to be changed to enhance recovery from the waste stream. Particularly the behavior of soft, ductile, and pliable materials needs to be investigated.

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