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A Model For the Effect Of Illegally Parked Cars On Street Cleaning*

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ABSTRACT

The effectiveness of urban street cleaning by mechanical sweepers is limited by the presence of illegally parked cars along curbs which are scheduled to be swept. Analytical and algorithmic models are developed and results are presented for both metered and non-metered zones, and for any distribution of illegally parked cars.

Introduction

The necessity for clean streets in the urban community has grown in importance in the last few years. The absence of litter and unsightly refuse is not only a significant factor in maintaining sanitary conditions, but it also adds to the quality of city life. For these reasons, the street cleaning operation carried out by the Department of Sanitation has long been a vital service to the people of New York City. However, while much is being done by the Department to increase the level of cleanliness of the streets, it is readily apparent that the final result is a complicated function of the various factors affecting this operation.

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Urban street cleaning is carried out by mechanical brooms routed so as to sweep the curbs in a district several times per week. For the sweeping to be effective, the curbs should be free of illegally parked cars. A set of no-parking regulations are put into effect that prohibit parking during the period a particular blockside curb is scheduled to be swept. However, the lack of an adequate number of legal parking spaces and the difficulty in enforcing the no-parking regulations have resulted in widespread noncompliance. Consequently, the operators of the sweepers are forced to maneuver around illegally parked cars and miss a substantial portion of the curb. The effectiveness of the cleaning operation is therefore sharply reduced. Specifically, the aim of this paper is to determine and analyze the factors affecting street cleaning and to develop a detailed simulation model to study the effects of these factors on the length of curb that can be swept.

Intuitively, the length of curb swept is a function of various parameters: These may be classified as machine-operator parameters and environmental ones. Those in the first group may include sweeper maneuverability and the operator's proficiency, while those in the second may include compliance to parking regulations, the distribution of illegally parked cars, the weather, etc.

In order to determine the effect of these parameters on the length of curb swept, it was necessary to develop an explicit mathematical model. The lack of any previous research in this area necessitated the development of the required relationships from fundamental considerations.

Model Parameters

The effects of machine-operator parameters on length of curb that can be swept are examined first. A key quantity in this analysis is the total clearance required by a sweeper to maneuver around one illegally parked car. This variable, denoted by TC, is a measure of the maneuverability of the sweeper and the skill of the operator. Expressed as the sum of the front and rear clearances (see Figure 1D), TC is also a measure of the curb missed for each car bypassed. Furthermore; for more than one car, the total missed curb is the sum of the clearance distances for each car plus the lengths of the cars (Figure 1D) provided the cars are sufficiently far apart.

Another important machine-operator parameter is intercar distance. The effect of intercar distance on swept curb length is determined by assuming first that two cars are illegally parked bumper to bumper. The amount of curb side missed by a sweeper bypassing the cars will be $L_1 + L_2 + FC + RC$, where L_1 and L_2 are the lengths of cars 1 and 2, and FC and RC are the front and rear clearances, respectively (Figure 1A). As the distance between the cars increases, the amount of unswept curb also

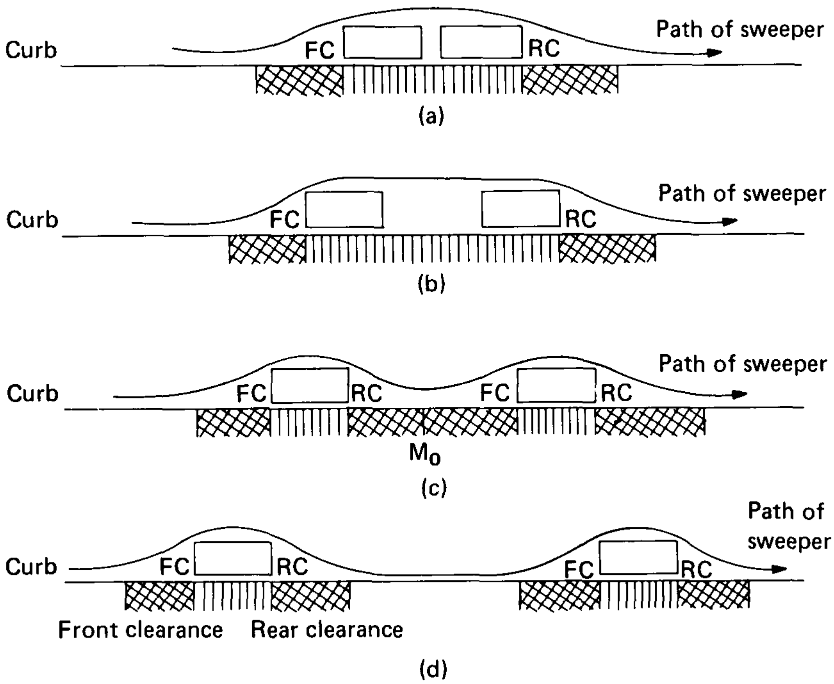


Figure 1. The effect of machine-operator parameters on curblength swept. (Shaded area = unswept curb.)

increases in proportion to the intercar distance (Figure 1B). At some critical distance, M_0, the sweeper will begin to maneuver between the two cars (Figure 1C). From this point on, the unswept distance decreases as the sweeper is capable of maneuvering more and more efficiently. Finally, when the cars are sufficiently far apart to be considered independently, the unswept distance will level off to a constant value $L_1 + L_2 + 2FC + 2RC$ (Figure 1D).

For a block of length L, the amount of street swept is L minus the unswept distance. The resulting relationship between curblength swept, YF, and intercar distance, DI, is shown in Figure 2, where ϵ is the additional distance necessary for the cars to be considered independently.

The critical intercar distance, M_0, is ideally equal to the total clearance (Figure 1C); however, the minimum distance for which the sweeper can maneuver between two cars depends upon the skill of the operator. Moreover, by definition, the total clearance cannot be greater than the critical intercar distance.

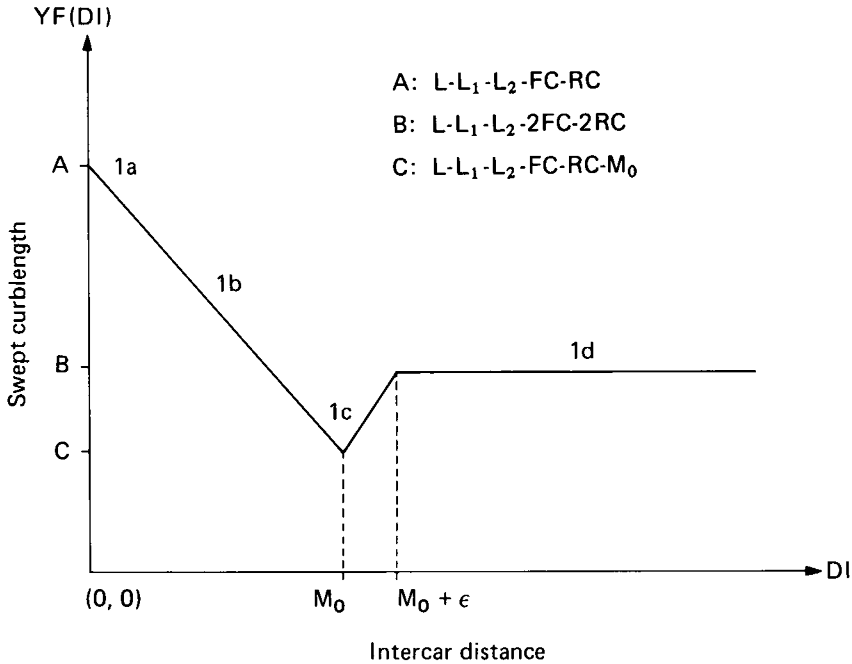


Figure 2. The effect of intercar distance on swept curblength.

In considering the environmental parameters, emphasis is placed on the compliance to parking regulations. As the number of illegally parked cars increases, the amount of curb side accessible to the sweeper decreases. Therefore, the amount of curb that can be swept also decreases. Furthermore, the density of illegally parked cars has a limiting effect on the maneuverability of the sweeper, which results in an additional decrease in curb length which is swept. The last parameter considered is the distribution of illegally parked cars. For a fixed number of cars, the location of each on the block, and the position of each with respect to the others, has an effect on the curb length swept.

Metered Parking

THE MATHEMATICAL MODEL

Metered parking zones were used first to study the effects of the various parameters on the expected length of curb that can be swept.[1] This choice facilitated the mathematical modeling and the computation of the expected values of swept curb. By considering each space as a discrete entity, the swept distance was determined by simple summation (this will become clearer in the sequel). To simplify further the development of this relationship, it was initially assumed that the sweeper could maneuver between two cars as long as the intercar distance was at least as great as one metered space ($M_0 \leq 1$). Furthermore, whenever two or more cars were parked in tandem, bumper to bumper, the missed curb was just the sum of the lengths of the cars plus the front clearance of the first car and the rear clearance of the last car (Figure 1A).

Expressing the variables FC and RC as fractions of a metered parking space, the actual curb length that could be swept was tabulated for a block with N consecutive spaces as the number of illegally parked cars, I, varied from zero to N. (A summary of symbols is given in Table 1.) These numbers represented the sum of the empty spaces minus the appropriate sums of front and rear clearances. For each value of I, the expected

Table 1. Summary of Symbols

Symbols	
N	number of available spaces on a block
I	number of illegally parked cars on a block
FC	front clearance in bypassing a parked car
RC	rear clearance in bypassing a parked car
TC	total clearance, FC + RC
S	per cent of curb (block) swept
C	compliance to parking regulations
$\bar{C}$	noncompliance, 1 − C
L	total length of a block (in feet)
L_i	length of car i (in feet)
DI	intercar distance
D	intercar distance expressed in units of a metered space
M_0	critical (minimum) intercar distance for which a sweeper maneuvers between two cars
ϵ	the additional distance necessary for two cars to be considered independently
Y	swept curblength expressed in units of a metered space
YF	swept curblength expressed in feet
UYF	unswept curblength expressed in feet
LE	effective length of a block, $L - \sum_{i=1}^{I} L_i$

(average) length of curb swept, $\overline{Y}$, expressed in metered space units, was computed using the formula

$$\text{Average curblength swept} = \sum_{i-1}^{N^{C_I}} P_i \times (\text{curblength swept})_i \qquad (1)$$

where $_NC_I$ represents all the possible sets of I occupied, distinguishable parking spaces on a block of size N without regard to the identity of the cars parked in those spaces, i.e., it is the collection of sets into which I elements may be grouped from N choices without regard to the order of arrangement within the group.[2] Furthermore, P_i is the probability of the ith set. If all the metered spaces are initially assumed occupied and have to be vacated at the time of the parking regulation, each arrangement resulting from (N - I) cars leaving metered spaces at random is equally likely with probability $1/N^C I$.

The resulting expression for the expected length of curb that can be swept, in units of metered spaces, is given by Equation 2.

$$E(Y) = \frac{N-1}{N}(N - I - TC) \qquad (2)$$

where TC is the total clearance expressed as a fraction of a metered space.

Since the number of empty parking spaces is the difference between the number of available spaces, N, and the number of illegally parked cars, I, *the compliance to parking regulations* can be expressed as:

$$C = \text{Compliance} = \frac{N-I}{N}\ ; \qquad (3)$$

in similar fashion, noncompliance is written as:

$$\overline{C} = \text{Noncompliance} = 1 - C = \frac{I}{N} \qquad (4)$$

Substituting Equations 3 and 4 into Equation 2, dividing the result by the number of available spaces and multiplying by one hundred yields the following relationship between percentage swept and compliance:

$$S = 100 \times [(TC)C^2 + (1 - TC)C] \qquad 0 < M_0 \leqslant 1 \qquad (5)$$

Although Equation 5 exhibits the quadratic nature of the relationship between S and C, it is Equation 2 that leads to a natural interpretation.

The expected curblength that can be swept given N spaces and I illegally parked cars is proportional to the total length adjusted for the clearance required to maneuver around each illegally parked car. The constant of proportionality is, naturally, the compliance. Note, that for perfect maneuverability, TC = 0, the average curb swept is simply

$$E(Y) = N \cdot C$$

The preceding development is valid for $0 < M_0 \leqslant 1$, where the value of

M_0 is expressed in metered space units. Now let $1 < M_0 \leqslant 2$. Since metered parking is assumed, values of M_0 in this range imply that the sweeper is unable to maneuver between two cars separated by a distance of less than two spaces. Therefore, the expected curblength swept between pairs of cars with such a separation is zero.

The average length of curb swept is the same for $0 \leqslant M_0 \leqslant 1$ and $1 < M_0 \leqslant 2$ when the number of illegally parked cars is zero or one. When I is greater than one, the two values are different with the one for $1 < M_0 \leqslant 2$ less than the one for $0 \leqslant M_0 \leqslant 1$. The discrepancy appears because every time the configuration of two cars separated by one space occurs, there is a contribution of (1 - TC) units of swept length to the calculation of the average for $0 \leqslant M_0 \leqslant 1$, while the contribution is zero for $1 < M_0 \leqslant 2$.

For I illegally parked cars on a block with N spaces, the number of possible arrangements with a distance D between two cars in tandem is given by

$$N - 1 - D^{C}I - 1$$

where N - I - D reflects the reduction in the number of spaces that can be occupied and the term I - 1 results from considering the j^{th} and $(j + 1)^{st}$ car and the distance between them as one entity. The probability for one such event to occur is given by

$$\frac{N - 1 - D^{C}I - 1}{N^{C}I}$$

Finally, for I cars, there are exactly I - 1 intercar distances, each of which may have length varying from zero to N - I. Therefore, the expected value of the discrepancy is given by

$$(1 - TC)\left[\frac{N - 1 - D^{C}I - 1}{N^{C}I}\right](I - 1)$$

For the case $1 < M_0 \leqslant 2$, D = 1, the above expression reduces then to

$$(1 - TC)\left[\frac{I(N - I)}{N(N - 1)}\right](I - 1) \qquad (6)$$

By subtracting Equation 6 from Equation 2 and collecting terms, the average length of curb swept is obtained

$$E(Y) = \frac{N - I}{N}\left[N - \frac{I(I - 1)}{N - 1} - TC\,\frac{I(N - I)}{N - 1}\right] \qquad (7)$$

Using the relationships for compliance and noncompliance (Equations 3 and 4), after much algebraic manipulation a relationship between per cent swept and per cent compliance is obtained:

$$S = 100 \times \left[C^2 \left[\frac{N(2 - TC) - 1}{N - 1} \right] - NC^3 \left[\frac{1 - TC}{N - 1} \right] \right] = 100 \times \; [E(Y/N)] \quad (8)$$

Proceeding as above, the expression for expected curblength swept can be generalized.

For $A < M_0 \leqslant B$, where A and B are the largest and smallest integers bounding M_0, respectively, the expected value of curblength swept is given by

$$E(Y) = \frac{N - I}{N} \; (N - I \cdot TC) - \sum_{D=1}^{A} (D - TC) \left[\frac{N - 1 - D^C I - 1}{N^C I} \right] (I - 1) \quad (9)$$

RESULTS

With the relationships derived in the previous section, it is now possible to examine the effects of the various parameters on the amount of street swept. First, the linear dependence of the per cent of street swept, S, on the total clearance, TC, is shown in Figure 3 for a realistic value of intercar distance (M_0 is equal to two parking spaces). The plot drawn for N = 10 and N = 20, demonstrates that the relationship is virtually independent of the size of the block. In addition, it shows that the slopes of these curves are also a function of the compliance.

In Figure 4, the relationship between curb swept and compliance to parking regulations is plotted for various values of total clearance with $M_0 = 2$. As the total clearance increases, the compliance needed to achieve a desired level of sweeping also increases. In addition, the level of compliance (measured in per cent) is always greater than the per cent of street than can be swept. Furthermore, since the slope of the curves rises sharply for compliance greater than fifty per cent, the increase of per cent swept per unit rise in compliance will be greater for high compliance values.

In Figure 5, the effect of critical intercar distance is shown for blocks with ten and twenty spaces (TC = 1.25). The result is a set of piecewise linear curves with the value of each step decrease a function of compliance and the respective discrepancy. The value of the per cent swept at the discontinuities is equal to the higher value. Note that blocksize is now an important variable. For a block with twenty spaces, a given compliance implies a greater number of illegally parked cars than a ten space block with the same compliance. This, in turn, implies a greater number of intercar distances. Thus, the effect of M_0 on the N = 20 block will be greater than that on the N = 10 block for a given compliance.

Finally, per cent swept versus per cent compliance for different values of M_0 (TC = 1.25), is plotted in Figure 6 for N = 10. Again, the similarity of the curves indicates the slight effect the size of the block has on per

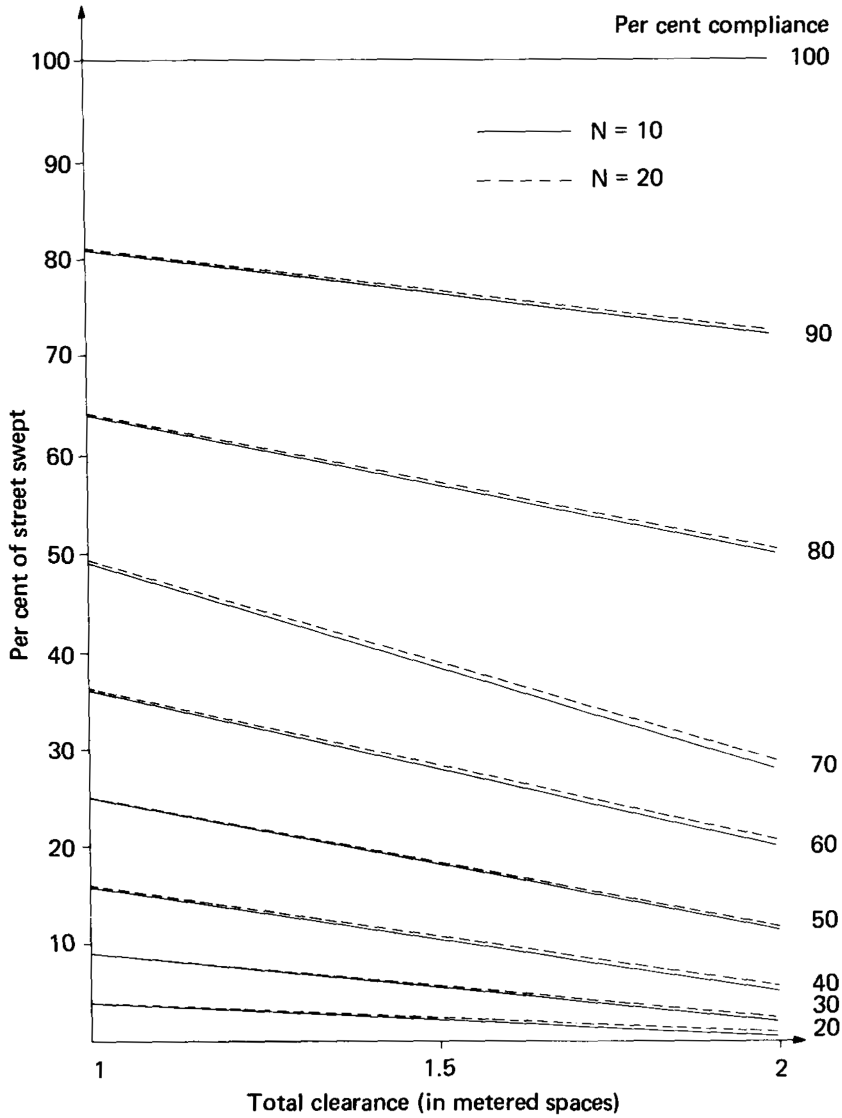


Figure 3. Per cent of street swept versus total clearance (M_0 = two spaces.)

cent of curb swept. For each case, as the critical intercar distance increases, the per cent of compliance needed to achieve a desired level of sweeping also increases. The insensitivity of per cent swept to changes in M_0 for high compliance results because these compliance values imply only one illegally parked car.

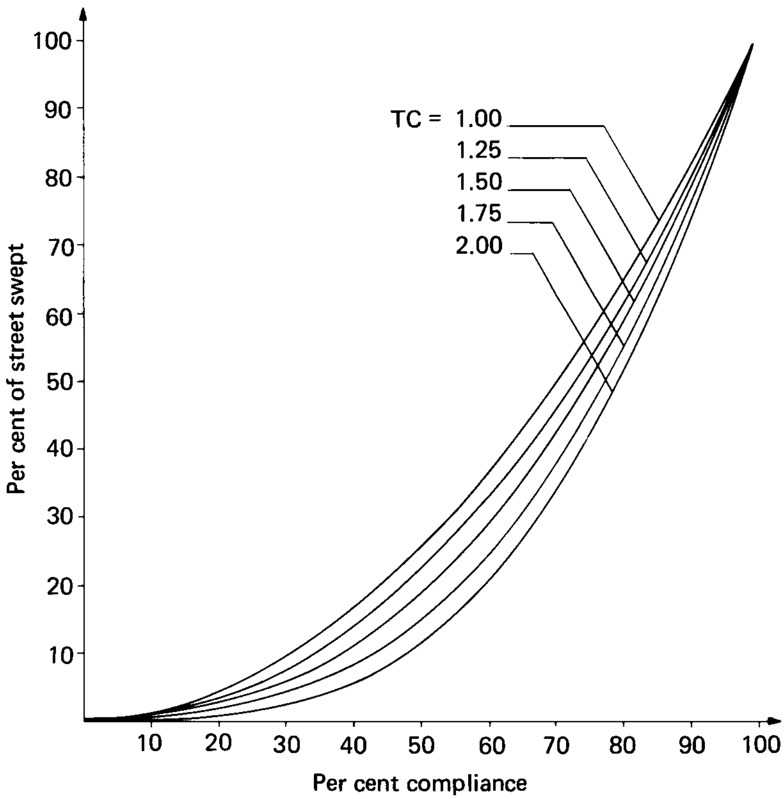


Figure 4. Per cent of street swept versus per cent compliance. (Metered Parking; M_0 = two spaces, N = 10.)

Non-Metered Parking

Curbs with continuous parking zones are next used to study the effects of the various parameters on the expected length of curb that can be swept. Continuous zones imply that the cars can be situated anywhere along a prescribed section of a block. However, the exact location of a car depends on the location of all other cars on the block. An analytical model based on the conditional probability that a car be at a certain location was developed for the two car case and an exact solution was obtained. This mathematical model is analogous to the one for the metered case.

Since the extension of this model to the many car case yielded unwieldy expressions for determining the expected length of curb swept, it became necessary to develop a computationally efficient algorithmic model to approximate the continuous parking case.

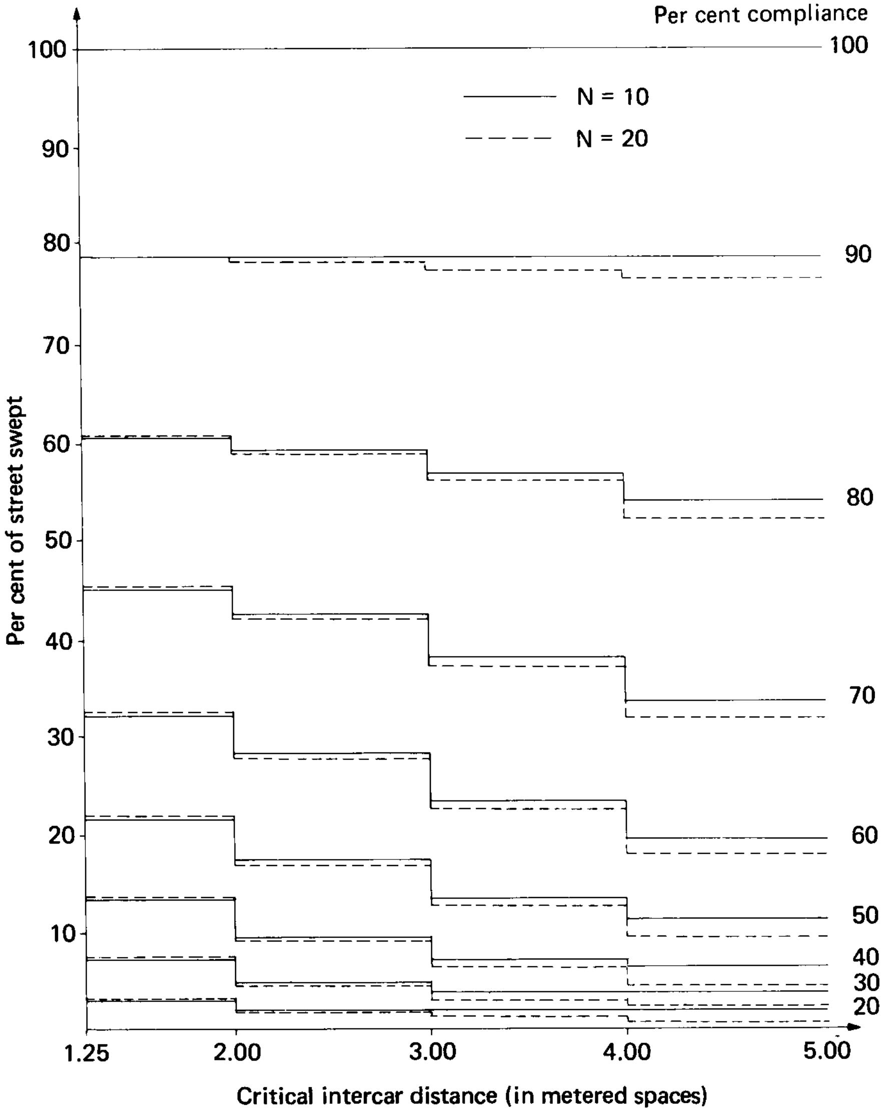


Figure 5. Per cent of street swept versus critical intercar distance. (TC = 1.25 spaces.)

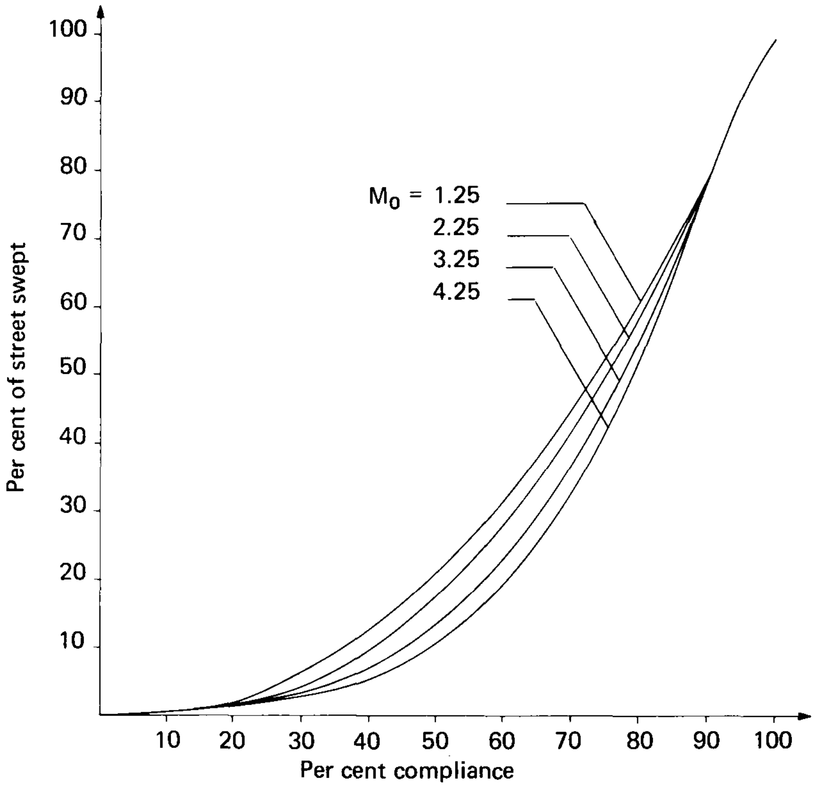


Figure 6. Per cent of street swept versus per cent compliance. (Metered Parking; TC = 1.25 spaces, N = 10.)

THE MOMENT MATCHING APPROXIMATION

In the development of the metered case the distribution described by a probability function f(x) that characterizes the relative position of I illegally parked cars on a block was assumed uniform, i.e., the probabilities of a car occupying any available space are equally likely. If, however, these distributions are interpreted as mass densities, where f(x) is a continuous function representing the "force" exerted on a block by any number of parked cars, then it is possible to determine discrete probability mass functions whose overall effect on the block is the same. This is accomplished by equating the first I moments of the continuous and the discrete distributions. Furthermore, the discrete points associated with the probability mass function (discrete) will correspond to the location of the cars on the block. In effect, this method determines the expected positions of the cars as opposed to the expected length of curb swept. With these positions, however, it is then possible to compute directly the corresponding curblength swept.

In order to locate I cars of zero length, i.e., I points, so that they are a discrete equivalent of a continuous distribution, it is necessary to determine an effective blocklength, LE, to compensate for the actual lengths of the cars L_i. This is accomplished by subtracting the length of the cars from the total length of the block, L

$$\text{Effective Blocklength} = LE = L - \sum_{i=1}^{I} L_i \tag{10}$$

If a new variable is defined

$$v = \frac{x}{LE} \tag{11}$$

so that $0 \leqslant v \leqslant 1$ when $0 \leqslant x \leqslant LE$, then the normalized moment matching equations become

$$\int_0^1 v^k f(v)dv = \sum_{i=0}^{I} w_i v_i^k \quad ; \quad k = 0,1,2,\ldots,I \tag{12}$$

where the v_i are the normalized unknown equivalent locations of the cars. The weighting coefficients, w_i, can be interpreted as a measure of the relative size of the cars.

Equations of this form belong to the class of equations known as Gaussian Quadrature.[3] The general formula for this class is given by

$$\int_a^b h(x)g(x)\,dx = \sum_{i=1}^{I} w_i g(x_i) \tag{13}$$

where $h(x) > 0$ and w_i and x_i are unknown parameters.

There is a well-known method[4] for solving this particular system of nonlinear equations.

By defining a sample polynomial, $\pi(x)$, as

$$\pi(x) = \prod_{i=1}^{I} (x - x_i) = \sum_{k=0}^{I} C_k x^k = C_0 + C_1 x + C_2 x^2 + \ldots + C_I x^I \tag{14}$$

where $\pi(x_i) = 0$; $i = 1, 2, \ldots I$, and where the C_k are computed from the equations

$$\sum_{k=0}^{I} C_k \, m_{k+j} = 0 \quad ; \quad j = 0,1,\ldots I - 1 \tag{15}$$

it is possible to solve for the values of x_i by setting Equation (14) equal to zero.

Thus the procedure for determining the per cent of curblength swept using the moment matching model can be summarized as follows:

1. compute 2I moments of x
2. solve a system of simultaneous linear equations for C_k
3. determine the roots of an I^{th} degree polynomial
4. compare the positions of the cars ($LE - x_i$) with M_0, FC and RC to determine the unswept curblength
5. subtract unswept curblength from length of block
6. divide swept curblength by length of block for per cent swept.

A computer program was written to carry out the computations indicated above.

Using the results of the exact solution for the continuous case as reference, the moment matching approximation and the metered solution for one and two illegally parked cars were compared: both models were good approximations to the continuous case. Further comparison of the two models reveals that the moment matching approximation converges more rapidly on the exact solution than the metered one. In addition, the rate of convergence is affected by the critical intercar distance and the total clearance. It is noted, however, that for small size blocks the metered solution provides a better approximation. These cases are analogous to blocks with a high density of illegally parked cars; hence the availability of parking spaces is greatly reduced. Thus a metered solution is the logical choice.

In order to establish a criterion for selecting the best approximation, the per cent differences for the metered and moment matching cases are plotted in Figure 7 for two illegally parked cars. A study of these curves suggests that if the following inequality is satisfied then the metered solution should be used;

$$L \leqslant [\sum_{i=1}^{I} L_i + I(TC) + (I-1)(M_0)] \tag{16}$$

otherwise the moment matching approximation should be used.

In the region where this inequality holds (as an example consider L = 80 feet in Figure 7), the single biased position of the cars as found in the moment matching case gives a lower value of area swept than in the metered case where all combinations are examined.

RESULTS

By combining the criterion for selecting the best approximation with an algorithm for metered and moment-matching solutions, the effects of the

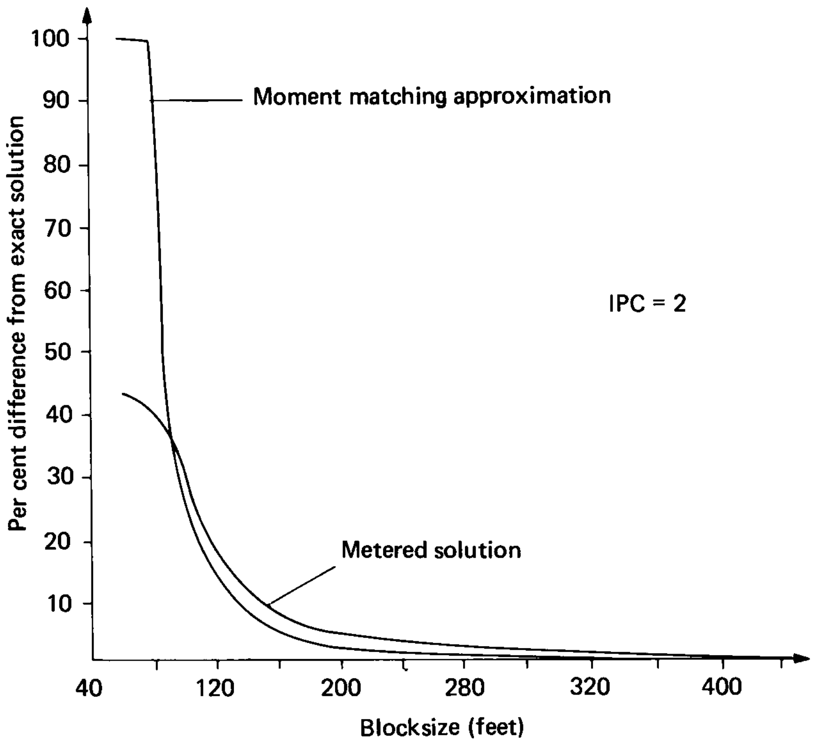


Figure 7. Comparison of moment matching approximation with metered solution. (M_0 = 25 feet, TC = 20 feet.)

various parameters on the relationship between per cent compliance and per cent swept was examined.

A study of the results indicates that, as in the metered case, the relationships are independent of changes in FC and RC as long as the total clearance remains the same. The dependence of the per cent of street swept on total clearance is illustrated in Figure 8. The plot, drawn for L = 200 and 400 feet, demonstrates that the relationship is independent of the size of the block. In addition, it shows that the exact slope of the curves is also a function of the compliance.

In Figure 9, the relationship between street swept and compliance for various values of total clearance is shown (M_0 = 40 ft.). As the total clearance increases, the per cent compliance needed to achieve a certain level of sweeping also increases. In addition, the per cent compliance is always greater than the per cent of street that can be swept.

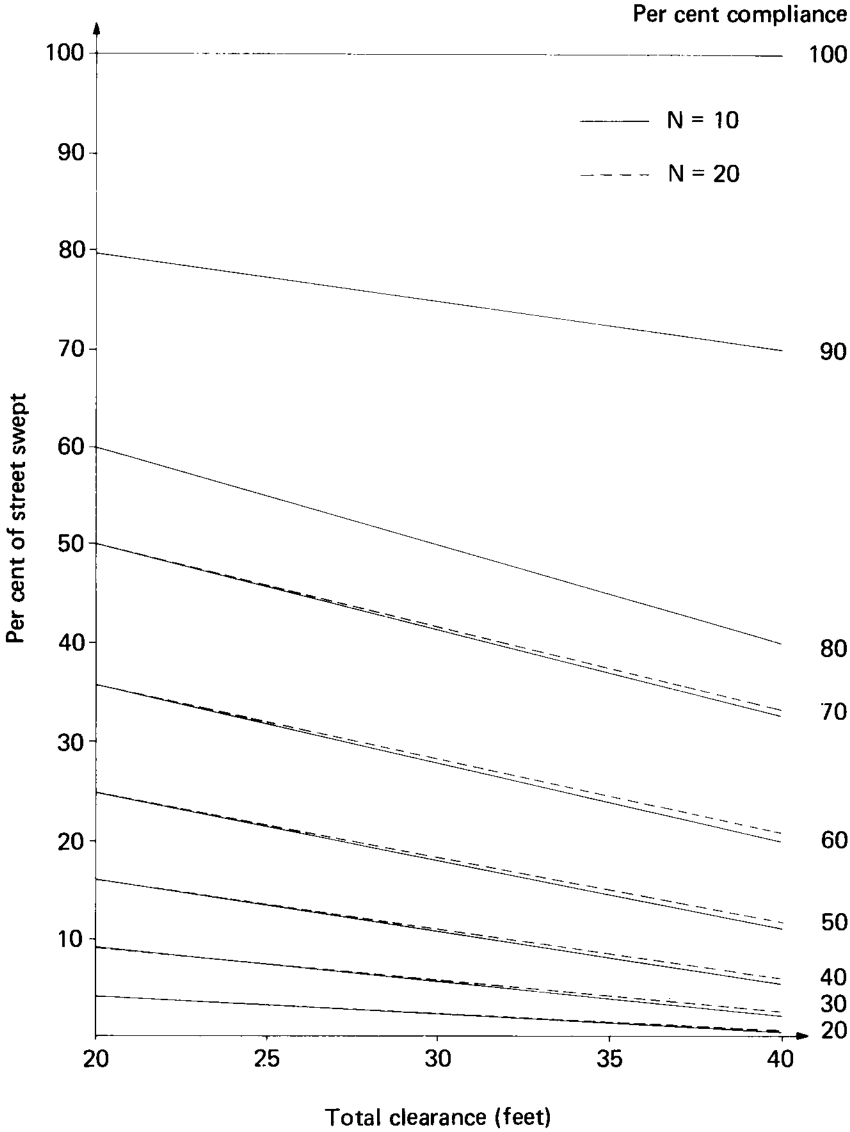


Figure 8. Per cent of street swept versus total Clearance.
(M_0 = 40 feet.)

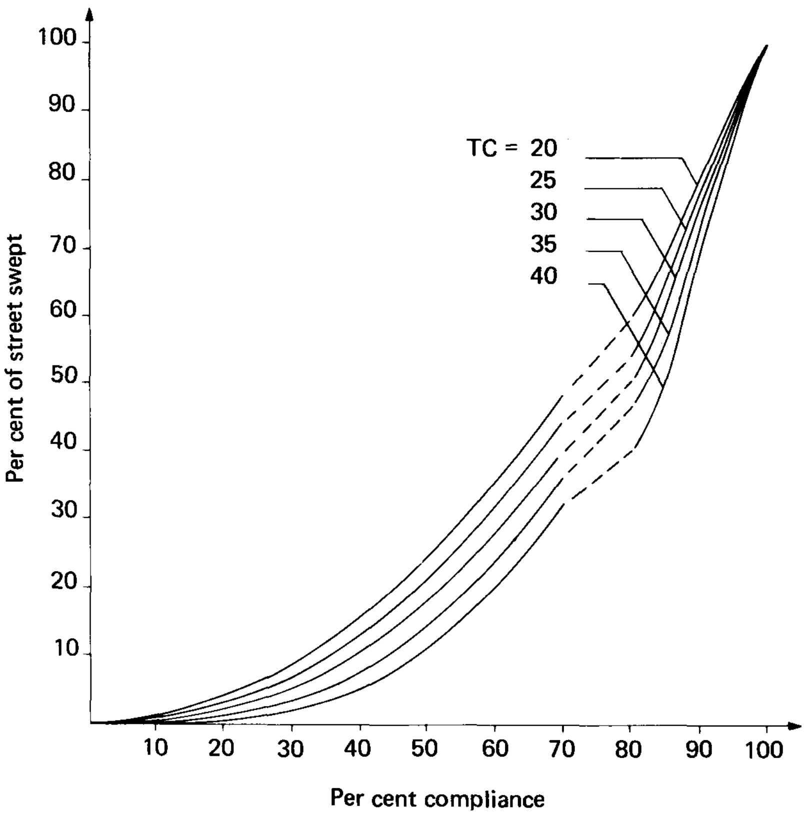


Figure 9. Per cent of street swept versus per cent compliance. (Non-metered Parking, M_0 = 40 feet, N = 10.)

In Figure 10 the effect of critical intercar distance on per cent swept is shown for blocks 200 and 400 feet long. In the region where the moment matching model is used, the per cent swept for each value of compliance is constant for the range of M_0 considered, whereas when the metered-zone approximation is used the result is the expected piecewise linear curve. These constant values occur because the moment matching solution at the respective levels of compliance positions the cars far enough apart so that only very large (and impractical) critical intercar distances M_0 will affect the per cent swept. In addition, as in the metered case, block size is a parameter that cannot be neglected.

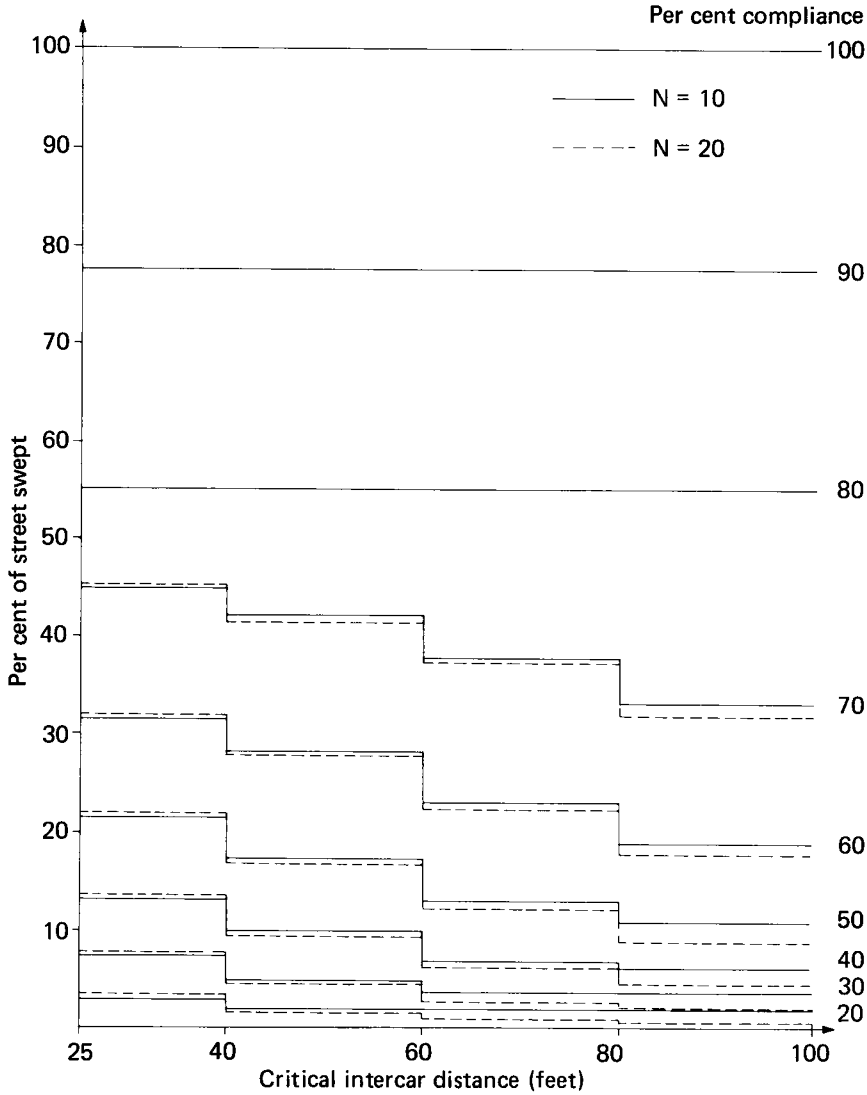


Figure 10. Per cent of street swept versus critical intercar distance. (TC = 25 feet.)

Per cent swept versus per cent compliance for various values of M_0 (TC = 25) is plotted for a block of length 200 feet in Figure 11. Once again, the dashed-line regions correspond to the changing of the models. These curves verify the conclusions of the preceding paragraph. In addition, as the critical intercar distance increases, the level of compliance needed for a given level of sweeping also increases.

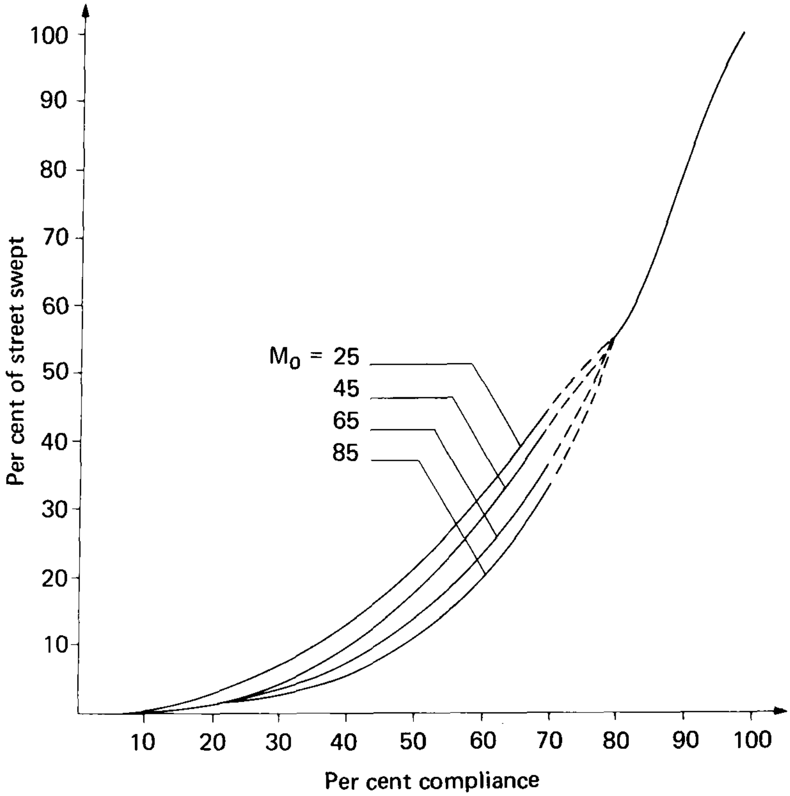
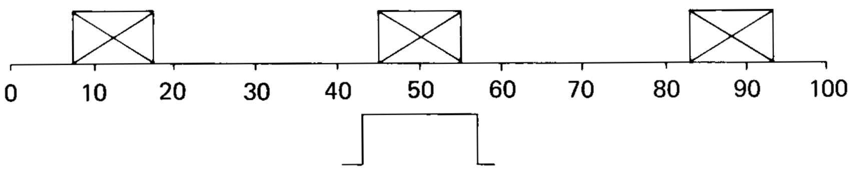
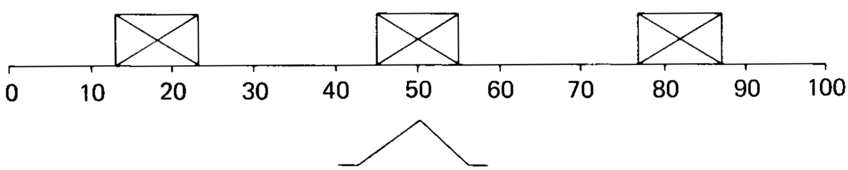


Figure 11. Per cent of street swept versus per cent compliance. (Non-metered Parking; TC = 25 feet, N = 10.)

Finally, the effect of different probability density functions for the distribution of the illegally parked cars was examined. Two triangular PDF's are considered, one corresponding to the assumption that the parked cars tend to bunch up in the middle of the block (peak at 0.5, f(x) = Δ) and the other that the cars tend to park at one end of the block (peak at 1, f(x) = ◿). A study of the results indicated that the different PDF's had little effect on the per cent of street swept. This can be explained by looking at the positions of three cars on a 200 foot block for the various distributions considered (Figure 12). The shifting of the positions for each distribution is small enough so that the model will have transferred to the metered solution before any deviation due to position changes occurs. Thus, the two-region model of the continuous parking case is insensitive to the distribution of illegally parked cars assumed. Note that the last distribution is non-symmetric and that the resulting equivalent lengths of the cars vary.

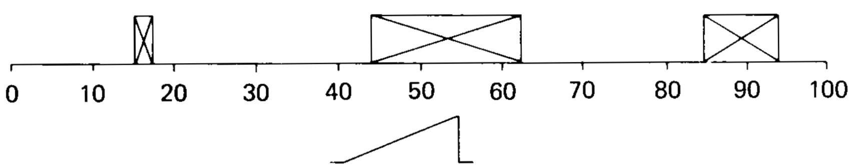


(a) Uniform distribution



(b) Triangular distribution

(peak at .5)



(c) Triangular distribution

(peak at 1.)

Figure 12. Effect of various distributions on the positions of illegally parked cars. (N = 10, I = 3.)

CONCLUSIONS

The results that have been presented demonstrate unequivocally the well known dependence of street cleaning by mechanical brooms on the compliance to parking regulations. For the first time, a simple analytical model was constructed that permits the quantitative evaluation of the degradation of performance due to illegally parked cars on both metered and non-metered streets. These results are virtually independent of the length of the block. Even in the unrealistic case of perfect maneuverability, i.e., no clearances are necessary, a compliance higher than 30 per cent is required for more than 15 per cent of the curb to be swept (on the average). For realistic sweeper maneuverability, 50 per cent compliance yields at best 25 per cent of curb swept. Since for a street to be clean at least 70-80 per cent of the curb must be swept (if not more in areas of high litter generation), the compliance to parking regulations must be kept at 85 per cent or higher.

These results also demonstrate the rapid degradation in street cleaning performance if the maneuverability of a machine is impaired or if the driver is not skilled. The effect of an unskilled or inattentive operator would be to decrease the curblength swept for a given level of compliance. Hence, the best results are obtained by skilled drivers intent on utilizing the machine's maneuverability to advantage.

Finally, the results presented here form a part of a larger research program on urban street cleaning that includes the analysis of the effect of enforcement on compliance, the determination of enforcement policies, and the routing of mechanical sweepers.

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