

ESTIMATION OF AIR TEMPERATURE INDUCED BY A HEAT SOURCE IN A COMPARTMENT WITH DISPLACEMENT VENTILATION*

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ABSTRACT

A simple model is proposed to calculate the air temperature rise induced by a heat source in a compartment with displacement ventilation. The model is based on the mixing process in ventilation theory. Expressions for the hot air temperature are derived by considering the cases with and without a thermal plume region. Numerical studies in a chamber with displacement ventilation were performed with computational fluid dynamics to justify the predicted results.

1. INTRODUCTION

Forced ventilation systems are installed in most of the commercial buildings in Hong Kong [1]. Displacement ventilation [2] is also used; in this design, fresh air is supplied at a lower position and the room air is extracted at the upper location. Any heat source in the building induces a thermal plume and mix with the hot air. Eventually, a stable upper hot layer is formed that can be extracted by a fan near the ceiling. The ventilation rate in the room depends on the use and would be quite high, on the order of up to fifty air changes per hour for a small toilet, for example. At such a high ventilation rate, not all the intake air from the lower level would mix with the thermal plume and a proportion of it would move directly to the upper layer. On the other hand, part of the air at the upper part would move down to the lower part of the chamber, depending on the interactions of the intake air and the thermal plume induced air motions. The mixing theory proposed by

*The CFD package Phoenix was used in this study.

Sandberg [3] can be applied to study the resultant air flow and calculation of the hot air temperature, as reported for a forced ventilation fire [4] with stronger heat sources. This simple model is further extended to study the removal of heat in a displacement ventilation system. The computational fluid dynamics (CFD) technique is also used to justify the results [5-7].

2. THE MODEL

A simple mathematical model is developed for modeling the temperature rise due to a heat source in a chamber with displacement ventilation systems. A room of volume V_{RM} with a heat source of thermal power Q is equipped with a displacement ventilation system. There is a fresh air intake fan at a lower level and an exhaust fan at a higher level but on the opposite wall as in Figure 1. Assume that the heat source induces a thermal plume and yields a rather stable thermally stratified upper hot layer. There are two zones with the upper hot zone at temperature T_H and the lower cool zone at temperature T_L . Apart from the heat and mass transfer process through the plume, there is some form of mixing between the two zones. Suppose that the air intake rate is \dot{m}_{in} and the exhaust rate is \dot{m}_{out} and that in the lower zone, a fraction of air α is entrained into the thermal plume with temperature T_p . The outward mass flow rate \dot{m}_{out} has two components: the mass flow rate of air which is not entrained $(1 - \alpha) \dot{m}_{in}$ and the mass flow rate of air in the plume \dot{m}_p . By conservation of mass:

$$\dot{m}_{out} = (1 - \alpha) \dot{m}_{in} + \dot{m}_p \approx \dot{m}_{in} \quad (1)$$

or

$$\dot{m}_p \approx \alpha \dot{m}_{in} \quad (2)$$

A mixing factor β [3] can be defined to describe the exchange flow rate between the two zones. By using the conservation law of enthalpy at the steady-state with the flow diagram shown in Figure 2, three equations on the lower zone temperature T_L , plume temperature T_p , and upper zone temperature T_H can be derived:

Lower zone:

$$\dot{m}_{in} T_{in} + (1 - \alpha) \beta \dot{m}_{in} T_H = (1 - \alpha)(1 + \beta) \dot{m}_{in} T_L + \alpha \dot{m}_{in} T_L \quad (3)$$

Plume:

$$Q = \alpha \dot{m}_{in} C_p T_p - \alpha \dot{m}_{in} C_p T_L \quad (4)$$

where C_p is the specific heat of air

Upper zone:

$$\alpha \dot{m}_{in} T_p + (1 - \alpha)(1 + \beta) \dot{m}_{in} T_L = \dot{m}_{in} T_H + (1 - \alpha) \beta \dot{m}_{in} T_H \quad (5)$$

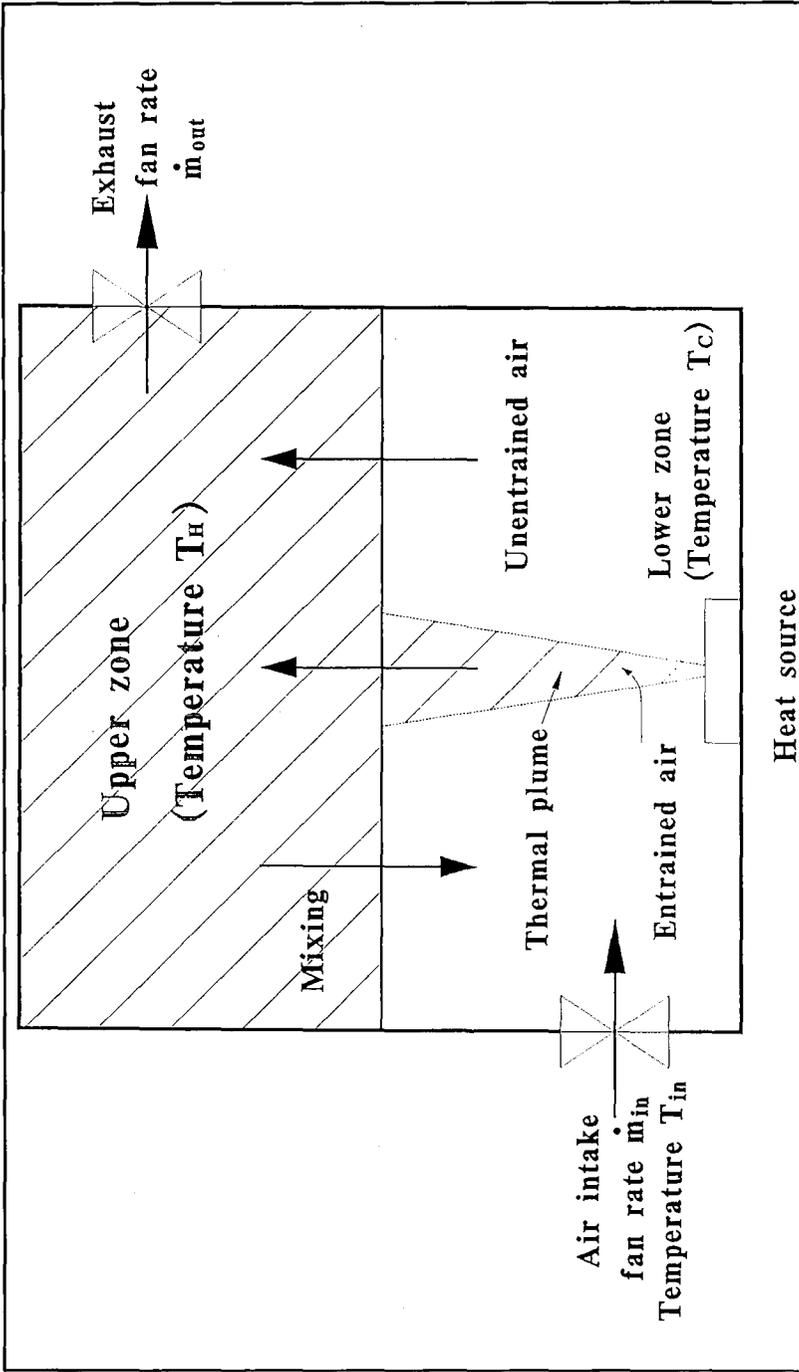


Figure 1. Principle of displacement ventilation.

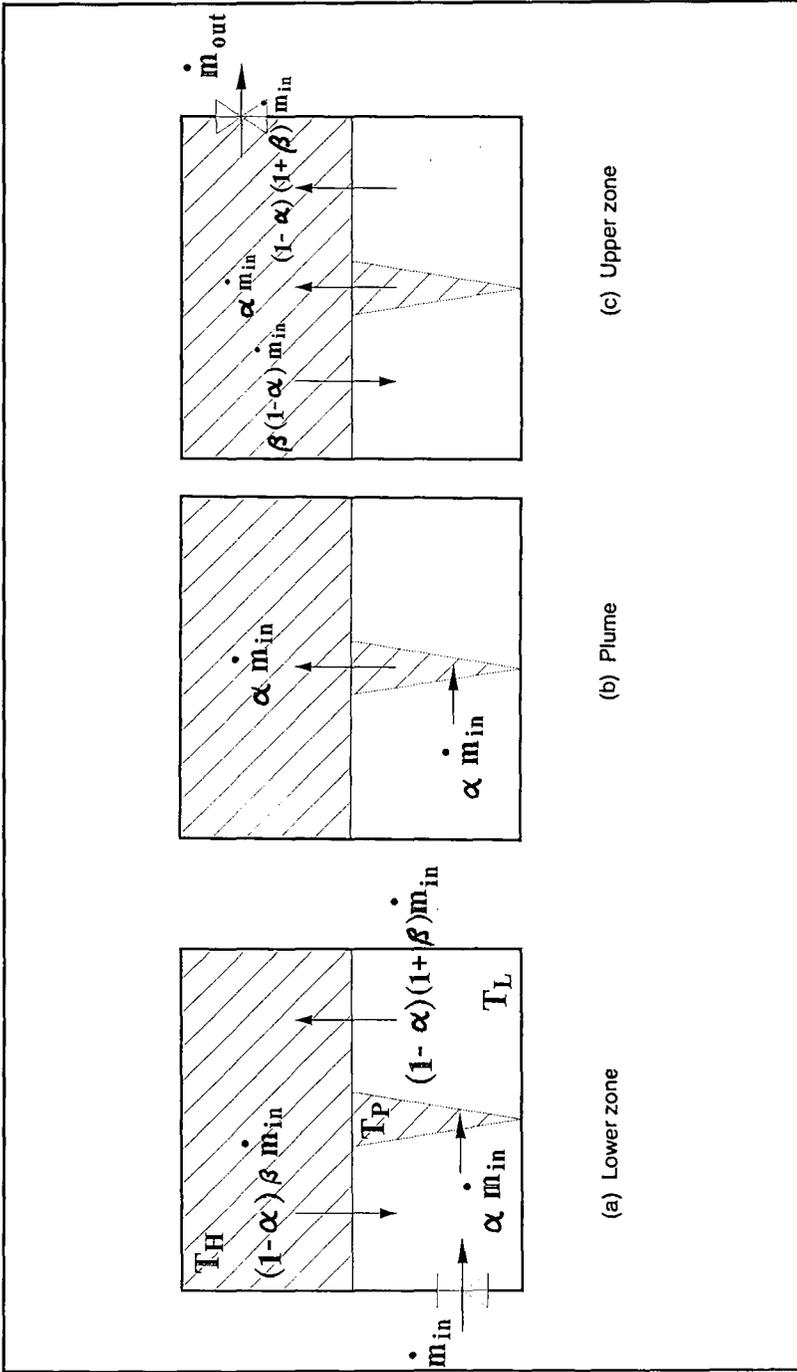


Figure 2. Flow diagram.

Simplifying expressions (3) to (5) gives:

$$T_L = \frac{\frac{Q}{\dot{m}_{in}C_p} (1 - \alpha)\beta + [1 + (1 - \alpha)\beta]T_{in}}{[\alpha + (1 - \alpha)(1 + \beta)]} \quad (6)$$

$$T_p = \frac{Q}{\dot{m}_{in}C_p\alpha} + \frac{\frac{Q}{\dot{m}_{in}C_p} (1 - \alpha)\beta + [1 + (1 - \alpha)\beta]T_{in}}{[\alpha + (1 - \alpha)(1 + \beta)]} \quad (7)$$

$$T_H = \frac{Q}{\dot{m}_{in}C_p} + T_{in} \quad (8)$$

Note that values of T_H do not depend on the mixing factor. The mixing would affect the lower zone and plume temperature. Putting the expression for T_H into equations (5) and (6) gives:

$$T_L = \frac{T_H(1 - \alpha)\beta + T_{in}}{[\alpha + (1 - \alpha)(1 + \beta)]} \quad (9)$$

$$T_p = \frac{(T_H - T_L)}{\alpha} + \frac{T_H(1 - \alpha)\beta + T_{in}}{[\alpha + (1 - \alpha)(1 + \beta)]} \quad (10)$$

Relatively simple expressions for T_L and T_p can be derived when there is no mixing between the two zones.

Taking $\beta = 0$ gives $[\alpha + (1 - \alpha)(1 + \beta)] = 1$, and substitution into equations (6) and (7) gives:

$$T_L = T_{in} \quad (11)$$

$$T_p = \frac{Q}{\dot{m}_{in}C_p\alpha} + T_{in} \quad (12)$$

3. RESULTS

The above set of equations is evaluated by studying the air flow in a chamber with displacement ventilation. The room is of length 4 m, width 4 m, and height 3 m (Figure 3). There are two axial fans installed at two square vents, each of size 0.5 m by 0.5 m and located opposite to each other. The exhaust fan is 0.5 m below the ceiling, the intake fan is 0.5 m above the floor.

Ten cases of different values of heat sources and ventilation rates (labeled as from A to F) were studied. The ventilation rates \dot{m}_{in} (ms^{-1} or air changes per hour ACH) in the experiments were set from $0.133 \text{ m}^3\text{s}^{-1}$ (10 ACH) to $2.66 \text{ m}^3\text{s}^{-1}$

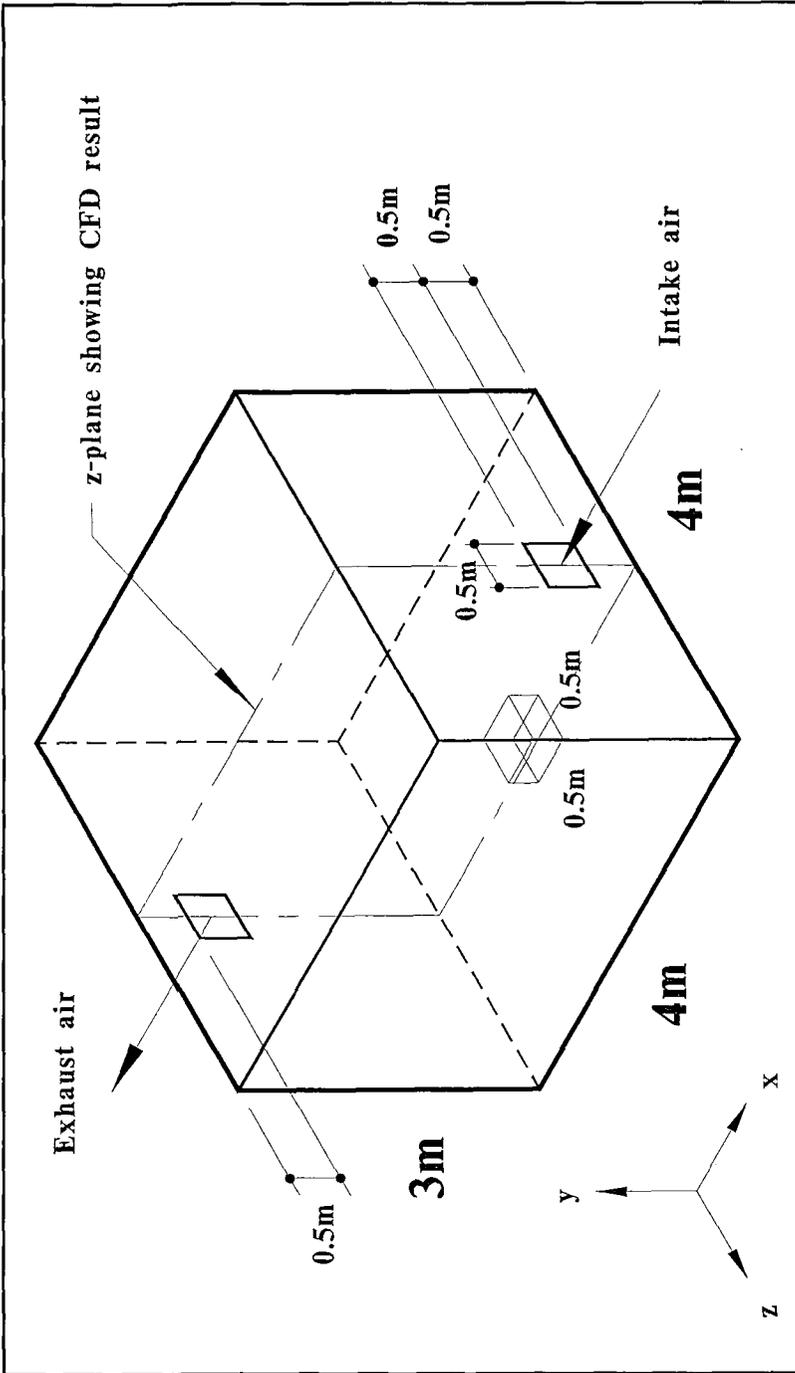


Figure 3. Geometry of the scale model chamber.

(200 ACH) by varying the face velocity at the opening. A heat source of size 0.5 m by 0.5 m and thermal power varying from 1- kW to 1000 kW was used. The initial temperature was taken to be 20°C. A summary of the ventilation conditions for the ten cases is shown in Table 1.

Values of the temperature T_H with the mixing factor β equal to zero are calculated by taking the density of air to be 1.12 kg s^{-1} . Values of T_p cannot be calculated unless the entrainment factor α is known, but the value $(T_p - T_{in})$ is simply $(T_H - T_{in})/\alpha$. Values of the temperature T_L and T_p are calculated from the model with a plume region using equations (9) and (10) for mixing factors β from 0.1 to 0.3, and factors of air entrainment to the plume α from 0.7 to 0.9 (Table 1).

4. COMPARISON WITH COMPUTATIONAL FLUID DYNAMICS

The above ten cases of ventilation operation conditions were also simulated by the Computational Fluid Dynamics (CFD) package PHOENICS version 2.1 [8] with the FLAIR manual [9]. Results on the velocity vectors and temperature central z-plane in Figure 3 are shown in Figures 4 to 13. It is observed that values of T_H predicted by the crude model and the CFD model are very similar with the exception of case H, reflecting the mean value of T_H predicted by CFD.

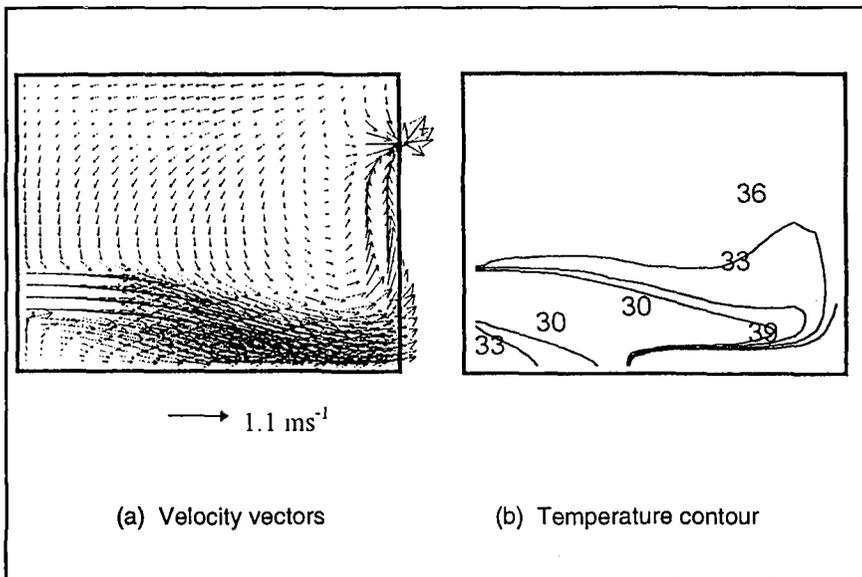


Figure 4. Results for case A.

Table 1. Results

Case	Ventilation Rates		Thermal Power of Source	Crude Model ($\beta = 0$)			Crude Model ($\beta = 0.1$)						
	\dot{m}_v	ACH		T_H /°C	T_p /°C			$\alpha = 0.9$		$\alpha = 0.8$		$\alpha = 0.7$	
	$/m^3s^{-1}$			$\alpha=0.9$	$\alpha=0.8$	$\alpha=0.7$	T_U /°C	T_p /°C	T_U /°C	T_p /°C	T_U /°C	T_p /°C	
A	0.5	37.5	10	36	38	40	43	21	38	21	41	22	44
B	0.5	37.5	1	22	22	22	22	21	22	21	22	21	22
C	0.5	37.5	100	182	200	223	252	22	202	24	226	26	257
D	0.133	10	50	325	359	401	456	23	362	27	407	31	465
E	0.133	10	100	630	698	783	892	27	704	33	795	40	910
F	0.133	10	200	1240	1376	1546	1764	33	1388	46	1570	59	1799
G	1.33	100	100	26	27	28	29	20	27	21	28	21	29
H	1.33	100	500	325	359	401	456	23	362	27	407	31	465
I	1.33	100	1000	630	698	783	892	27	704	33	795	40	910
J	2.66	200	100	51	54	58	64	21	54	21	59	22	64

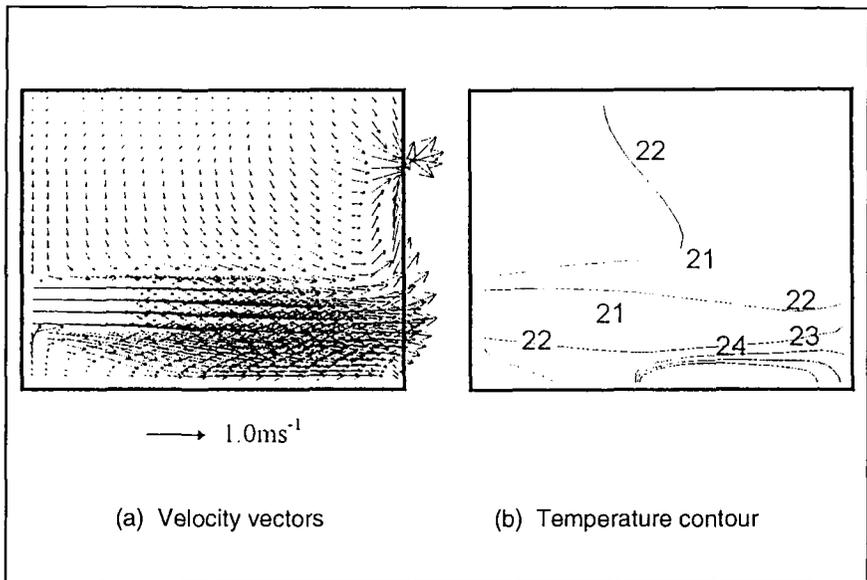


Figure 5. Results for case B.

Crude Model ($\beta = 0.2$)						Crude Model ($\beta = 0.3$)						CFD
$\alpha = 0.9$		$\alpha = 0.8$		$\alpha = 0.7$		$\alpha = 0.9$		$\alpha = 0.8$		$\alpha = 0.7$		Mean
$T_U/^\circ\text{C}$	$T_P/^\circ\text{C}$	$T_U/^\circ\text{C}$	$T_P/^\circ\text{C}$	$T_U/^\circ\text{C}$	$T_P/^\circ\text{C}$	$T_U/^\circ\text{C}$	$T_P/^\circ\text{C}$	$T_U/^\circ\text{C}$	$T_P/^\circ\text{C}$	$T_U/^\circ\text{C}$	$T_P/^\circ\text{C}$	$T_H/^\circ\text{C}$
20	38	21	41	24	44	20	39	21	41	21	45	36
20	22	20	22	23	22	20	22	20	22	20	22	21
23	204	26	229	33	261	25	205	29	232	33	265	200
26	365	32	413	42	473	29	368	37	419	45	481	320
32	710	43	806	62	926	38	716	55	817	70	942	600
44	1400	67	1593	100	1833	56	1412	89	1615	121	1864	1200
20	27	20	28	23	29	20	27	20	28	21	29	40
26	365	32	413	42	473	29	368	37	419	45	481	150
32	710	43	806	62	926	38	716	55	817	70	942	600
21	55	21	59	25	65	21	55	22	60	23	66	60

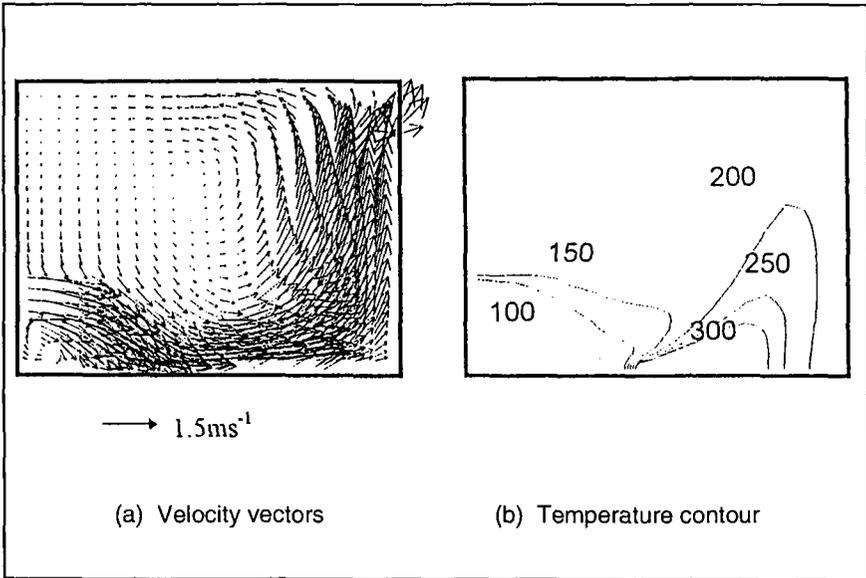


Figure 6. Results for case C.

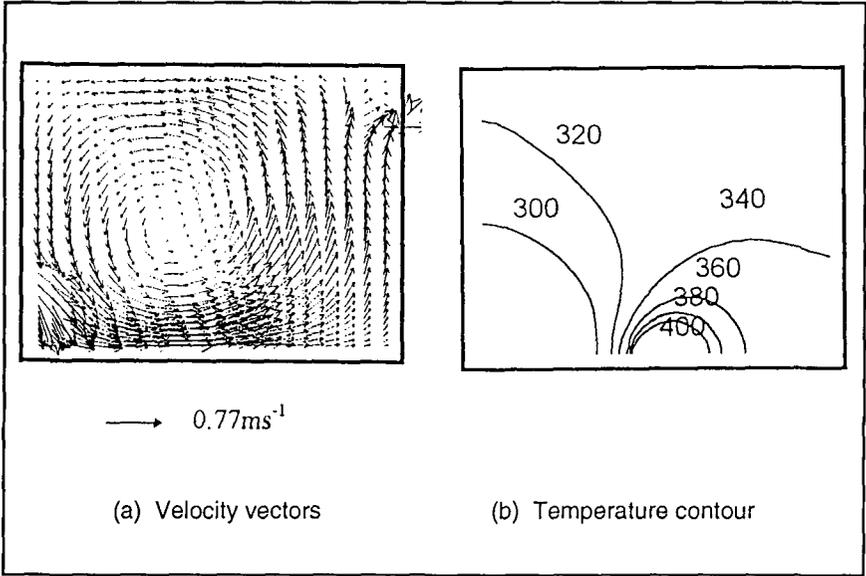


Figure 7. Results for case D.

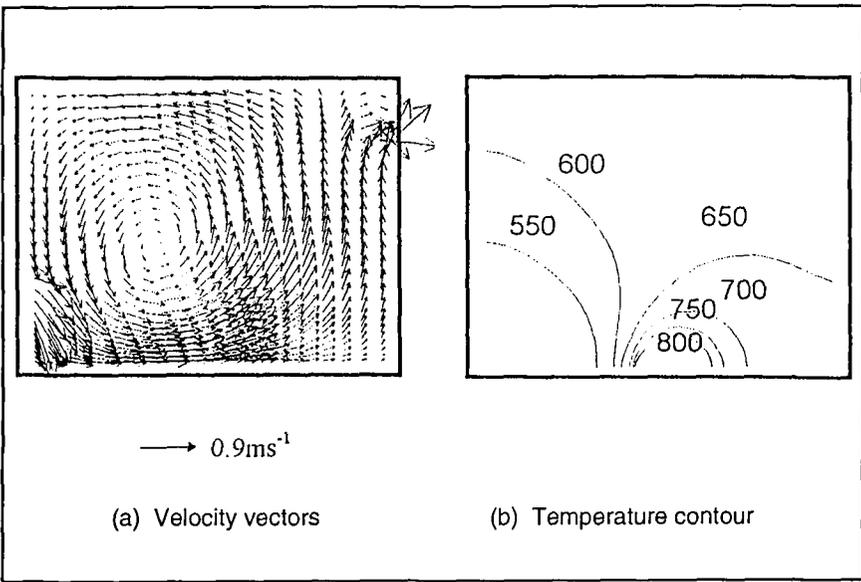


Figure 8. Results for case E.

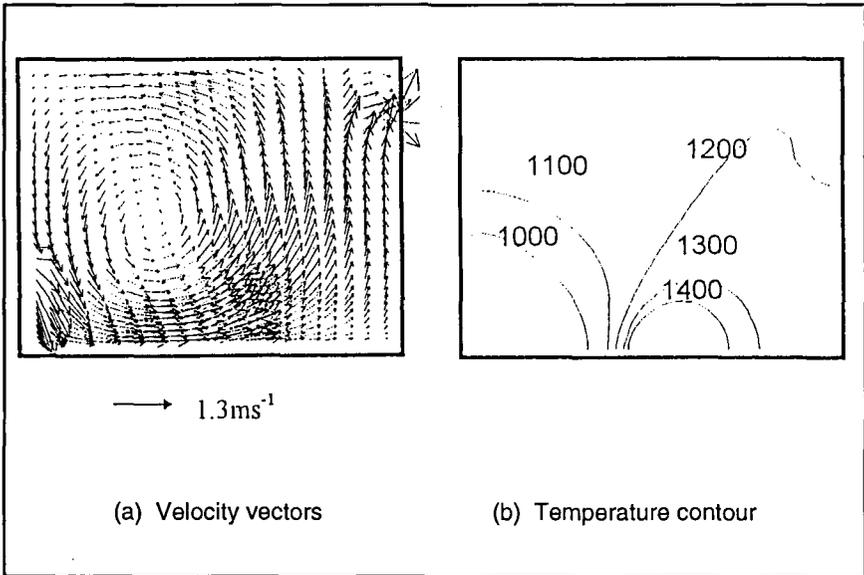


Figure 9. Results for case F.

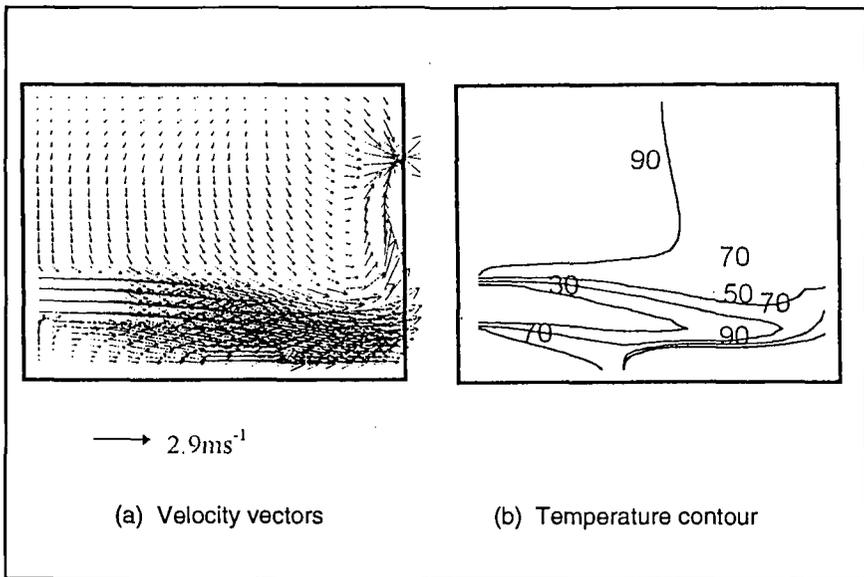


Figure 10. Results for case G.

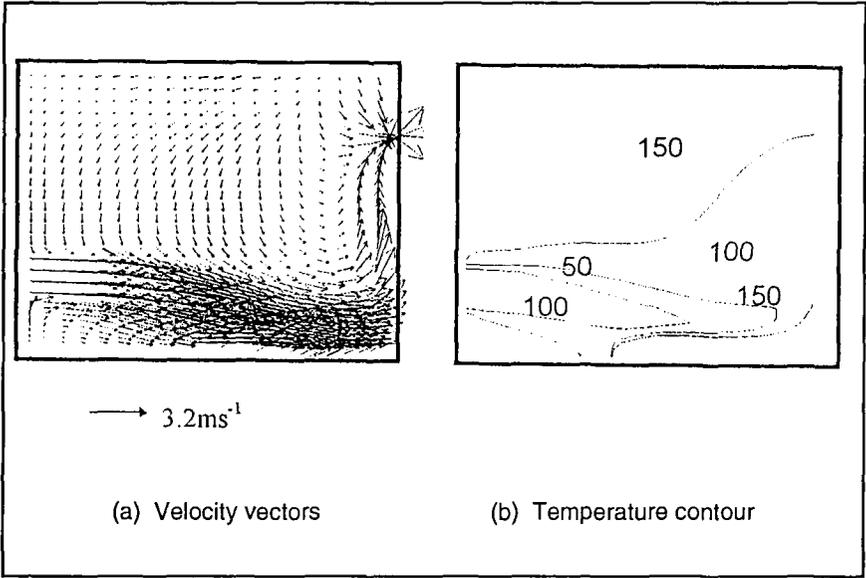


Figure 11. Results for case H.

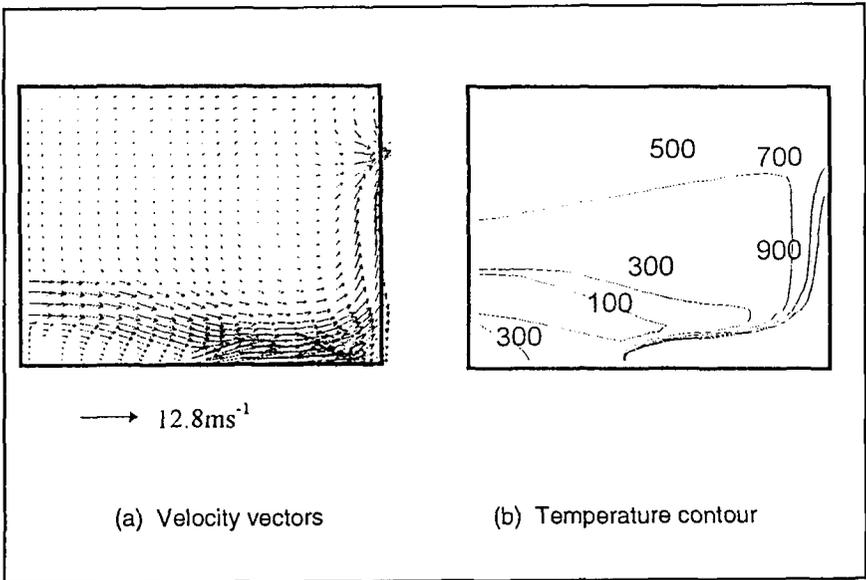


Figure 12. Results for case I.

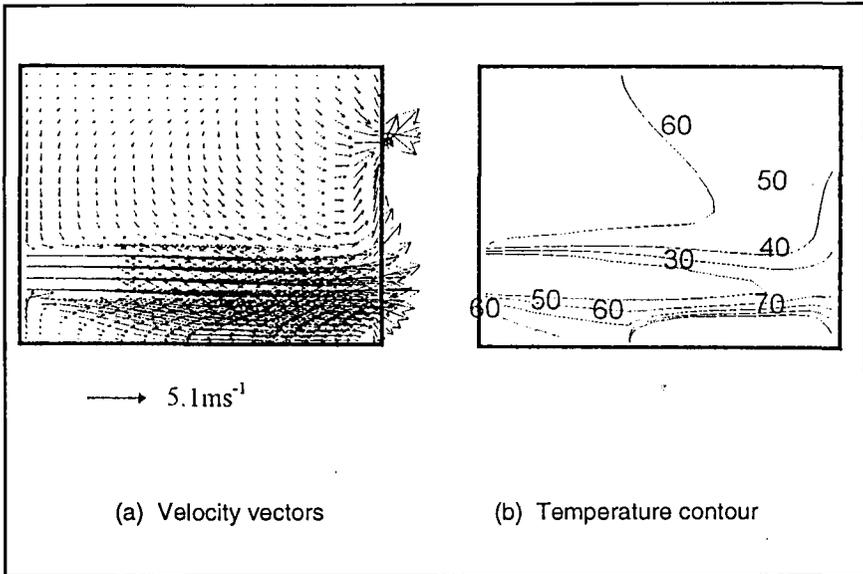


Figure 13. Results for case J.

Concerning values of T_L and T_p , good agreement with the values predicted by CFD can be obtained by selecting suitable values of α and β . These two parameters depend on the air flow pattern determined by the ventilation operating conditions. Comparison of the temperature pattern with the CFD results can also be made by assuming a value for the thickness of the hot layer. This cannot be calculated as mass transfer is not considered. An assumption can be made by taking the hot layer interface height to be just underneath the extraction fan, i.e., 1.0 m below the ceiling. Using case E as an example to illustrate the comparison of the predicted temperature, schematic diagrams of the temperature of all the twelve cases of different values of α and β are shown in Figure 14, with values of α from 0.7 to 0.9 and β from 0 to 0.3. The best match can be found by either comparing the temperature contour diagrams predicted by the CFD, or getting the mixing parameters α and β from the flow patterns predicted by the CFD [10].

5. CONCLUSIONS

A simple model is proposed to study the temperature induced by a heat source in a compartment with displacement ventilation. The hot gas temperature in a chamber with air intake at a lower position and air exhaust at higher position is predicted. A mixing factor as proposed by Sandberg can be defined [3]. Based on this concept, expressions for the hot layer temperature, cool layer temperature, and

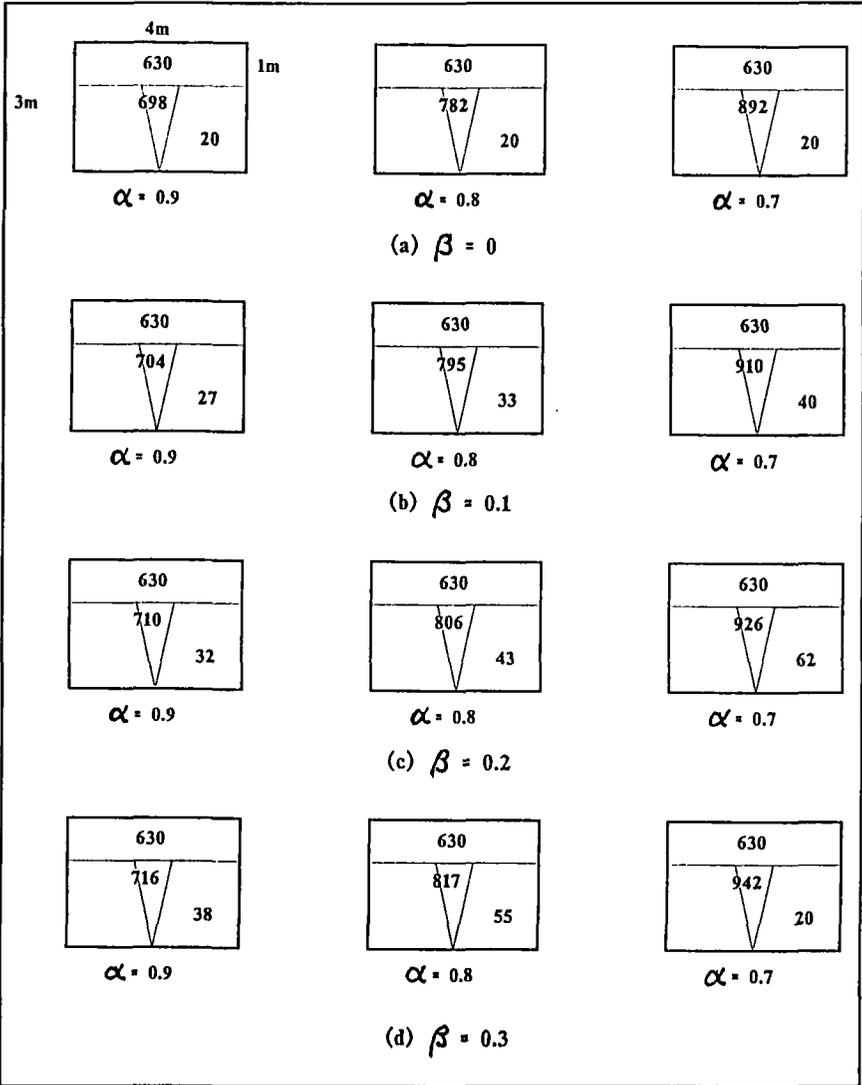


Figure 14. Thermal environments simulation for case E.

the plume temperature are derived for cases with and without considering heat and mass transfer in the plume region. The choice of the mixing factor is important and this mixing factor depends on the ventilation rates, locations of the air intake and exhaust, and the thermal power of the heat source. Numerical experiments with a CFD model [8, 9] were performed in a chamber with displacement ventilation to confirm the predicted results. Fairly good agreements were

obtained. This supports the argument that the choice of mixing factor is important. However, this approach deals with heat transfer only, and so the thickness of the hot layer cannot be determined. The location of the exhaust fan, to some extent, would affect the width of the hot layer.

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