J. ENVIRONMENTAL SYSTEMS, Vol. 28(4) 257-269, 2000-2001

LINGUISTIC ORDERED WEIGHTED AVERAGING OPERATORS: POSSIBILITIES FOR ENVIRONMENTAL PROJECT EVALUATION

P. N. SMITH
The University of Queensland, Australia

ABSTRACT

This article outlines some aspects of ordered weighted averaging (OWA) operators as a framework in the evaluation of alternative environmental projects. OWA operators are considered in the context of the linguistic or non-numeric aggregation of factors and the importance weight of those factors. A simple example drawn from Horsak and Damico [1] is given which involves the location of a hazardous waste disposal facility at one of three sites based on ten factors. OWA aggregation operators are considered in the context of the above illustrative example. It is concluded that non-numeric OWA operators have considerable potential in providing a framework for the aggregation of linguistic labels in the evaluation of projects with environmental consequences.

INTRODUCTION

The numeric aggregation of fuzzy sets in the context of environmental project evaluation has been considered elsewhere [2]. Here, the aggregation of fuzzy sets defined in non-numeric or linguistic terms is considered. Often, in the context of the evaluation of environmental projects, impacts and factors are either non-quantifiable or not easily quantified (e.g., wildlife impact) or precise quantitative information is unavailable or the cost of acquiring it is too high. In these cases approximate values are often used. For example, the speed of a car might be categorized in linguistic terms as "fast," "slow," "very slow," "moderately

© 2001, Baywood Publishing Co., Inc.

fast," etc. rather than in precise numerical terms, "160 k/hr," "100 k/hr," "60 k/hr," etc. In project evaluation, the use of words or sentences rather than numbers provides a less specific, more flexible approach to express qualitative aspects of projects.

Though some of the applications on non-numeric aggregation have been devoted to the evaluation of projects characterized along multiple dimensions [3-6], more relate to group decision-making [7, 8] and the aggregation of evidence from multiple experts [9, 10]. However, many of the principles in these latter contexts may be adapted to the context of aggregating the factors/impacts characteristic of environmental projects assessed in linguistic terms.

FUZZY EVALUATION OF PROJECTS IN A LINGUISTIC ENVIRONMENT

The basic structure for the environmental evaluation of environmental projects is an *outcome matrix,* $\phi = [\phi_{ij}]$, in which ϕ_{ij} denotes the *outcome* or *performance* of project P_i with respect to factor/impact F_j. $P = \{P_1, P_2, ..., P_1\}$ is a set of I mutually exclusive projects and $F = \{F_1, F_2, ..., F_1\}$ is a set of J factors/impacts.

	F_1	F_2	…	F_J
P_1	ϕ_{11}	ϕ_{12}	…	ϕ_{1J}
P_2	ϕ_{21}	ϕ_{22}	…	ϕ_{2J}
⋮	⋮	⋮	…	⋮
P_I	ϕ_{I1}	ϕ_{I2}	…	ϕ_{IJ}

In terms of fuzzy set theory, $\mathbf{F}_j$ may be construed as a fuzzy subset of P, representd as $\mathbf{F}_j = \{F_j(P_1)|P_1, F_j(P_2)|P_2, ..., F_j(P_1)|P_1\}$, where $F_j(p)$ indicates the degree to which $p \epsilon P$ belongs to factor F_j. Note that $\phi_{ij} = F_j(P_i)$. Typically, weights $w = \{w_1\ w_2, ..., w_j\}$ are introduced to represent the differential importance of factors/impacts.

Project evaluation typically involves the identification of a "best" project which satisfies as much as possible each factor/impact. Here "satisfies" implies lower values of negative factors/impacts (e.g., cost, wildlife impact) and higher values of positive factors/impacts (e.g., accident reduction, aesthetics). Rarely will any real project completely satisfy all factors/impacts. For brevity, the term "factor" will be used below, where possible, to include also impacts.

LINGUISTIC VARIABLES

A *linguistic variable* is one whose values are words or sentences in a natural or artificial language [11]. The concept of a linguistic variable provides a means for the approximate characterization of phenomena too complex or too ill-defined for description by conventional quantitative terms.

The choice of a linguistic term set with its semantic is a first step in any linguistic approach. The terms of a linguistic variable provide the words by which those involved in the process of project evaluation may express their information. One approach is to define the linguistic term set by means of an ordered structure of linguistic terms. The semantic of linguistic terms is derived from their own ordered structure.

Consider a set of linguistic labels $S = \{s_0, s_1, ..., s_{max}\}$ where $s_i, s_k \in S$, $s_i < s_k$ if $i < k$ and s_0, and s_{max} are the lowest and highest elements respectively. Here, max = #S – 1 where #S is the *cardinality* (the number of elements) of S. For example, let $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ = {*none, very low, low, medium, high, very high, outstanding*} be a set of linguistic labels. The cardinality of S is 7. Since max = #S – 1 = 7 – 1 = 6, $s_{max} = s_6$.

An important issue in the context of a linguistic approach is the "granularity" of the uncertainty by which is meant the cardinality of the term set S. Usually, sets with odd cardinality are adopted. For example, in the context of linguistic expressions of probability, a middle term of "about 0.5" was used with the remaining terms placed symmetrically around it [12]. In addition, a limit of granularity was around 11 or 13 terms. The linguistic term set $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$ = {*none, very low, low, more or less low, medium, more or less high, high, very high, outstanding*} has cardinality 9. The former set where #S = 7 will be used in the example developed below.

In the above term set, $s_i < s_j$ if $i < j$. Usually it is required that the linguistic term set satisfy the following conditions that $s_i \vee s_j = s_i$ if $s_i \geq s_j$ and that $s_i \wedge s_j = s_i$ if $s_i \leq s_j$. In addition, a negation operator for a linguistic label is defined as $neg(s_i) = s_{max-i}$. Thus, for example, $neg(s_2) = s_{6-2} = s_4$, (i.e., neg(*low*) = *high*) and $neg(s_0) = s_{max}$ (i.e., neg(*none*) = *outstanding*).

EXAMPLE OF ENVIRONMENTAL PROJECT EVALUATION

Consider an example adapted from Horsak and Damico [1] (also considered in [2] and by Anandalingam and Westfall [13]) involving the location of a hazardous waste disposal facility with three possible sites assessed against ten factors: 1) *air quality* (dispersive capabilities of site/plant and degree to which waste emissions could concentrate onsite and offsite, F_1); 2) *surface water quality* (potential for surface water degradation due to spills associated with handling storage and waste, F_2; 3) *groundwater quality* (potential for groundwater degradation due to spills associated with handling and storage of waste, including leaching into aquifer, F_3);

4) *impact on ecology* (potential impact on ecological resources of area due to routine operations or emergency conditions, F_4); 5) *impact on aesthetics* (visual impacts of hazardous waste management operations, including handling, storage, and disposal, F_5); 6) *impact on population* (potential long-term exposure to emissions due to routine operations or emergencies; F_6); 7) *impact on surrounding land use* (compatibility of surrounding land use with the hazardous waste operation, F_7); 8) *possibility of emergency response* (ability of a response team to combat an emergency associated with a spill or other exposure, F_8); 9) *distance from sources of waste* (distance through which the waste should travel to get to the site, F_9); and 10) *political opposition* (political or other organized intervention or opposition to the hazardous waste operation, F_{10}). Factors are fuzzy subsets of the projects (sites), for example $\mathbf{F}_1 = \{s_6|P_1, s_4|P_2, s_2|P_3\}$ for *air quality* (F_1) where a conversion from numeric membership grades (in [12]) to linguistic labels is achieved using the Label(•) function (see below). The matrix $\phi = [\phi_{ij}]$, is given as follows

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
P_1	s_6	s_5	s_6	s_6	s_5	s_6	s_5	s_5	s_6	s_3
P_2	s_4	s_6	s_6	s_6	s_6	s_3	s_4	s_3	s_4	s_6
P_3	s_2	s_1	s_6	s_1	s_6	s_6	s_1	s_1	s_2	s_2

Thus, $F_j(p) \in S = \{s_0\ s_1, s_2, s_3, s_4, s_5, s_6\}$. In terms of linguistic data, project P_1 (site 1) is a strong competitor as the "best" site. Further assume weights as follows $w = \{s_6, s_6, s_6, s_5, s_4, s_4, s_3\ s_2, s_2, s_1\}$, again derived using the Label(•) function from numeric weights, {1, 0.969, 0.919, 0.714, 0.689, 0.658, 0.460, 0.323, 0.286, 0.193} in [12]. Therefore, $w_j \in S$ also. It is clear that *air quality, surface water quality,* and *groundwater quality* are of fundamental importance.

LINGUISTIC QUANTIFIED STATEMENTS

In classical logic, quantifies in statements or propositions, in particular "for all" and "there exists" ("not none"), may be used to represent the number of items satisfying a given predicate. Zadeh introduced fuzzy subsets as the basis for *linguistically quantified statements* or *propositions* [14]. The general form of a quantified statement is "Q F's are **A**," where Q is a linguistic quantifier (e.g., "few" "most," "at least n"), F is a class of objects and **A**, a fuzzy subset of F, is some property associated with the objects. For example, in the quantified statement "*most* local roads are *short,*" the quantifier Q is "most," F is a set of "local roads," and "short" is a property/characteristic of local roads.

Two types of quantifiers, *absolute* and *proportional,* were introduced by Zadeh [14]. Absolute quantifiers are used to represent amounts that are absolute in nature ("about 5," "more than 10") and are defined on the set of non-negative reals, $\mathbb{R}^+$. They are closely related to the concept of the count or number of elements. Proportional quantifiers ("most," "few," "at least half") represent relative amounts and are defined on the unit interval, [0, 1].

In the context of project evaluation, "Q F's are $\mathbf{A}_p$" where Q is a linguistic quantifier, $\{F_1(p), F_2(p), ..., F_J(p)\}$ is a set of factors against which a project $p \in P$ is assessed and $\mathbf{A}_p$ is a fuzzy subset of F indicating the predicate "satisfied by p." "Satisfied" is interpreted as above. Examples of linguistic quantified statements in the context of fuzzy evaluation of projects include "*most* factors are satisfied by project p" or "*at least n* factors are satisfied by project p." This is a *type I* statement [15].

An extension of this quantified statement is "Q B F's are $\mathbf{A}_p$" where B = $\{B(F_1)|F_1, B(F_2)|F_2, ..., B(F_J)|F_J\}$ is a fuzzy subset of F, such that $B(F_j)$ indicates the importance of factor F_j. Examples of this type of quantified statement are "*most important* factors are satisfied by project p" and "*at least n important* factors are satisfied by project p." These are *type II* statements. Zadeh has presented approaches to establishing the truth of such statements [14].

Yager defines *regular increasing monotone* (RIM) quantifiers (e.g., "all," "most," "many," "at least x percent") such that $Q(0) = 0$, $Q(1) = 1$, and $Q(r) \geq Q(s)$ if $r > s$ [16]. One particular family of RIM quantifiers is $Q(r) = r^{\xi}$ ($\xi \geq 1$).

NON-NUMERIC ORDERED WEIGHTED AVERAGING AGGREGATION OPERATORS

Ordered weighted averaging operators introduced in a numeric environment [17] have also been considered in a non-numeric or ordinal environment [18]. The ordinal OWA operator involves weights $\alpha = \{\alpha_1, \alpha_2, ..., \alpha_J\}$ in which $\alpha_j \in S$ and $\alpha_1 \leq \alpha_2 \leq ... \leq \alpha_J$ and $\vee_{j=1,J}\{\alpha_j\} = s_{max}$ is defined as

$$OWA = \vee_{j=1,J}\{\alpha_j \wedge b_j\}$$

$\{b_1, b_2, ..., b_J\}$ is associated with $\{F_1(p), F_2(p), ..., F_J(p)\}$ such that b_j is the jth largest value $F_j(p)$. As an illustrative example, consider outcomes $\{s_2, s_4, s_6\}$ and OWA operator weights $\{s_2, s_5, s_6\}$. Then $b_1 = s_6$, $b_2 = s_4$, $b_3 = s_2$, and $OWA = (s_2 \wedge s_6) \vee (s_5 \wedge s_4) \vee (s_6 \wedge s_2) = s_4$.

The OWA operator with weights, $\alpha_\star = \{s_0, s_0, ..., s_{max}\}$ is denoted $OWA_\star$ and the OWA operator with weights, $\alpha^\star = \{s_{max}, s_{max}, ..., s_{max}\}$, is dnoted $OWA^\star$. It can be shown that $OWA_\star \leq OWA \leq OWA^\star$ [18]. The "orness" of an OWA operator is given as

$$\text{orness}(\alpha) = \vee_{j=1,J}(\alpha_j \wedge \text{Label}((J - j)/(J - 1))$$

where $\alpha_j \in S$ and the function, Label(•), defined as

$$\text{Label}(x) = s_i \quad i/\#S \leq x < (i + 1)/\#S \quad i = 0, ..., \max$$

together with Label(1) = s_{max}, maps a numeric value $x \in [0, 1]$ to a linguistic label $s_i \in S$ (see Figure 1).

"Orness" is an ordinal indication of the inclination for the OWA operator to impart more weight to higher (ordinal) membership grades than lower ones. It is easy to show that the "orness" of $\alpha^\star = \{s_{max}, s_{max}, ..., s_{max}\}$ is orness($\alpha^\star$) = s_{max} whereas the "orness" of $\alpha_\star = \{s_0, s_0, ..., s_{max}\}$ is orness($\alpha_\star$) = s_0. Note that Label(s_{max}) = 1 and Label(s_0) = 0.

Bordogna et al. [19] have considered two methods, one which uses an ordinal OWA operator which aggregates values in a non-numeric environment based on [18] and one which uses an OWA operator to aggregate linguistic values in a numeric environment. The latter involves a *weighted ordered weighted averaging* (WOWA) operator.

In the non-numeric method, consider a set of linguistic labels, S. Then an implementation of the quantified statement "Q B F's are $\mathbf{A}_p$" is given by an OWA operator associated with a RIM quantifier, Q, (represented at OWA_Q). OWA operator weights are given as α_j = Label(Q(j/J)) (j = 1, ..., J) where the Label(•) function converts the membership grade of the proportion of factors satisfied to an element of S. Note that in the quantified statement, $A_p(F_j) \equiv F_j(p)$ (j = 1, ..., J) which are drawn from S. Thus,

$$OWA_Q = \vee_{j=1,J}\{\alpha_j \wedge b_j\} = \vee_{j=1,J}\{\text{Label}(Q(j/J)) \wedge b_j\}$$

$\{b_1, b_2, ..., b_J\}$ is associated with arguments $\{F_1(p), F_2(p), ..., F_J(p)\}$ such that b_j is the jth largest value of $F_j(p)$.

The importance of factors is included by modifying the values to be aggregated. One possibility has been given as follows [18]:

$$H(F_j(p),w_j,\text{orness}(\alpha)) = (w_j \vee (\text{neg}(\text{orness}(\alpha)))) \wedge (F_j(p) \vee \text{neg}(w_j)) \wedge (F_j(p) \vee (\text{neg}(\text{orness}(\alpha))))$$

where $w_j \in S$, $F_j(p) \in S$ and orness(α)$\in$S. When orness(α) = s_{max}, $H(F_j(p),w_j,s_{max}) = w_j \wedge F_j(p)$ and the OWA operator reduces to $\vee_{j=1,J}(w_j \wedge F_j(p))$ [3] and when orness(α) = s_0, $H(F_j(p),w_j,s_0) = F_j(p) \vee \text{neg}(w_j)$ and the OWA operator reduces to $\wedge_{j=1,J}(\text{neg}(w_j) \vee F_j(p))$ [20]. Note that the (ordinal) membership grades of fuzzy subset, B, are factor importance weights, that is, $B(F_j) \equiv w_j \in S$ (j=1, ..., J).

Now consider this method in the context of the above illustrative example. Assume a RIM quantifier, "most," defined by $Q(r) = r^2$. OWA operator weights are given by α = {Label(Q(1/10)), Label(Q(2/10)), ..., Label (Q(10/10))} = $\{s_0, s_0,$

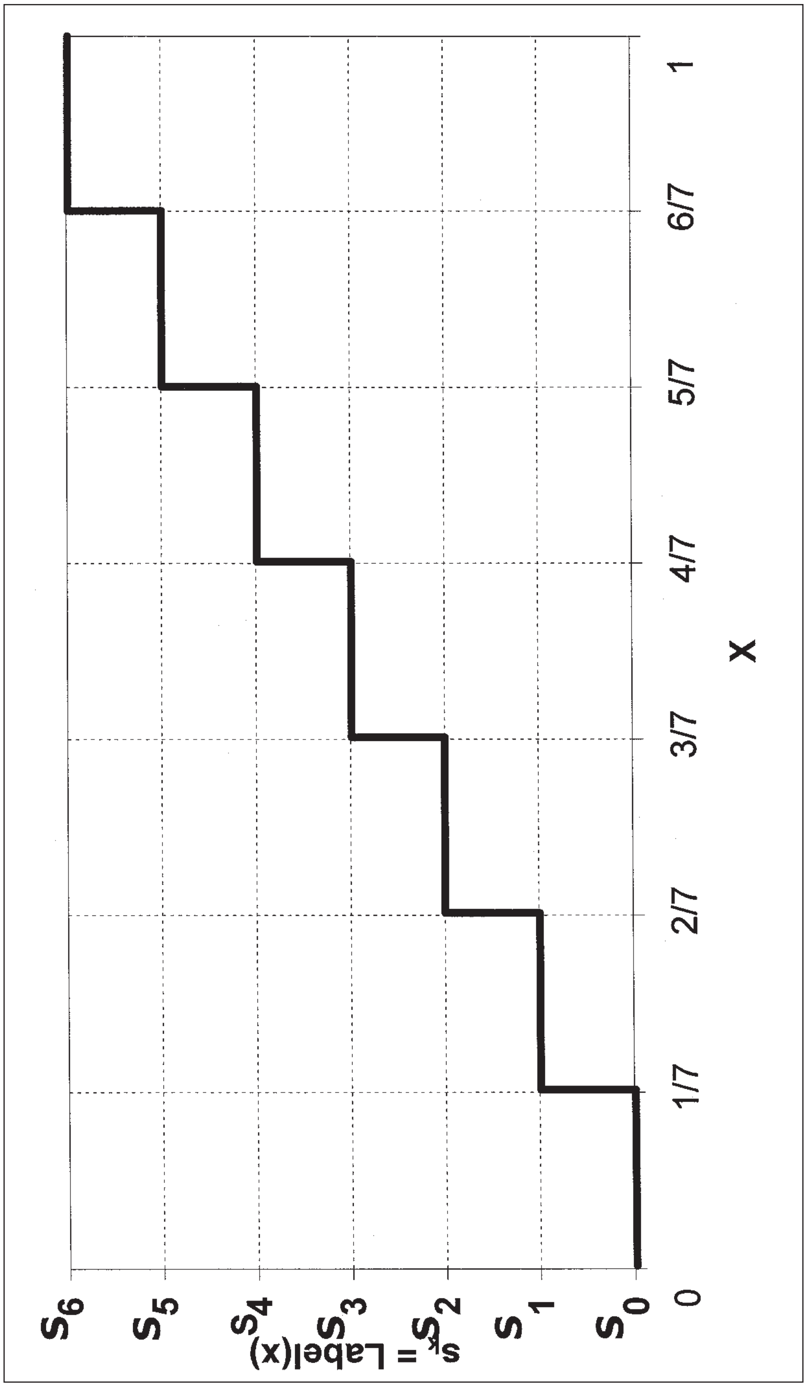

Figure 1. Label function.

s_0, s_1, s_1, s_2 s_3, s_4, s_4, s_6} where α_j = Label(Q(j/J)) = Label($(j/J)^2$) (j = 1, ..., J). The level of "orness" of OWA operator weights is orness(α) = $\vee_{j=1,10}(\alpha_j \wedge$ Label((J – j)/(J – 1)) = s_2, so that H(F_j(p),w_j,s_2) is given as

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
P_1	s_6	s_5	s_6	s_5	s_4	s_6	s_4	s_4	s_4	s_4
P_2	s_4	s_6	s_6	s_5	s_4	s_3	s_4	s_4	s_4	s_4
P_3	s_2	s_1	s_6	s_1	s_4	s_4	s_3	s_4	s_4	s_4

where, for example, H($F_4(P_1)$,w_4,s_2) = H(s_6,s_5,s_2) = ($s_5 \vee$ neg(s_2)) $\wedge$ ($s_6 \vee$ neg(s_5)) $\wedge$ ($s_6 \vee$ (neg(s_2)) = ($s_5 \vee s_4$) $\wedge$ ($s_6 \vee s_1$) $\wedge$ ($s_6 \vee s_4$) = $s_5 \wedge s_6 \wedge s_6$ = s_5. In terms of "effective satisfaction," it is clear that P_1 dominates P_3 in the sense that P_1 performs equally or better than P_3 with respect to each factor. Thus, $OWA_Q(P_1)$ = s_4, $OWA_Q(P_2)$ = s_4 and $OWA_Q(P_3)$ = s_3. Projects P_1 and P_2 are "best" from this perspective.

WEIGHTED ORDERED WEIGHTED AVERAGING AGGREGATION OPERATORS

A weighted OWA (WOWA) operator which generalizes the numeric OWA operator has also been defined as

$$\text{WOWA} = \Sigma_{j=1,J}\ \beta_j b_j$$

where b_j is the jth largest F_j(p) (j=1, ..., J) [21-23]. Weights, w_j, are such that $w_j \in [0,1]$ and $\Sigma_{j=1,J} w_j = 1$ reflect the importance of F_j(p). WOWA operator weights are defined as $\beta_j = W(\Sigma_{k=1,j} u_k) - W(\Sigma_{k=1,j-1} u_k)$ where u_j is the weight associated with b_j. Thus, if $b_1 = F_3(p)$, then $u_1 = w_3$. W(•) is a monotone non-decreasing function that interpolates the points (j/J, $\Sigma_{k=1,j}\alpha_k$) together with point (0,0), if the weights used in the OWA operator, {α_1, α_2, ..., α_J}, are given. If factor weights are all equal (i.e., w_j = 1/J; j=1, ..., J), then the WOWA operator reduces to the OWA operator with weights $\beta_j = \alpha_j$ (j=1, ..., J). Alternatively, given a monotonic non-decreasing function, W(•), it is possible to derive β_j (j = 1, ..., J) from W(•) without the initial step of defining OWA operator weights [24].

The numeric method considered by Bordogna et al. [19] implements the quantified linguistic statement "Q B F's are $\mathbf{A}_p$" and involves the aggregation of linguistic values in a numeric environment based on a WOWA operator. That is, linguistic performance values $F_j(p) \in S$ and linguistic expressions of the importance of factors, $w_j \in S$ are mapped into numbers in the [0,1] interval by applying the inverse (linguistic label to numeric function), $Label^{-1}(\bullet)$, defined as $Label^{-1}(s_i) = i/max$ (i = 0, 1, ..., max) (see Figure 2). The WOWA operator is $WOWA_Q = \Sigma_{j\epsilon 1,J} \beta_j b_j$, with operator weights determined by a RIM quantifier, Q, so that $\beta_j = Q(\Sigma_{k=1,j} u_k) - Q(\Sigma_{k=1,j-1} u_k)$.

Numeric equivalents for linguistic factor importance weights are given by w = {1, 1, 1, 0.833, 0.667, 0.667, 0.5, 0.333, 0.333, 0.167}. The linguistic performances of projects are transformed using the label to numeric function, $Label^{-1}(\bullet)$ yielding

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
P_1	1	0.833	1	1	0.833	1	0.833	0.833	1	0.5
P_2	0.667	1	1	1	1	0.5	0.667	0.5	0.667	1
P_3	0.333	0.167	1	0.167	1	1	0.167	0.167	0.333	0.333

where, for example, $F_2(P_1) = 0.883$ (since $Label^{-1}(F_2(P_1)) = Label^{-1}(s_5) = 5/(\#S - 1) = 5/6 = 0.883$). Note that this transformation will be used here even though numeric data for both outcomes and weights was originally given by Horsak and Damico [1]. Numeric WOWA operator weights are given by $\beta_j = Q(\Sigma_{k-1,j} u_k) - Q(\Sigma_{k=1,j-1} u_k)$, where the weight u_j is associated with b_j (the jth largest element of $\{Label^{-1}(F_j(P_1)\}$ (j=1, ..., 10)). Thus, $WOWA_Q(P_1) = 0.872$. Further, Label(0.872) $= s_6$, $WOWA_Q = 0.725$ (Label(0.725) $= s_5$), $WOWA_Q = 0.311$ (Label(0.311) $= s_2$). Here, project P_1 is "best." Note that the WOWA weights will, in general, be different for each project as will orness(β).

LINGUISTIC OWA AND WOWA AGGREGATION OPERATORS

Again assume $F_j(p) \epsilon S$ to be aggregated. Then a *linguistic ordered weighted averaging* (LOWA) operator [25-28] which combines linguistic values by direct computation of the labels (based on the OWA operator [17] and on the *convex combination* of linguistic labels [24]) has been defined as

$$LOWA = \mathbb{C}^J\{\alpha_j, b_j, j = 1, ..., J\}$$
$$= \alpha_1 \odot b_1 \oplus (1 - \alpha_1) \odot \mathbb{C}^{J-1}\{\alpha_h/\Sigma_{k=2,J}\, \alpha_k, b_h, h = 2, ..., J\}$$

where $\{\alpha_1\, \alpha_2, ..., \alpha_J\}$ are numeric LOWA operator weights (such that $\alpha_j \epsilon [0,1]$ and $\Sigma_{j=1,J}\alpha_j = 1$). b_j is the jth largest $F_j(p)$ (j=1, ..., J). If J = 2, then $\mathbb{C}^2\{\alpha_j, b_j, j = 1,2\} = \alpha_1 \odot b_1 \oplus (1 - \alpha_1) \odot b_2 = \alpha_1 \odot s_j \oplus (1 - \alpha_1) \odot s_1 = s_k$, where $s_j \geq s_i$ and for s_k, index k is calculated as $\min(s_{max}, i + round(\alpha_1(j - i)))$. For example, consider outcomes $\{s_1, s_2, s_5\}$ and weights $\alpha = \{0.7, 0.2, 0.1\}$. Then

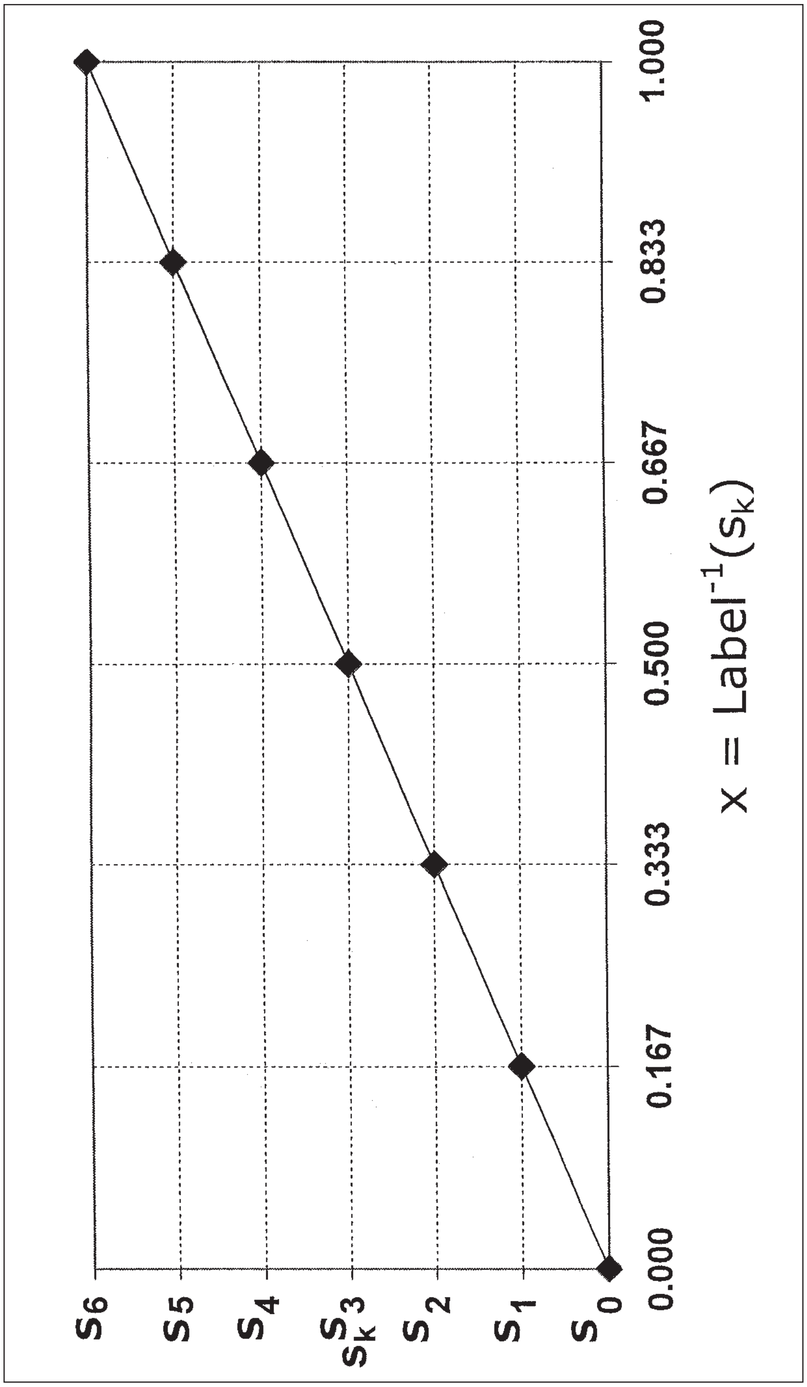

Figure 2. Inverse label function.

$$\text{LOWA} = \mathbb{C}^3\{0.7, 0.2, 0.1, s_5, s_2, s_1\}$$
$$= 0.7 \odot s_5 \oplus (1 - 0.7) \odot \{0.2/(0.2 + 0.1) \odot s_2 \oplus 0.1/(0.2 + 0.1) \odot s_1\}$$

and since for $\mathbb{C}^2\{0.67, 0.33, s_2, s_1\}$, k = min(6, 2 + round(0.67(2 – 1))) = min(6, 3) = 3. Thus

$$\text{LOWA} = 0.7 \odot s_5 \oplus (1 - 0.7) \odot s_3 = s_4$$

since k = min(6, 3 + round(0.33(5 – 3))) = min(6, 4) = 4. The weights in the LOWA may be derived from a quantifier as $\alpha_j = Q(j/J) - Q((j-1)/J)$. The LOWA differs from the ordinal OWA in terms of the nature of the operator weights which are numeric in the former and linguistic in the latter, and in terms of the different requirements on the weights in each. Appropriate inclusion of the factor weights within the LOWA requires further exploration. However, a linguistic WOWA (L-WOWA) operator [24] defined as

$$\text{L-WOWA} = C^J\{\beta_j, b_j, j = 1, ..., J\}$$
$$= \beta_1 \odot b_1 \oplus (1 - \beta_1) \odot C^{J-1}\{(\beta_h/\Sigma_{k=2,J}\, \beta_k), b_h, h = 2, ..., J\}$$

is applicable in this context. Again, b_j is the jth largest $F(_j(p)$ (j=1, ..., J). Numeric factor importance weights are defined such that $\Sigma_{j=1,J} w_j = 1$ and u_j is the factor weight associated with b_j. The L-WOWA operator weights are defined as $\beta_j = W(\Sigma_{k=1,j} u_k) - W(\Sigma_{k=1,\, j-1} u_k)$ where W(•) is a monotonic non-decreasing function satisfying the requirements of a WOWA.

In terms of the L-WOWA operator approach, again assume a quantifier "most" defined above and numeric importance weights. For project, P_1, β = {0.026, 0.05, 0.028, 0.117, 0.111, 0.204, 0.175, 0.09, 0.138, 0.061}. Then $\text{L-WOWA}_Q(P_1) = s_4$, $\text{L-WOWA}_Q(P_2) = s_4$, and $\text{L-WOWA}_Q(P_3) = s_1$. Here, projects P_1 and P_2 are "best." However, it is noted that a more discriminating ordering of projects results from a direct expansion of the LOWA rather than recursive implementation of the convex combination. In this case, the round(•) need only be implemented once to convert a numeric value to a linguistic label. Thus, $\text{L-WOWA}_Q(P_1) = s_5$, $\text{L-WOWA}_Q(P_2) = s_4$, and $\text{L-WOWA}_Q(P_3) = s_2$.

CONCLUSION

This article has outlined some of the significant features of non-numeric or linguistic OWA aggregation operators as a framework for the evaluation of alternative projects with significant environmental consequences. These methods facilitate the use of soft or linguistic expressions of project outcomes with respect to factors and impacts and linguistic expressions of the importance of those factors. An example adapted from Horsak and Damico [1] involving the location of a hazardous waste-disposal facility with three possible sites assessed against ten factors was considered in terms of the operators.

Various aggregation methods have been considered including the non-numeric (ordinal) OWA operator, the numeric WOWA operator based on a linguistic to numeric function, and the LOWA and L-WOWA operators. Inclusion of factor weights is achieved either by modification of project outcomes along each factor or by incorporation in WOWA operator weights guided by an appropriate linguistic quantifier. These methods are believed to have potential to assist in the assessment of projects where only imprecise or approximate data is available. In addition, the methods are applicable for the initial screening of a wide range of projects prior to more detailed examination of a selected subset.

REFERENCES

1. R. Horsak and S. Damico, Selection and Evaluation of Hazardous Waste Disposal Sites Using Fuzzy Set Analysis, *Journal of the Air Pollution Control Association, 35,* pp. 1081-1085, 1985.
2. P. N. Smith, Numeric Ordered Weighted Averaging Operators: Possibilities for Environmental Project Evaluation, *Journal of Environmental Systems, 28,* pp. 1-16, 2000-2001.
3. R. R. Yager, A New Methodology for Ordinal Multiple Aspect Decision Based on Fuzzy Sets, *Decision Sciences, 12,* pp. 589-600, 1981.
4. R. R. Yager, Non-Numeric Multi-Criteria Multi-Person Decision Making, *Group Decision and Negotiation, 2,* pp. 81-93, 1993.
5. R. R. Yager, Fuzzy Screening Systems, in *Fuzzy Logic,* R. Lowen and M. Roubens (eds.), Kluwer, Dordrecht, pp. 251-261, 1992.
6. R. R. Yager, An Approach to Ordinal Decision Making, *International Journal of Approximate Reasoning, 12,* pp. 237-261, 1995.
7. J. Kacprzyk and M. Fedrizzi, A 'Human-Consistent' Degree of Consensus Based on Fuzzy Logic with Linguistic Quantifiers, *Mathematical Social Sciences, 18,* pp. 275-290, 1989.
8. J. Kacprzyk and M. Fedrizzi, *Multiperson Decision-Making Models Using Fuzzy Sets and Possibility Theory,* Kluwer, Dordrecht, 1990.
9. R. R. Yager, General Multiple-Objective Decision Functions and Linguistically Quantified Statements, *International Journal of Man-Machine Studies, 21,* pp. 389-400, 1984.
10. R. R. Yager, Aggregating Evidence Using Quantified Statements, *Information Sciences, 36,* pp. 179-206, 1985.
11. L. A. Zadeh, The Concept of a Linguistic Variable and its Application to Approximate Reasoning, Part I, Part II, Part III, *Information Sciences, 8,* pp. 199-249, pp. 301-357, *9,* pp. 43-80, 1975.
12. P. P. Bonissone and K. S. Decker, Selecting Uncertainty Calculi and Granularity: An Experiment in Trading-Off Precision and Complexity, in *Uncertainty in Artificial Intelligence,* L. N. Kanal and J. F. Lemmer (eds.), Elsevier Science, pp. 217-347, 1986.
13. G. Anandalingam and M. Westfall, Selection of Hazardous Waste Disposal Alternatives Using Multiattribute Utility Theory and Fuzzy Set Theory, *Journal of Environmental Systems, 18,* pp. 69-85, 1988.

14. L. A. Zadeh, A Computational Approach to Fuzzy Quantifiers in Natural Language, *Computing Mathematical Applications, 9,* pp. 149-184, 1983.
15. R. R. Yager, Interpreting Linguistic Quantified Propositions, *International Journal of Intelligent Systems, 9,* pp. 541-569, 1994.
16. R. R. Yager, Families of OWA Operators, *Fuzzy Sets and Systems, 59,* pp. 125-148, 1993.
17. R. R. Yager, On Ordered Weighted Averaging Aggregation Operators in Multicriteria Decision Making, *IEEE Transactions Systems, Man and Cybernetics, 18,* pp. 183-190, 1988.
18. R. R. Yager, Applications and Extensions of OWA Aggregations, *International Journal of Man-Machine Studies, 37,* pp. 103-132, 1992.
19. G. Bordogna, M. Fedrizzi, and G. Pasi, A Linguistic Modelling of Consensus in Group Decision Making Based on OWA Operators, *IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans, 27,* pp. 126-152, 1997.
20. R. R. Yager, A Note on Weighted Queries in Information Retrieval Systems, *Journal of the American Society of Information Science, 38,* pp. 23-24, 1987.
21. V. Torra, Weighted OWA Operators for Synthesis of Information, *Proceedings of the Fifth International Conference on Fuzzy Systems,* FUZZ-IEEE '96, New Orleans, Louisiana, pp. 966-971, 1996.
22. V. Torra, The Weighted OWA Operator, *International Journal of Intelligent Systems, 12,* pp. 153-166, 1997.
23. R. R. Yager, Quantifier Guided Aggregation Using OWA Operators, *International Journal of Intelligent Systems, 11,* pp. 49-73, 1992.
24. F. Herrera and E. Herrera-Viedma, On the Linguistic OWA Operator and Extensions, in *The Ordered Weighting Averaging Operators: Theory and Applications,* R. R. Yager and J. Kacprzyk (eds.), Kluwer Academic, Boston, pp. 60-72, 1997.
25. F. Herrera and E. Herrera-Viedma, Aggregation Operators for Linguistic Weighted Information, *IEEE Transactions on Man, Systems and Cybernetics, Part A: Systems and Humans, 27,* pp. 646-656, 1997.
26. F. Herrera, E. Herrera-Viedma, and J. L. Verdegay, Direct Approach Processes in Group Decision Making Using Linguistic OWA Operators, *Fuzzy Sets and Systems, 79,* pp. 175-190, 1996.
27. F. Herrera, E. Herrera-Viedma, and J. L. Verdegay, Applications of the Linguistic OWA Operators in Group Decision Making, in *The Ordered Weighting Averaging Operators: Theory and Applications,* R. R. Yager and J. Kacprzyk (eds.), Kluwer Academic, Boston, pp. 207-218, 1997.
28. M. Delgardo, J. L. Verdegay, and M. A. Vila, On Aggregation Operations of Linguistic Labels, *International Journal of Intelligent Systems, 8,* pp. 351-370, 1993.

Direct reprint requests to:

P. N. Smith
Department of Geographical Sciences and Planning
The University of Queensland
St. Lucia, Queensland
Australia 4072