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ANALYZING ACTIVATED SLUDGE PROCESS PERFORMANCE DATA

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ABSTRACT

Because of their important impacts on the environment, the biochemical oxygen demand (BOD) and total suspended solids (TSS) concentrations in effluent streams are critical performance measures for wastewater treatment plants. Wastewater effluent standards often specify the concentration limit for each of these pollutants. Most often, the reported BOD of treatment plant effluent is the sum of the residual unassimilated dissolved BOD and the BOD embedded in escaping biological solids of the clarifier effluent. To meet this effluent standard, a treatment plant must have adequate aeration for substrate removal and sludge formation suitable for separation. This article proposes a method to analyze the activated sludge treatment performance data for the estimation of dissolved BOD and BOD embedded in TSS. The estimation is useful for better control of operation and for the mathematical modeling, by Monod's reaction formulation, that governs the relation between the substrate concentration (S) and bacteria concentration (X). In the secondary clarifier effluent, S is the dissolved BOD while X represents the escaping biological solids or TSS. The proposed method applies the log-linear regression to derive a regression of dissolved BOD on TSS. This basic model is then utilized in conjunction with bootstrap to estimate the dissolved BOD and the BOD embedded in TSS. The analysis of a set of sixty-seven secondary biological municipal wastewater treatment plants yields an estimate of 2.65 mg/ℓ with standard deviation of 1.18 mg/ℓ for dissolved BOD and 0.605 with standard deviation of 0.083 for the ratio of BOD and TSS.

1. INTRODUCTION

National wastewater effluent standards often specify the concentration limit for the BOD (biochemical oxygen demand) and TSS (total suspended solids). The

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removal of these "pollutants" is the primary performance measure for activated sludge treatment process. These two parameters relate to each other by the fact that a treatment plant is to facilitate the conversion of organic matters measured by BOD to biomass measured by TSS. An ideal aeration basin produces low dissolved BOD and settleable sludges. In practice, a treatment plant is always subject to the vagaries of natural environment, and performs far from ideally. In an effluent system, BOD reflects organic materials in two states. They are the residual unassimilated BOD in the reactor effluent and the BOD contained in escaping biological solids of the secondary clarifier effluent. Because of its cost, measurements are seldom made for these two species separately. This study proposes a method to estimate these constituent BODs by utilizing reported plant operating data.

2. MATHEMATICAL DESCRIPTION OF THE ACTIVATED SLUDGE PROCESS

Monod's reaction formulation provides a useful quantitative description of the assimilation taking place in an activated sludge process [1, 2].

$$\frac{dS}{dt} = -kXS/(K_S + S) \tag{1}$$

where S = the substrate concentration (BOD of dissolved wastes), mg/ℓ
X = concentration of volatile suspended solids in mixed liquor, mg/ℓ
k = maximum growth rate of microorganisms, day^{-1}
K_S = "half-velocity" constant, mg/ℓ

The net rate of change of the sludge biomass is equal to the rate of reproduction of organisms minus the decay rate:

$$\frac{dX}{dt} = -a\frac{dS}{dt} - bX \tag{2}$$

where a = the yield parameter (mass of bacteria produced per unit mass of substrate assimilated)
b = the organism decay parameter, day^{-1}

For a plant with design flow Q mgd and an aeration basin capacity V, in megagallons, equations (1) and (2) can be integrated simultaneously to estimate S and X. Alternatively, equation (1) may be rewritten as follows:

$$Q(S_0 - S) = kXVS / (K_S + S)$$

$$\text{or} \quad S = S_0 - kXTS / (K_S + S) \tag{3}$$

where T is the aeration time (days). Equation (3) may be solved for S as follows:

$$S = \frac{1}{2}[A + \sqrt{A^2 + 4k_S S_0}] \tag{4}$$

where $A = kXT + K_S - S_0$ (4a)

and equation (2) may be written as:

$$X = X_0 - a(S - S_0) - bT(X + X_0)/2 \tag{2a}$$

Equations (2a) and (4) are applicable to the "complete mix" type of activated sludge process in which return sludge and influent wastewater are rapidly dispersed to uniform concentration through the aeration basin. However, most of the plants in the field are not of this type. They fall in the zone between plug flow and complete mixing. To model the variations of mixing condition, one can assume the reactor consists of n compartments or cells, each with complete mixing. Apply equations (2a) and (4) to each cell for a total of 2n equations. Simultaneous solution of equations (2a) and (4) with given parameters value allows calculation of the concentrations of BOD and biological solids. Theoretically, n may vary from 1 for instantaneous complete mixing to infinity for pure plug flow. In practice, values of 2 to 8 can adequately model the mixing characteristics of most plants [3, 4]. Parameter (a,b,k and K_S) values are often estimated by pilot plant studies. Reported literature values [5] are very wide in range and can differ significantly from working values in actual treatment plants. Thus treatment performance (BOD and TSS) can also differ greatly from the design value. To verify the parameter values and the entire modeling approach, the actual data is compared with the assumed (and calculated) values. If observed BOD and TSS are not consistent with those simulated, one can adjust parameter values for better fitting.

3. DATABASE

To develop the estimation method, performance data for sixty-seven actual plants were compiled from three published studies [6-8] of activated sludge processes. Table 1 shows average daily data pertaining to BOD_5 and TSS observed in the same time period. Figure 1 depicts the same data. Since effluent limitations and discharge permits usually pertain to monthly average (or 30-day running averages) in the United States, it will be useful to develop an approximate statistical relationship between the daily and maximum monthly parameters. Table 2 contains the daily and maximum monthly data for nine secondary plants of the investigation by Roper et al. [8]. The median value of the variability factor, Med(y/x) for BOD is 1.98 and for TSS is 1.94. From these data it cannot determine whether the two variability factors differ significantly. Prior to adoption of 30/30 definition of secondary treatment based on maximum monthly averages, effluent quality control for seasonal and "wet weather" variation was less widely practiced and a considerable variation in the y/x ratio occurred from plant to plant. The stringent 30/30 requirement appears to have put pressure

Table 1. Effluent Characteristics: 67 Activated Sludge Plants (in mg/ℓ)

no.	BOD_5	TSS	from[a]	no.	BOD_5	TSS	from[a]	no.	BOD_5	TSS	from[a]
1	14	24	(1)	24	8	14	(1)	47	9	9	(2)
2	16	22	(1)	25	8	9	(1)	48	5	7	(2)
3	11	26	(1)	26	4	5	(1)	49	6	12	(2)
4	36	42	(1)	27	12	14	(1)	50	18	24	(2)
5	48	39	(1)	28	17	27	(1)	51	5	9	(2)
6	5	4	(1)	29	6	18	(1)	52	9	17	(2)
7	5	18	(1)	30	8	11	(1)	53	11	6	(2)
8	27	24	(1)	31	9	11	(1)	54	9	12	(2)
9	22	43	(1)	32	6	12	(1)	55	15	22	(2)
10	9	8	(1)	33	13	16	(1)	56	10	10	(2)
11	9	16	(1)	34	6	11	(1)	57	16	23	(2)
12	17	19	(1)	35	9	3	(1)	58	10	23	(2)
13	14	18	(1)	36	8	9	(1)	59	7	8	(2)
14	15	20	(1)	37	8	8	(1)	60	8	18	(3)
15	15	16	(1)	38	13	16	(2)	61	8	16	(3)
16	10	14	(1)	39	19	18	(2)	62	22	13	(3)
17	16	26	(1)	40	26	33	(2)	63	24	29	(3)
18	57	65	(1)	41	27	25	(2)	64	7	9	(3)
19	14	22	(1)	42	16	15	(2)	65	23	18	(3)
20	7	12	(1)	43	11	11	(2)	66	26	15	(3)
21	5	12	(1)	44	14	21	(2)	67	13	17	(3)
22	10	14	(1)	45	31	26	(2)				
23	6	7	(1)	46	17	12	(2)				

[a] Data source (1) = Reference [6]; (2) = Reference [7]; (3) = Reference [8].

on plant designers and operators to reduce both the average annual effluent concentration and seasonal peak concentration. An important design tradeoff exists between the cost of maintaining low annual averages and the cost of maintaining quality control by measures such as seasonal application of chemical coagulation. Log-linear regression of the daily mean BOD_5 of Table 3 on maximum thirty-day BOD_5 yields the equation:

$$x = 0.736y^{0.892} \qquad (r^2 = 0.91)$$

For a 30 mg/ℓ effluent limit the corresponding daily mean concentration is $x = 0.736(30)^{0.892} = 15.3$ mg/ℓ. Similar calculations for TSS yield:

$$x = 0.507y^{1.002} \qquad (r^2 = 0.58)$$

and with $y = 30$ mg/ℓ, $x = 0.507(30)^{1.002} = 15.3$ mg/ℓ. Figure 1 shows these levels dividing the data set into four categories with respect to compliance with effluent limits (assuming 30/30 as shown in Table 3.

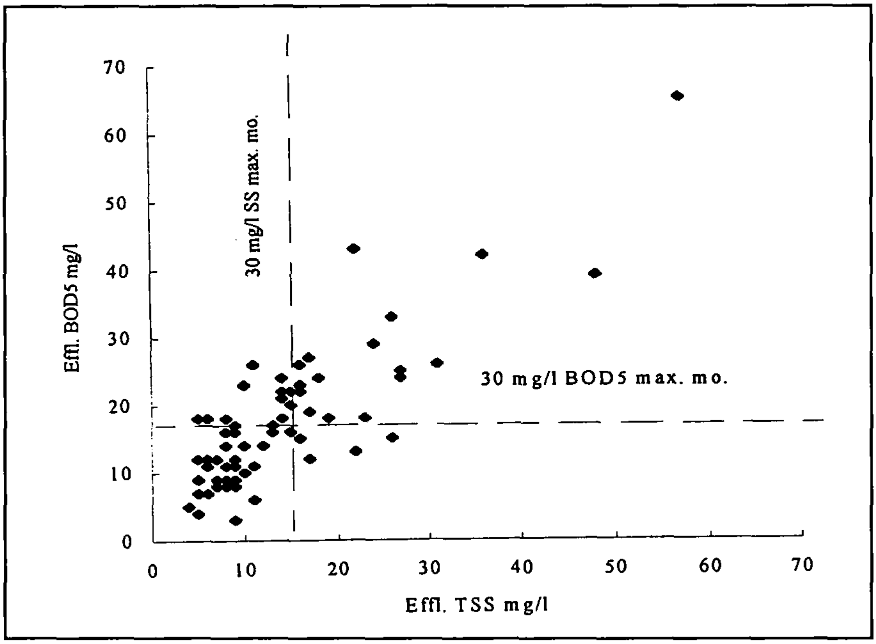

Figure 1. Sixty-seven activated sludge plants performance: BOD and TSS.

Table 2. Daily Average and Maximum Thirty-Day Average

	BOD5, mg/ℓ		TSS, mg/ℓ	
Plant No.	Daily Avg.	Mo. Max.	Daily Avg.	Mo. Max.
	x	y	x	y
1	7.6	14	18	35
2	8.4	17	15.5	26
3	21.6	47	13.8	30
4	23.6	35	28.5	49
5	7	12	9.2	24
6	23	61	18.2	43
7	25.8	42	14.9	35
8	10.1	20	21	31
9	13	28	16.6	27

Source: Reference [8].

Table 3. Compliance Status

Category	No. Plants	Percent
pass BOD, pass TSS	28	42
pass BOD, fail TSS	18	27
fail BOD, pass TSS	4	6
fail BOD, fail TSS	17	25
Total	67	100

With the variability factor of 30/15.3 = 1.96 for BOD and for TSS, only 42 percent of plants are in compliance. Forty-six plants (67%) passed the BOD test, but only thirty-two plants (46%) passed the TSS test. The stringency of TSS test during critical seasons is an important determinant of the marginal cost of treatment.

4. ESTIMATION OF DISSOLVED BOD

The BOD_5 of the plant effluent consists of two sources: 1) the residual unassimilated dissolved BOD in the reactor effluent and 2) the BOD_5 of the biological solids of the effluent:

$$y = S + \alpha X \tag{6}$$

where y = BOD_5 of plant effluent, mg/ℓ
S = BOD_5 of the residual dissolved organic substrate, mg/ℓ
X = TSS of plant effluent, mg/ℓ

The parameter α represents the BOD/TSS ratio of escaping solids. Simple regression yields $\alpha = 0.79$ and $S = 0.25$ mg/ℓ. The dissolved BOD of 0.25 mg/ℓ is too low for an activated sludge plant. Application of weighted least square [9] and major axis regression [10] yields estimates that are also implausible in reality. Thus, a simple transformation is exercised for further analysis. Log-linear regression of effluent BOD_5, y, on TSS, X yields:

$$y = 1.46X^{0.770} \qquad (r^2 = 0.555) \tag{7}$$

It may be noted that geometric means of BOD, Mg(y) and TSS, Mg(X), if plotted on Figure 1, appear in the center of the small subset of plants at the margin of compliance with the 30/30 effluent limitation. It is in this region also that the regression is most precisely determined. The equation of the tangent to the regression equation (7) is

$$y = M_g(y) + \frac{dy}{dX}[X - M_g(X)] = 2.71 + 0.60X \tag{8}$$

where $\frac{dy}{dX}$ is evaluated at the geometric mean point $M_g(X) = 15.07$ mg/ℓ and $M_g(y) = 11.80$ mg/ℓ. Thus regression yields an estimate of $\alpha = 0.60$ for plants at the margin of compliance. This analysis also yields a very low value of S (2.71 mg/ℓ) indicating that for these plants the process is close to its ultimate capability for treatment. Metcalf and Eddy [11] use $\alpha = 0.60$ and 0.65 while McGhee [12] uses a value of $\alpha = 0.65$ in analysis of activated sludge process design. Randtke and McCarty report a value of 1.9 mg/ℓ of dissolved BOD_5 in the effluent of a complete-mix plant. For the same plant in the study period, the effluent BOD and TSS averaged 12 mg/ℓ and 16 mg/ℓ respectively [13]. Using equation (6), a value of about 0.63 can be estimated for α. In view of favorable comparison of calculated results with literature values, the log-linear model is useful for the present study. To account for uncertainty inherent in estimating parameter values, sensitivity tests are desirable to delimit the range of uncertainty associated with estimation of dissolved BOD and the BOD included in TSS.

5. BOOTSTRAPPING

What does the sample distribution of $\hat{S}$ and $\hat{\alpha}$ look like? More specifically, what are their variances? One way to answer these questions is by bootstrapping [14, 15], which involves resampling the data from observed samples by Monte Carlo techniques. Two methods of bootstrapping are applied to generate synthetic BOD and TSS effluent concentration data for sets of samples of *n* = 67 plants. The first method (called random sampling of cases) resamples sixty-seven cases with replacement from original sample (Table 1) by regarding each observed case (X = BOD, y = TSS) has a probability of occurrence of 1/67. These sixty-seven cases make up a random sample. A log-linear regression equation is calculated for each set of sixty-seven pairs (X_i, y_i). That is, for sample j, a regression equation similar to equation (7), $y_{ij} = a_j(X_{ij})b_j$, is fitted. Tangents to the regression curve at point $M_g(X)$ and $M_g(y)$ are calculated as before: $y_{ij} = S_j + \alpha_j X_{ij}$ for estimates, $\hat{S}$ and $\hat{\alpha}$. In all, the method repeats 299 times. With observed sample included for a total 300 samples, estimated mean($\hat{S}$) = 2.65 mg/ℓ, std($\hat{S}$) = 1.16 mg/ℓ, mean($\hat{\alpha}$) = 0.605, and std($\hat{\alpha}$) = 0.081. Figure 2 and Figure 3 display the sample distribution of these estimates. The second method (called random sampling of residuals) generates synthetic data by random sampling from log-normal populations having the same moments and cross-correlation as the actual sample set. In algebraic form, generating functions for synthetic data are:

$$\ell n(X_i) = \mu_{\ell n(x)} + (\sigma_{\ell n(x)}) \cdot u_i = 2.7130 + 0.5600u_i \quad \text{for } i = 1, 2, ---, 67 \qquad (9)$$

$$\begin{aligned} \ell n(y_i) &= \mu_{\ell n(y)} + \rho * \sigma_{\ell n(y)} (u_i + c * v_i) \\ &= 2.4685 + 0.4280\, u_i + 0.3834 v_i \qquad \text{for } i = 1,2,\ldots, 67 \qquad (10) \end{aligned}$$

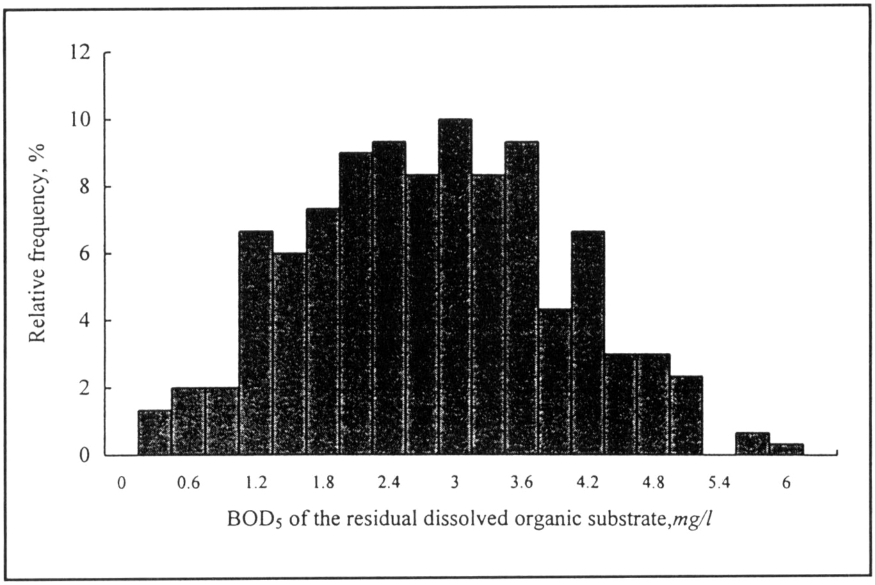

Figure 2. The frequency distribution of dissolved BOD_5.

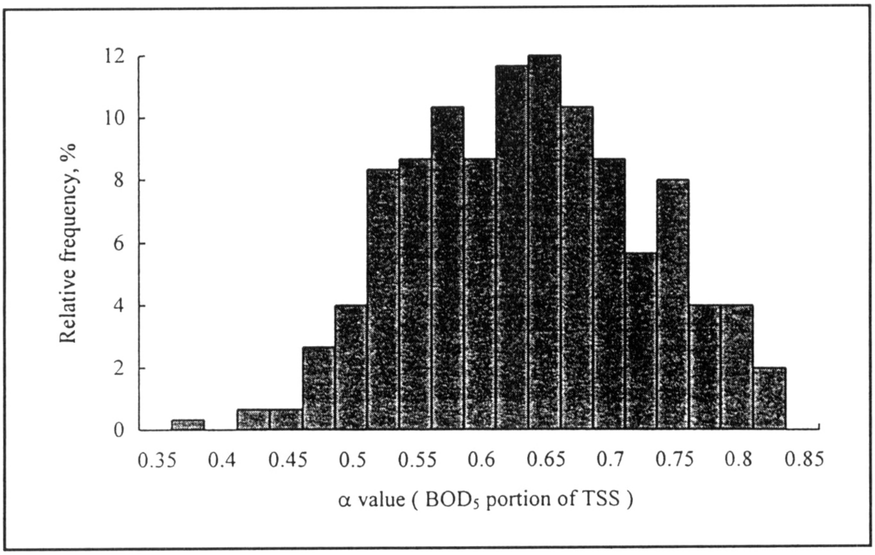

Figure 3. The frequency distribution of α Value.

Here μ and σ stand for sample mean and standard deviation; ρ is the correlation coefficient of $\ell n(x)$ and $\ell n(y)$; $c = [(1/\rho^2) - 1]^{0.5}$; u_i and v_i are independent standard normal deviates. For each sample sixty-seven computer generated random normal deviates, u_i and v_i are substituted in equations (9) and (10) to yield sixty-seven pairs (X_i, y_i). Each sample is then treated by equations (7) and (8) as in the first method. For 300 samples (including the actual observation), mean($\hat{S}$) = 2.66 mg/ℓ, std($\hat{S}$) = 1.21 mg/ℓ, mean($\hat{\alpha}$) = 0.606, and std($\hat{\alpha}$) = 0.086. Figure 2 and Figure 3 display the sample distribution of these estimates. These results compare well with those of the actual sample; $S = 2.71$ vs. 2.66 and $\alpha = 0.60$ vs. 0.61.

6. SUMMARY

In order to meet stringent effluent limitations, a wastewater treatment designer will take account of all possibilities for attaining the objective. These include not only the treatment capability and balance of the major components ("core") of the plant but also the utility of several "add-on" processes, structures, controls and equipment of the "augmented" plant needed for site-specific conditions. In deciding whether each of these should be incorporated in the design, the primary consideration is their cost-effectiveness relative to cost of increase in treatment efficiency of the core plant. In the balancing of tradeoffs it is seldom possible to estimate cost-effectiveness of alternative designs with precision. Engineering design decisions must account for many factors that may be difficult to evaluate, such as reliability of design data, projections of flow, composition and strength of wastewater from different sources (sampling frequency and procedure, etc.), significance of laboratory and pilot plant studies to ascertain sludge characteristics, the skill and experience of future operators and the quality of maintenance. As may be seen from plant performance records shown in Figure 1, errors of under- and over-design with respect to effluent limitations are the rule rather than the exception. For most designers precise balance of tradeoffs is an ideal rarely achieved. But with the accumulation of experience over the years and maturation of skill, some plants come close. The proposed method regards those plants at the margin of compliance with effluent limitation as having nearly balanced design.

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