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ESTIMATE OF SOIL PERMEABILITY AND POROSITY USING AN ANALYTICAL SOLUTION CONSIDERING GAS COMPRESSIBILITY

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ABSTRACT

An analytical solution considering gas compressibility is developed for radial gas flow in the vadose zone. Simulation results indicate that a solution not considering the gas compressibility provides a close approximation only when the change in pressure is less than 0.2 atmosphere. Error increases with the increase of pressure variation, and the error can reach about 20 percent when the pressure variation is 0.5 atmosphere. This article also presents two methods for the estimate of the soil parameter values from pneumatic test data using the developed analytical solution. The first method is a computer automatic fitting procedure using time-pressure data from a single observation well, and the second method requires gas pressure data from two observation wells and steady-state gas flow. Numerical experimental examples show that the inversely estimated permeability and porosity values considering the gas compressibility are different from these estimated not considering the gas compressibility.

1. INTRODUCTION

Soil vapor extraction (SVE) is a cost-effective technique for the cleanup of soils contaminated by volatile and semi-volatile compounds. Hydrocarbons are one such contaminant source. A basic soil vapor extraction system consists of vapor extraction wells, a pump, and a vapor treatment facility on the ground surface. Vapor is extracted out of the contaminated zone, which is replenished by air. The addition of air to the contaminated zone dilutes the vapor concentration, increases

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the volatilization rate of contaminants in the soils, and enhances the aerobic biodegradation rate.

Analytical solutions for gas flow in the vadose zone are simple but useful. They can be used: 1) to analyze pneumatic test data to derive soil parameter values, 2) to estimate the radius of influence of a vapor extraction well, 3) to estimate the gas flow velocity and travel time, and 4) to verify a numerical model. Soil permeability and porosity estimated from pneumatic test data are useful parameters in the design and evaluation of an SVE system. For example, soil permeability is used to determine the radius of influence and porosity for the calculation of gas flow velocity, which is an important variable in the estimate of vapor concentration around an extraction well.

Because the gas is compressible, equations governing gas flows are nonlinear. Modification or linearization is often applied to the initial governing equations prior to the development of an analytical solution. Johnson et al. [1] linearized the gas flow equation and adapted the Theis solution [2] for the transient gas flow in a nonleaky confined zone. This solution was given for a linearized transient gas flow equation which actually neglects the compressibility of a gas flow. It was applied by [1] in the analysis of an SVE system. The solution provided by [1] can be used in the estimation of soil permeability (k) and porosity (ε), but using this solution may result in some error when gas compressibility considered. Baehr and Hult [3], Baehr and Joss [4] and Shan et al. [5] developed analytical solutions for steady-state gas flow over a vertical profile considering the gas compressibility. The analytical solutions for steady-state gas flow, however, do not provide information about soil porosity.

The objectives of this study are: 1) to develop an analytical solution for transient gas flow considering the gas compressibility, and 2) to provide the techniques of using the developed analytical solution in the determination of soil permeability and porosity from pneumatic test data. This article is organized in the following format. First, it will present the equation governing compressible gas flow. It will then develop an analytical solution for the transient gas flow considering gas compressibility. Results from the developed solution will be compared with ones from the solution not considering the gas compressibility [1]. Finally, this article presents two techniques in the derivation of soil permeability and porosity values from pneumatic test data.

2. GOVERNING EQUATION

The transient gas flow equation for compressible gas flow in radial coordinates can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{k}{\mu}r\frac{\partial p^2}{\partial r}\right)=\frac{\varepsilon}{p}\frac{\partial p^2}{\partial t} \tag{1}$$

where

k = soil gas permeability along the r direction, $[L^2]$ or [darcy], (1 darcy = 10^{-8} cm^2);
p = gas pressure $[M/L\text{-}T^2]$ or [atm] (1 atmosphere = 1.013×10^6 g/cm-s^2);
μ = gas viscosity [M/L-T];
ε = gas-filled porosity of soil; and
t = time [T]

It is assumed that gas flow in porous media follows Darcy's law and the ideal gas law. Pore water is assumed to be immovable with gas flow and the Klinkenberg effects [3] are negligible. Equation (1) is nonlinear. The nonlinearity causes difficulty in the development of analytical solutions. Linearization, however, will make the equation solvable.

Johnson et al. [1] provided an approximate analytical solution to equation (1) such that

$$p = p_{atm} - \frac{Q\mu}{4\pi bk} W(u) \tag{2}$$

where Q = gas injection or extraction rate $[L^3/T]$; p_{atm} = ambient pressure (1 atmosphere); b = thickness of a confined vadose zone [L]; $W(u)$ is the well function and is defined as

$$W(u) = \int_u^\infty \frac{1}{y} e^{-y} dy; \tag{3}$$

and u is defined as

$$u = \frac{r^2 \varepsilon \mu}{4kp_{atm}t} \tag{4}$$

The solution described by equation (2) is identical in form to the Theis solution for transient groundwater flow. Equation (2) is a solution limited to the condition where the gas compressibility is negligible, for example, where the gas pressure drawdown is less than 0.2 atmosphere. When the gas pressure drawdown is greater, this solution can cause significant error. Johnson et al. used this solution to analyze the parameters of a vapor extraction system without mentioning this limitation [1].

3. SOLUTIONS CONSIDERING GAS COMPRESSIBILITY

Kidder [6] indicated that the coefficient of the time derivative term in equation (1) can be approximated by $\frac{\varepsilon}{p_{atm}}$. Equation (1) can, therefore, be linearized into the form of

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{k}{\mu}r\frac{\partial p^2}{\partial r}\right)=\frac{\varepsilon}{p_{atm}}\frac{\partial p^2}{\partial t} \tag{5}$$

Using a dummy variable H to represent p^2, a solution considering the gas compressibility for the following boundary conditions

$$H(r,t)=H_0=(p_{atm})^2,\ t>0,\ \text{and}\ r\to\infty$$

$$H(r,t)=H_0=(p_{atm})^2,\ t=0,\ \text{and}\ r>0$$

$$\lim_{r\to 0}\left(r\frac{\partial H}{\partial r}\right)=\frac{Q\mu p_{atm}}{\pi bk},\ t>0 \tag{6}$$

can be derived for equation (5) as follows

$$H=H_0-\frac{Q\mu p_{atm}}{2\pi bk}W(u) \tag{7}$$

That is

$$p^2=(p_{atm})^2-\frac{Q\mu p_{atm}}{2\pi bk}W(u) \tag{8}$$

The boundary condition described by equation (6) represents the gas flux to the well and it can be derived from the assumption that the vapor behaves as an ideal gas [1], so that

$$\rho=\rho_{atm}\frac{p}{p_{atm}} \tag{9}$$

where ρ_{atm} = gas density at the reference pressure p_{atm}; and ρ = gas density at pressure p. Equation (8) is similar in form to equation (2). But notice that the coefficient of the well function in equation (8) is $\frac{Q\mu p_{atm}}{2\pi bk}$ instead of $\frac{Q\mu}{4\pi bk}$ of equation (2). When u is less than 0.01 equation (8) can be approximated by

$$p^2=(p_{atm})^2-\frac{Q\mu p_{atm}}{2\pi bk}(-0.577216-\ln(u)) \tag{10}$$

Simulations are performed to compare the results from equations (2) and (8). Table 1 summarizes the parameter values for the simulation. The results from both equations are plotted in Figure 1. These results are the pressures at a distance of 10 feet from an extraction well. The difference between the two solutions is small when the pressure p is greater than 0.8 atm (Figure 1). When the time increases, as pressure drawdown p′ does ($p' = p_{atm} - p$), the results from the equation (8) deflect away from the results of equation (2). At a time of 360 seconds, the difference is trivial, but it reaches nearly 20 percent at a time of

Table 1. Parameter Values Used for the Simulation of Equations (2) and (8)

$P_{atm} = 1.013 \times 10^6$ g/cm-s^2
$\mu = 1.8 \times 10^{-4}$ g/cm-s
r = 10 feet = 304.8 cm
k = 10 darcy = 10^{-7} cm^2
$\varepsilon = 0.4$
b = 20 feet = 609.6 cm
Q = 500 scfm = 235,974 cm^3/s

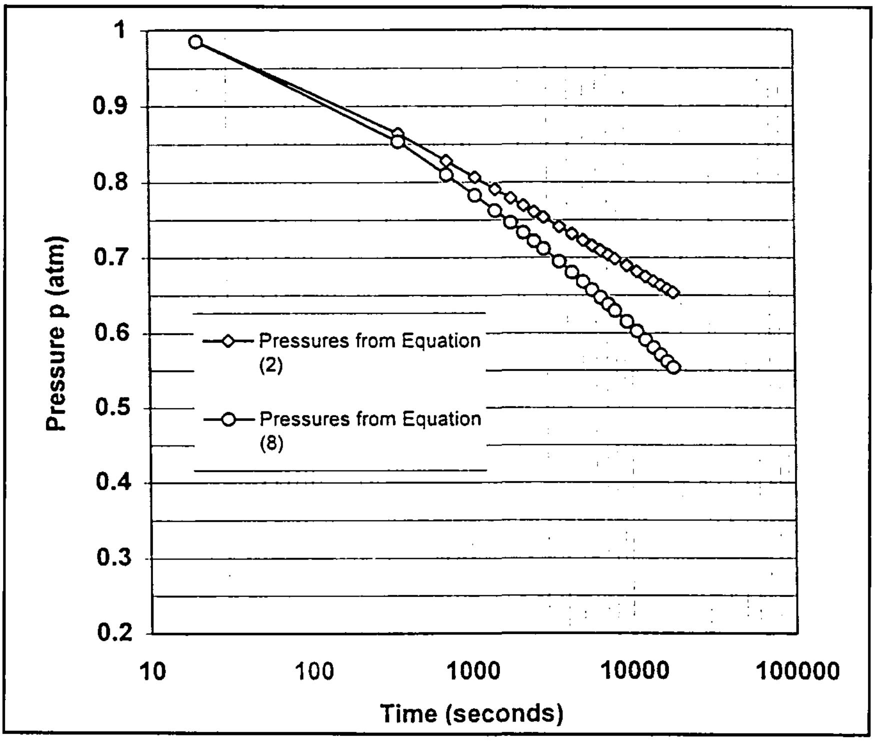

Figure 1. Transient pressure profiles at a distance of 10 feet from the well. Results from equation (8) consider the gas compressibility and results from equation (2) do not consider the gas compressibility.

16,000 seconds when the p is about 0.56 atmosphere. Notice that the pressure profile of solution (8) is not a straight line but curved.

This simulation example clearly indicates that a difference between equations (2) and (8) exits in the calculation of gas pressures. For an inverse problem, i.e., in deriving permeability (k) and porosity (ε) values from pneumatic test data, a difference is also expected between these two solutions. We will develop algorithms based on equation (8) for identification of k and ε values.

4. METHODS FOR ANALYSIS OF PNEUMATIC TEST DATA

Pneumatic tests can provide useful information about in situ soil properties, including soil permeability and air-filled porosity, which are crucial in the design of a vapor extraction system. This section will discuss two methods to estimate these parameter values from pneumatic test data based on equation (8).

4.1 Estimation of Soil Properties (k and ε) Using One Observation Well

Graphic procedure is a commonly used method to estimate the aquifer properties from pumping-test data [7, 8]. A graphic procedure can be easily adapted for the estimate of soil parameter values through equation (8). However, using a graphic method is subject to the expertise of a hydrogeologist and therefore to the judgment of the analyst. McElwee [9], Chander et al. [10], and Yeh [11] presented techniques to automatically fit pumping-test data to the Theis equation by obtaining the "best" estimate of transmissivity and storage coefficients. The method of nonlinear least squares was used for each of the techniques [9-11]. Chander, Kapoor, and Goyal used the Marquardt algorithm to analyze pumping-test data [10]. This algorithm can be extended for the automatic analysis of pneumatic test data.

Using s to represent the difference of $(p_{atm})^2 - p^2$, equation (8) can be written as

$$s = \frac{Q\mu p_{atm}}{2\pi bk} W(u) \tag{11}$$

Assume that there are N observations of s in a pneumatic test and s_i^0 is the observation at time t_i. We use the staring values of variables k^* and ε^* to compute the theoretical solution s_i^* at time t_i using equation (11). Varying the parameters k and ε by small amount Δk and $\Delta\varepsilon$, respectively, the new solution of s at time t_i can be approximated using Taylor series [12] such that

$$s_i = s_i^* + \frac{\partial s_i^*}{\partial k^*} \Delta k + \frac{\partial s_i^*}{\partial \varepsilon^*} \Delta\varepsilon \tag{12}$$

Minimizing the error between observed and theoretically calculated s by taking derivatives with respect to k and ε, respectively, from the following function

$$\text{error} = \sum_{i=1}^{N} (s_i^0 - s_i)^2 \tag{13}$$

yields a pair of algebraic equation as follows

$$\sum_{i=1}^{N}\left[\left(\frac{\partial s_i^*}{\partial k^*}\right)^2 \Delta k + \left(\frac{\partial s_i^*}{\partial k^*}\right)\left(\frac{\partial s_i^*}{\partial \varepsilon^*}\right)\Delta\varepsilon\right] = \sum_{i=1}^{N} (s_i^0 - s_i^*)\left(\frac{\partial s_i^*}{\partial k^*}\right) \tag{14}$$

and

$$\sum_{i=1}^{N}\left[\left(\frac{\partial s_i^*}{\partial k^*}\right)\left(\frac{\partial s_i^*}{\partial \varepsilon^*}\right)\Delta k + \left(\frac{\partial s_i^*}{\partial \varepsilon^*}\right)^2 \Delta\varepsilon\right] = \sum_{i=1}^{N} (s_i^0 - s_i^*)\left(\frac{\partial s_i^*}{\partial \varepsilon^*}\right) \tag{15}$$

The derivatives in equations (14) and (15) are derived from equation (11) and are expressed as

$$\frac{\partial s_i^*}{\partial k^*} = \frac{Q\mu p_{atm}}{2\pi k^{*2} b}\left[-W(u_i) + \exp(-u_i)\right] \tag{16}$$

and

$$\frac{\partial s_i^*}{\partial \varepsilon^*} = \frac{-Q\mu p_{atm}}{2\pi b k^* \varepsilon^*}\exp(-u_i) \tag{17}$$

where $u_i = \dfrac{r^2\mu\varepsilon^*}{4k^* p_{atm} t_i}$.

Δk and Δε are computed by solving equations (14) and (15) and the estimated k and ε values are

$$k^{*i+1} = k^{*i} + \Delta k \tag{18}$$

and

$$\varepsilon^{*i+1} = \varepsilon^{*i} + \Delta\varepsilon \tag{19}$$

k^{*i+1} and ε^{*i+1} are the values calculated in the current iteration and k^{*i} and ε^{*i} are the values from the previous iteration. After a number of iterations, Δk and Δε diminish and k and ε are estimated. A computer program has been written to perform the calculation of k and ε values.

Numerical experimental examples are used to compare the estimated permeability (k) and porosity (ε) values using equations (2) and (8). Figures 2 and 3 show the k and ε values, respectively, inversely calculated based on the two solutions. As indicated by Figure 2, the estimated k values using equation (2) is

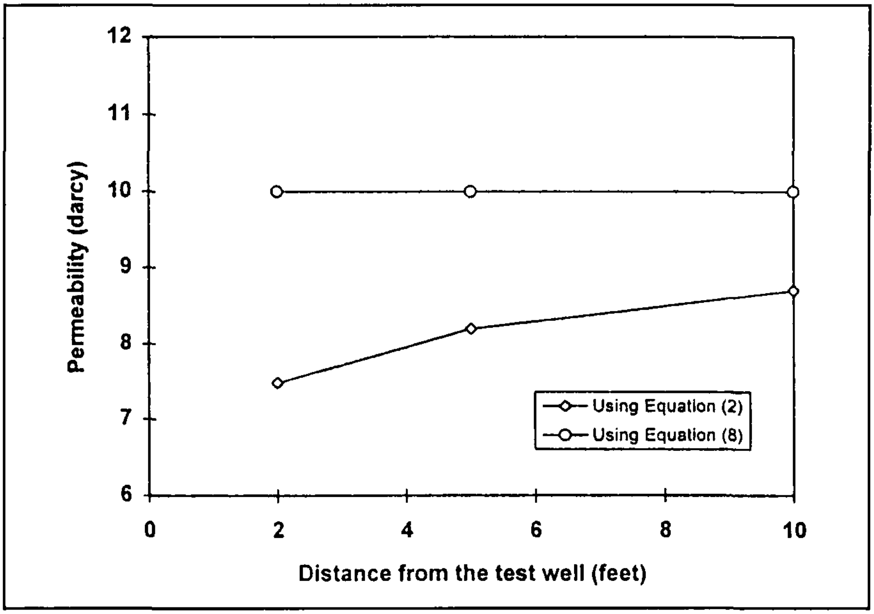

Figure 2. Estimated permeability values using equations (2) and (8).

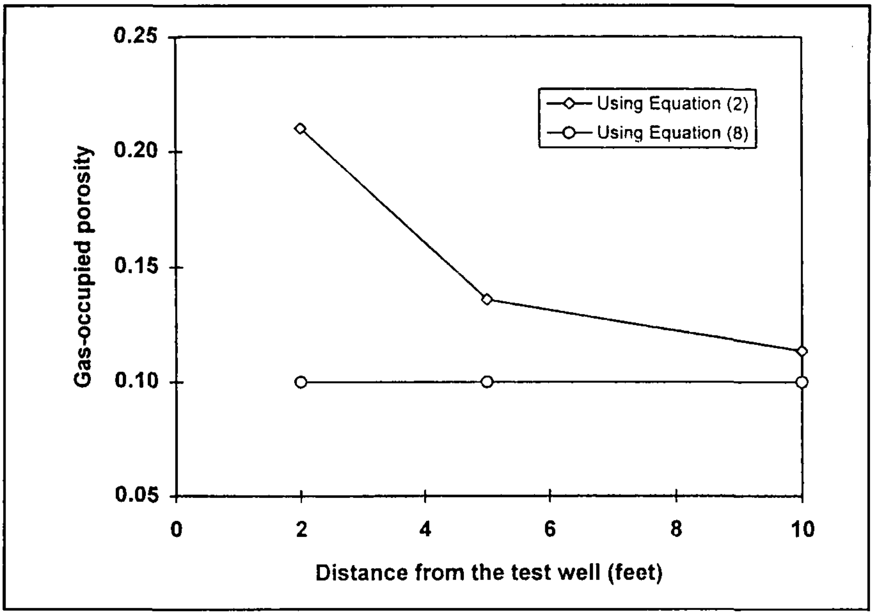

Figure 3. Estimated porosity values using equations (2) and (8).

less than the values estimated using equation (8), while the estimated ε values using equation (2) is greater than the values estimated using equation (8) as indicated by Figure 3.

4.2 Estimation of Soil Properties (k and ε) Using Two Observation Wells

Gas flow in the field can approach a steady-state condition in a relatively short time. For example, a nearly steady-state gas flow condition at the extraction and observation wells can often be observed in less than an hour for highly permeable soils during a pneumatic test. Figure 4 shows an air pressure profile collected from an observation well 20 feet from an injection well. This is a multiple-step air injection test, conducted in the Southern San Joaquin Valley, California, by the author in early 1994. As indicated in Figure 4, a steady-state gas flow condition for each injection rate was established in less than thirty minutes.

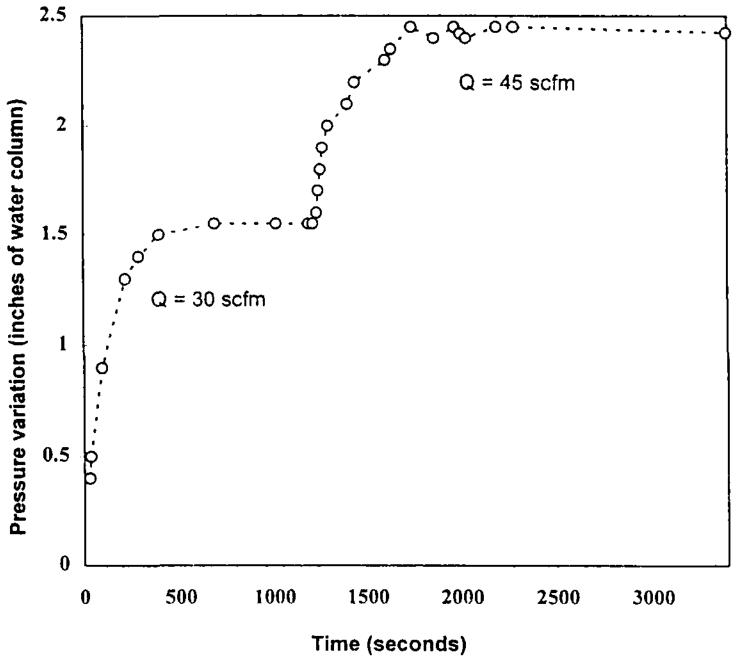

Figure 4. Pressure data collected from an observation well 20 feet from the air injection well.

The solution for the steady-state condition for equation (1) is

$$s = \frac{Q\mu p_{atm}}{\pi bk} \ln\left(\frac{r_e}{r}\right) \tag{20}$$

where b = thickness of the vadose zone [L]; r = radial distance from the gas extraction or injection well in radial coordinates [L]; r_e is the distance where the pressure is equal to the initial reference pressure p_{atm} (1 atmosphere) and is called the radius of influence in this article; and $s = (p_{atm})^2 - p^2$.

Assume that two observation wells are located at distance r_1 and r_2 from the vacuum well, respectively, and the measured pressure variations under a steady-state condition are $s_1 = (p_{atm})^2 - p_1^2$ for well 1 and $s_2 = (p_{atm})^2 - p_2^2$ for well 2. Substituting the collected data r_1, r_2, s_1, and s_2 into equation (20), we are able to obtain the following equation for the estimation of k value

$$k = \frac{Q\mu p_{atm}}{\pi b(p_1^2 - p_2^2)} \ln\left(\frac{r_1}{r_2}\right) \tag{21}$$

After the k value is estimated, air filled porosity ε can be calculated using

$$\ln(\varepsilon) = -s\frac{2\pi bk}{Q\mu p_{atm}} - \ln\left(\frac{1.781r^2\mu}{4kp_{atm}t}\right) \tag{22}$$

for $u < 0.01$, where t is the time when pressure p is measured. Otherwise, ε can be calculated from equation (11).

Organizing equation (11) in form of

$$f(\varepsilon) = \frac{2\pi bks}{Q\mu p_{atm}} - W(u) \tag{23}$$

allows us to use Newton's method [13] to find ε from the following iterative equation

$$\varepsilon_{i+1} = \varepsilon_i - \frac{f(\varepsilon_i)}{f'(\varepsilon_i)} \tag{24}$$

where $f'(\varepsilon_i) = e^{-u_i}\frac{1}{u_i}$. Well function W(u) can be calculated using approximate polynomials [14] or using Simpson's rule to evaluate the integral [13].

Soil permeability (k) is the function of porous media only, while the hydraulic conductivity (K) is the function of both media and fluid properties. The relationship between K and k is given by [15]:

$$K = \frac{\rho gk}{\mu} \tag{25}$$

By giving proper values of the fluid density and viscosity, and the value of soil permeability k, the hydraulic conductivity K value can be calculated using equation (25).

5. SUMMARY AND CONCLUSIONS

Simulation results indicate that the solution for incompressible gas provides a close approximation for gas flow when pressure drawdown is less than 0.2 atmosphere. The result from the linear differential equation deflects from the squared pressure equation when the pressure variation is greater than 0.2 atmosphere. The difference between these two solutions can reach about 20 percent at the pressure variation of 0.5 atm.

Although pressure drawdown may be less than 0.2 atm in most of the area within the radius of influence r_e, it can be often greater than 0.2 atmosphere around the extraction or injection well. The pressure around the well head is an important parameter in the sizing of a vacuum or a blower. In that case, solutions considering the compressibility of gas flow should be used.

A nonlinear least-square method combining the Taylor series is used for the analysis of pneumatic test data. This algorithm allows us to automatically fit the observation data to the developed analytical solution [equation (8)] to determine the k and ε values. When pressure data are collected from two observation wells and gas flow reaches steady-state condition, an alternative method described by equations (21), (22) or (24) can be used to estimate the k and ε values. Numerical experimental examples show that the inversely estimated permeability and porosity values using equation (8) are different from these estimated using equation (2).

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