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ALTERNATIVE FORMS OF AGGREGATION IN THE ANALYTIC HIERARCHY PROCESS: ORDERED WEIGHTED AVERAGING OPERATORS

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ABSTRACT

The analytic hierarchy process (AHP) methodology was introduced by Thomas Saaty and has had numerous applications in a wide range of contexts. In a common three-level hierarchy, AHP involves the aggregation of criterion importance weights (or priorities) and the performance scores (or priorities) of alternatives as the sum of the products of weights and scores for each alternative. The multiplicative AHP (MAHP) involves the multiplicative aggregation of performance scores raised to the power of the criterion weights and has been seen by many individuals (notably, Freerk Lootsma and John Barzilai) as an alternative more desirable structure. It is suggested that alternative forms of aggregation of performance scores and criterion weights might be more useful, in particular the ordered weighted averaging (OWA) operator introduced by Ronald Yager. The choice of weights in an OWA operator may be guided by a linguistic quantifier involving the importance weights associated with each criterion. The geometric ordered weighted averaging (GOWA) operator is also considered as a possibility for aggregation in the MAHP. An example is given relating to the location of a Green Bridge Link (for public transport and non-motorized modes of transport) in Brisbane, Queensland, Australia. The four approaches (alternatives) to the identified site of the Green Bridge (Dutton Park) are a Full Busway, Boggo Road, Cornwall Street, and Kent Street.

INTRODUCTION

The analytic hierarchy process (AHP) methodology [1-5] was introduced by Thomas Saaty in the 1970s and has had numerous applications in a wide range of contexts. A recent review of some AHP applications is given by Vaidya and Kumar [6]. Earlier reviews of applications were given by Zahedi [7] and Shim [8]. The AHP has also been the subject of special issues of journals such as *Mathematical Modelling* (Vol. 3-5, 1987), *European Journal of Operational Research* (Vol. 48, No. 1, 1990), and *Socio-Economic Planning Sciences* (Vol. 20, No. 6, 1986; Vol. 25, No. 2, 1991).

The AHP aggregates criterion weights and project performance scores in a simple additive weighting format. It is suggested that alternative forms of aggregation might be more useful, in particular, the ordered weighted averaging (OWA) operator introduced by Yager [9] and discussed (both in a numeric and a linguistic context) in this journal [10, 11]. The choice of weights in an OWA operator may also be guided by a linguistic quantifier involving the importance weights associated with each criterion.

ANALYTIC HIERARCHY PROCESS (AHP)

The AHP explicitly recognizes the hierarchical structure of decision problems. A hierarchical structure comprising homogeneous clusters of elements is a means of coping with complexity [12]. However, in addition to hierarchical structuring, the AHP is based on two other compelling concepts, namely, the use of pairwise, relative comparisons, and the use of redundancy in judgments (that is, more pairwise comparisons are obtained than necessary to identify a "priority" for each element in the hierarchy). Relative judgments (comparisons of an element relative to information about another element held in "short-term memory") are assumed to be more easily generated and more meaningful than absolute judgments (that is, ratings, which, in a sense, involve comparison of an element relative to information about another element stored in "long-term memory," and perhaps in new situations relative to no information) [4]. Redundancy reduces errors and provides a measure of consistency of judgments.

The limited "cognitive capacity" of individuals in terms of both "short-term memory" and "discriminability" (channel capacity) identified by Miller [13] is cited in support of the AHP and the need to hierarchically structure a decision problem into manageable "chunks" Miller [13] conjectured that there is an upper limit to an individual's capacity to process information on simultaneously interacting elements with reliable accuracy and with validity (7 plus or minus 2 elements). In the light of this "magic number," Saaty and Ozdemir [14] suggest that the number of elements in each group should be no more than 7. This recommendation is based on the consistency of information derived from relations among the elements.

Thus, the AHP represents a decision/evaluation problem hierarchically and involves pairwise comparisons of elements (projects, criteria, sub-criteria, etc.) at each level with respect to elements at the adjacent higher level. In a three-level hierarchy, each project is compared to each other project with respect to each criterion, and each criterion is compared relative to each other criterion with respect some overriding goal. Comparison is in terms of the extent to which one element (project, criterion) "dominates" another element within the same "cluster" of elements. Such subjectively determined pairwise comparisons (judgments) are commonly expressed on a 1-9 (*response*) scale of "dominance" (e.g., importance, performance, preference). For example, if project A performs outstandingly relative to project B with respect to criterion C_1 then "9" might be used to represent this dominance in terms of performance. If A and B perform equally with respect to C_1, then a score of "1" would be used, and other scores used as appropriate to represent intermediate degrees of dominance. Pairwise comparisons are considered to be *reciprocal* such that, for example, if the dominance of A relative to B for C_1 is say "5," then the dominance of B relative to A for C_1 must be "1/5." Numbers 1, 3, 5, 7, 9 are associated with verbal expressions of dominance (respectively, "equal," "'weak," "strong," "very strong," "absolute") and the numbers 2, 4, 6, 8 represent intermediate values between adjacent scale values. Criteria are then compared to each other in terms of their importance in achieving some overall goal (e.g., select a "best" project), again using a 1-9 scale. Numbers 1, 3, 5, 7, 9 are now associated with verbal expressions of relative importance (respectively, "equal," "wealth," "strong," "very strong," "absolute") and the numbers 2, 4, 6, 8 represent intermediate values between adjacent scale values. With reciprocal values, the complete Saaty response scale is thus (1/9,9).

For each reciprocal pairwise comparison matrix of Q elements, $\underline{A} = [a_{ij}]_{QxQ}$, "scores" or "priorities" representing the "dominance" of elements may be derived by solving for $\underline{z}^T = (z_1, z_2, ..., z_Q)$ in the matrix equation [1-5].

$$\underline{A}\ \underline{z} = \lambda^{max}\ \underline{z}$$

$\underline{z}$ is the *eigenvector* associated with the largest (*Perron-Frobenius*) (right) *eigenvalue,* λ^{max}, of $\underline{A}$. The right eigenvector is selected because of the nature of the dominance in the pairwise comparison matrix (dominance of row over column). These scores are normalized to $\underline{w} = (w_1, w_2, ..., w_Q)$ where $w_i = z_i/\Sigma_{k=1,Q}\, z_k$ (I = 1, ...,Q). Thus

$$\underline{w} = \lim_{k\to\infty}\left(\frac{\underline{A}^k \underline{e}^T}{\underline{e}\underline{A}^k \underline{e}^T}\right)$$

Here, $\underline{e}$ = (1, 1, ..., 1) is of dimension Q. The *Perron-Frobenius* theorem on non-negative square matrices, $\underline{A}$, states that $\underline{A}$ has a non-negative real eigenvalue (λ^{max}), no real eigenvalue of $\underline{A}$ can have an absolute value larger than the largest real eigenvalue and at least one right eigenvector and one left eigenvector associated with λ^{max} are *semipositive* (that is, all elements are non-negative and at least one element is positive) [15].

Each pairwise comparison a_{ij} is an estimate of the ratio w_i/w_j. Thus, given priorities, $a_{ij} \approx w_i/w_j$, and in the perfectly consistent case, $a_{ij} = w_i/w_j$. Normalized scores (priorities) associated with each pairwise comparison matrix are concatenated throughout the hierarchical structure to form scores for each lowest level project. That is, for a three-level hierarchy,

$$u_i = \sum_{j=1,J} w_j p_{ij}$$

where u_i is the overall performance score of alternative I (I = 1, 2, ..., I), w_j is the weight (priority) of criterion j (j = 1, 2, ..., J) and p_{ij} is the performance score (priority) of alternative I with respect to criterion j.

Various approximations have been proposed as an alternative to the calculation of the principal eigenvector of a pairwise comparison matrix, e.g., [16]. Three common approximations for finding the priorities of $\underline{A} = [a_{ij}]_{QxQ}$ are the *average of the normalized columns, the sum of the normalized rows,* and the *normalized geometric row means,* respectively, as follows

$$w_i = \frac{1}{Q} \sum_{j=1,Q} \left(\frac{a_{ij}}{\sum_{r=1,Q} a_{rj}} \right) \qquad w_i = \sum_{j=1,Q} \left(\frac{a_{ij}}{\sum_{r=1,Q} \sum_{s=1,Q} a_{rs}} \right) \qquad w_i = \frac{\left(\prod_{j=1,Q} a_{ij} \right)^{1/Q}}{\sum_{r=1,Q} \left(\prod_{s=1,Q} a_{rs} \right)^{1/Q}}$$

Each of these yields correct results when consistency prevails, that is when $a_{ij} = w_i/w_j$. The average of the normalized columns is commonly presented as a good approximation to the Perron-Frobenius right eigenvector (e.g., [17, 18]).

However, the geometric mean has been advocated as more than an approximate solution to the principal right eigenvector. Crawford and Williams [19] show the geometric mean is a unique solution to the minimization of a logarithmic least squares function (see below). Barzilai, Cook, and Golany [20] and Barzilai [21] derive the geometric mean from an axiomatic basis as the only acceptable solution to estimating priorities from a pairwise comparison matrix.

However, evidence in support of the principal eigenvector method has been given by Kumar and Ganesh [22] using a simulation analysis based on the concept of approximating a continuous pairwise comparison matrix by its closest discretized pairwise comparison matrix. Saaty and Hu [23] show that for inconsistent judgments, their transitivity effects the final result, and that the eigenvector captures the transitivity uniquely and is the only way to obtain a correct ranking. Saaty [24] further supports the principal eigenvector as the only acceptable method of deriving priorities from pairwise comparison matrices.

Nevertheless, various other methods for deriving priorities from pairwise comparison matrices have been explored (e.g., [25-29]). Jensen [30] strongly advocates a least-squares approach to priority estimation. Recently, Gass and Rapcsák [31] have recently advocated the *singular value decomposition* of the pairwise reciprocal matrix, using the left and right *singular vectors* belonging to the largest *singular value* of matrix $\underline{A}$.

Priorities based on $\underline{A}_{QxQ}$ can be approximated by a uniquely determined normalized positive weight vector

$$w_i = \frac{u_i + \frac{1}{v_i}}{\sum_{j=1,Q} \left(u_j + \frac{1}{v_j} \right)}$$

Here u_i and v_i are left and right singular vector values, respectively, and w_i is the priority weight vector. This result was obtained by solving a distance minimization and a measure theoretic (*Kullbach-Leibler I-divergence*) minimization problem with a unique solution for each. Srdjevic [32] reviews many alternative prioritization methods and has advocated combining different priority methods in AHP.

A further aspect of the AHP is the calculation of the consistency of pairwise comparison judgments. Saaty [2] recommends a check for the consistency using the index $\mu = (\lambda^{max} - Q)/(Q - 1)$. The consistency index, μ, is compared to a random index and Saaty suggests that this ratio should be <0.1 for acceptable consistency of pairwise comparisons. That is, inconsistency is considered unacceptable if it is above that found in 90% of random matrices of the same size. However, Peláez and Lamata [33] propose a different method based upon the average of the consistency index of all matrix transitivities. This involves a comparison of elements (e.g., criteria) E_i, E_j, E_k, by examining the determinant of the sub-matrix formed from these three elements of a_{ij} and the product $a_{ik}a_{kj}$ for all $Q!/(Q - 3)3!$ $(Q \geq 3)$ possibilities. The determinant is zero for perfect transitivity ($aij = a_{ik}a_{kj}$), but greater than zero otherwise.

Lootsma and Schuijt [34] outline a multiplicative variant of AHP which adjusts for contended flaws in AHP using a rating on a logarithmic response scale (which replaces (1/9-9) Saaty response scale), eigenvector calculation replaced by the geometric mean, and aggregation of scores by weighted sum replaced by the product of the relative scores of each alternative weighted by the power of weights obtained from analysis of hierarchical elements (criteria) above the alternatives. Thus the multiplicative AHP (MAHP) involves

$$u_i = \prod_{j=1,J} p_{ij}^{\ w_j}$$

Here u_i is the overall performance score of alternative I (I = 1, 2, ..., I), w_j is the weight of criterion j (j = 1, 2, ..., J) and p_{ij} is the performance score of alternative I with respect to criterion j. The MAHP is based on minimizing a logarithmic least squares function

$$l = \sum_{i=1,Q} \sum_{j>i} \ln r_{ij} - \ln v_i + \ln v_j)$$

where $w_j = \ln v_j$. The solution to this problem is,

$$w_j = \left(\prod_{j=1,Q} r_{ij} \right)^{\frac{1}{j}}$$

Thus, priorities from pairwise comparison matrices are based on the geometric mean of rows.

Further, a geometric scale ($e^{\lambda(-8)}$,$e^{\lambda(-7)}$, $e^{\lambda(-6)}$, ..., $e^{\lambda(0)}$, ..., $e^{\lambda(6)}$, $e^{\lambda(7)}$, $e^{\lambda(8)}$), is used to determine pairwise comparisons rather than the (1/9,9) Saaty response scale, and a scale values are moderated by a scale factor, λ, which is different for performance and criterion priorities [35, 36]. Lootsma recommends that in $r_{ij} = e^{\lambda\delta j}$, $\lambda = \ln(\sqrt{2}) = 0.347$ in calculating criterion weights and $\lambda = \ln(2) = 0.693$ in calculating performance scores. δ_{ij} are integers in the range (–8,8) selected by the decision maker in the same way as Saaty response scale values (1/9, 9) are selected. Effectively, the scale values yield a shorter scale, approximately, $r_{ij \in} (2^{-4}$, $2^{-7/2}$, 2^{-3}, $2^{-5/2, \ldots}$ 2^0, ..., $2^{5/2}$, 2^3, $2^{7/2}$ 2^4), for comparing the relative importance of criteria and a longer scale, approximately, $r_{ij \in}$ (2^{-8}, 2^{-7}, 2^{-6}, 2^{-5}, ..., 2^0, ..., 2^5, 2^6, 2^7, 2^8), for comparing the performance of alternatives. Thus,

$$w_j = \left(\prod_{j=1,Q} r_{ij} \right)^{\frac{1}{j}} = \left(\prod_{j=1,Q} e^{(\lambda \delta_{ij})} \right)^{\frac{1}{j}}$$

There is no consistency measure in REMBRANDT, though it is possible to identify inconsistencies between pairwise judgments δ_{ij} and the resulting ratio of priorities, w_i/w_j.

The MAHP and REMBRANDT (Ratio Estimation in Magnitudes or decibels to Rate Alternatives which are Non-Dominated), involving a geometric response scale, are explored further by Lootsma [35, 36]. Debate on the merits or otherwise of the MAHP has been given in articles [37-39] and responded to by Lootsma and Barzilai [40]. Olson, Fliedner, and Currie [41] and Olson [42] compare REMBRANDT to AHP. Stam and Duarte Silva [43] outline some properties of MAHP that appear to have eluded attention and draw some analogy with the Cobb-Douglas production function used in macro-economic modelling to relate production factors (capital, labor) to output.

Various fuzzy extensions of AHP have been developed in recent years (e.g., [44-48]). van Laarhoven and Pedrycz [49] propose a fuzzy version of extensions to the AHP developed by de Graan [50]. Boender, de Graan, and Lootsma [51] propose a modification of the method. Chang [52] proposes *extent analysis* method for fuzzy AHP, elaborated by Enea and Piazza [53].

Fuzzy methods weaken the demands for precision is pairwise comparisons and are considered to be cognitively less demanding and more adequately reflect the decision maker's attitude with respect to risk and confidence in their subjective assessments [46].

APPLICATION OF AHP TO THE GREEN BRIDGE LINK PROJECT

The *Green Bridge Link* has been developed by Brisbane City Council (Brisbane, Queensland, Australia) to address some of the transport demands arising from this population growth in southeast Queensland. Though travel to Brisbane CBD remains important today as in the past, there is also significant and increasing demand for cross-city travel, which would be facilitated by orbital public transport linkages. Presently, the radial nature of Brisbane's transport network, limited river crossings, and the dispersion of non-CBD destinations has made it difficult for public transport to penetrate the cross-city travel market. The radial nature of the transport network generally requires travel into the CBD and then out on a different corridor to access a destination. The University of Queensland, in particular, is a major non-CBD destination that is difficult to access from the southeast sector of Brisbane on the radial transport network.

One key issue identified through various regional, city, and local planning exercises for Brisbane is that of providing a transport network, comprising private and public transport, cycling, and walking, that will remain sustainable given increased travel demand and a dispersed settlement pattern. The large, fast-growing population is creating increasing demand for travel on a transport network that has reached maturity and in many places is at capacity.

The Green Bridge Link aims to efficiently move high volumes of people along a purpose built public transport corridor. The corridor will run westwards from the Buranda Bus/Rail Interchange, crossing the Brisbane River at Dutton Park, and then entering the University of Queensland (UQ) St. Lucia campus [54].

Three possible locations for the Green Bridge Link were identified on the east bank of Brisbane River: Dutton Park, Yeronga, and West End. Twelve preliminary eastern approaches linking the bridge and the South East Busway were identified. After evaluating eastern approaches based on a number of criteria such as travel time, construction costs, operational costs, land use, rail links, pedestrian and cycle links, and engineering considerations, Dutton Park was selected as a preferred bridge location [54]. The four alternative approaches associated with

Dutton Park location are: (A_1) Full Busway approach; (A_2) Boggo Road approach; (A_3) Cornwall Street approach; (A_4) Kent Street approach.

This article will use these four alternatives evaluated against eight criteria as follows: C_1: Links with trip generators; C_2: Patronage growth potential; C_3: Value to the transport network; C_4: Local amenity and local impacts; C_5: Support for complementary development; C_6: Ease and success of implementation; C_7:Project cost; and C_8: Future opportunity.

In the impact study, these criteria were disaggregated and performance scores derived for each alternative with respect to each sub-criterion. Scores were then aggregated for each criterion, and these aggregate scores were used to guide the pairwise comparisons assigned to each of the four alternatives/approaches with respect to the eight criteria. These pairwise comparisons are shown in the Appendix. In addition, the pairwise comparisons of criteria with respect to the overall goal of selecting a good approach to the Green Bridge Link location are shown in the Appendix.

Based on the hierarchical structure in Figure 1 and the pairwise comparison matrices given in the Appendix, the conventional AHP yields the following results

Alternative/approach	Overall score
A_1 – Full Busway	**0.331***
A_2 – Boggo Road	0.264
A_3 – Cornwall Street	0.251
A_4 – Kent Street	0.153

*Bold indicates better performance.

These results indicate preference for A_1 (followed by A_2) based on the pairwise comparison matrices shown in the Appendix. Consistency indices (μ) are also shown in the Appendix. These are all acceptable, except for C_6, though as pairwise comparisons were selected to be as close as possible to the direct ratings in the Green Bridge Link impact study [54], they have not been adjusted. The MAHP (based on priorities calculated as the normalized geometric row means) yields the following results

Alternative/approach	Overall score
A_1 – Full Busway	**0.243***
A_2 – Boggo Road	0.235
A_3 – Cornwall Street	0.164
A_4 – Kent Street	0.109

*Bold indicates better performance.

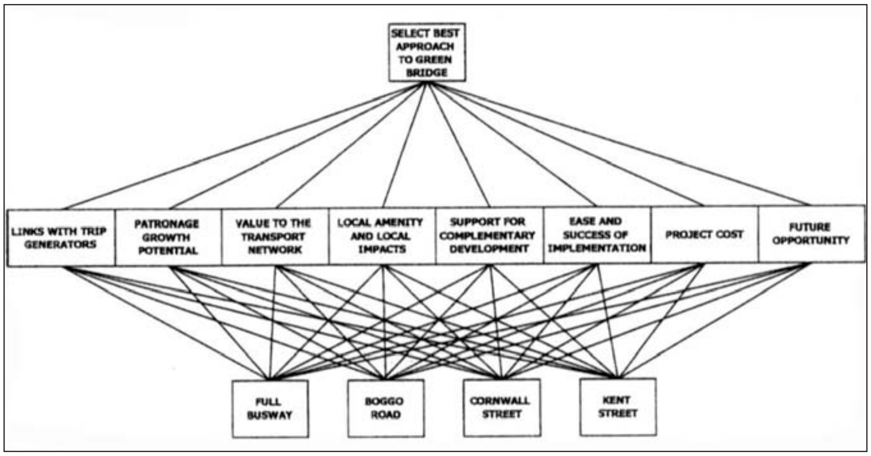

Figure 1. Hierarchical structure for Green Bridge Link.

These results, using pairwise comparisons drawn from the Saaty [1/9,9] response scale, indicate preference for A_1 (followed more closely by A_2).

ORDERED WEIGHTED AVERAGING AGGREGATION OPERATORS

The ordered weighted averaging (OWA) operator for aggregating fuzzy subsets was introduced by Yager [9]. It has been elaborated in this article [10, 11]. An OWA operator (of dimension J) is represented as

$$OWA(\underline{\alpha},\underline{a}) = \Sigma_{j=1,J}\alpha_j b_j$$

where b_j is the jth largest element of the values $\underline{a} = [a_1, a_2, ..., a_J]$.

OWA operator weights, $\underline{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_J]$ are associated with the position of b_j and are such $\alpha_j \in [0, 1]$ and $\Sigma_{j=1,J}\alpha_j = 1$. Note that α_j is associated with a particular ordered position j of the arguments (project performance along criteria) and is not a reflection of the importance (salience, significance) of criteria in the context of project evaluation.

The OWA operator includes the commonly used maximum and minimum operators [9, 10] and the arithmetic mean operator for appropriate choice of operator weights represented as $\underline{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_J]$. In particular, the OWA operator is bounded such that OWA $((0, 0, ..., 1), \underline{a}) \leq OWA(\underline{\alpha}, \underline{a}) \leq OWA([1, 0, ..., 0], \underline{a})$. Thus from the definition of the OWA operator, OWA $([0, 0, ..., 1], \underline{a}) = \min_{j=1,J} a_j$ and OWA $([1, 0, ..., 0], \underline{a}) = \max_{j=1,J} a_j$ so that extreme OWA operators are the "and" and "or" operators [9, 10]. The arithmetic average corresponds to the

OWA operator OWA ((1/J, 1/J, ..., 1/J), $\underline{a}$). The "and" (minimum) provides no compensation in that a high grade of membership with respect to one factor cannot offset (or compensate for) a low grade of membership with respect to another factor. The "or" (maximum) provides full compensation. "And" reflects a conservative/pessimistic attitudinal character on the part of the decision maker, "or" reflects a risk-taking/optimistic character.

OWA weights can be parameterized by a function Q: $[0,1] \rightarrow [0,1]$ having the properties: (1) $Q(0) = 0$; (2) $Q(1) = 1$; and (3) $Q(x) \geq Q(y)$ if $x > y$. Yager [55] refers to Q as a "basic unit-interval monotonic" (BUM) function. BUM functions are monotonic and fixed at the end points. Using a BUM function, we can obtain the quantifier-guided OWA weights as $\alpha_j = Q(j/J) - Q((j-1)/J)$. It can be shown that these weights satisfy the conditions $a_j \in [0,1]$ and $\Sigma_{j=1,J}\ \alpha_j = 1$ [10].

The linguistic quantifier, Q, may take a number of forms [9, 10]. The classical quantifier, "all" is $Q(j/J) = 0$ for $j < J$ and $Q(J/J) = Q(1) = 1$ in which case $\underline{\alpha} = (0, 0, ..., 1)$. The classical quantifier, "at least one," is $Q(j/J) = 1$ for $j \geq 1$ in which case $\underline{\alpha} = [1, 0, ..., 0]$. The quantifier "average" is defined as $Q(r) = r$ yielding $\underline{\alpha} = [1/J, 1/J, ..., 1/J]$. The quantifier, "most" is defined as a BUM function, $Q(r) = r^2$, $r \in [0,1]$.

When criterion weights, w_j, are used,

$$\alpha_j = Q\left(\frac{\sum_{k=1,j} u_k}{\sum_{k=1,J} u_k}\right) - Q\left(\frac{\sum_{k=1,j-1} u_k}{\sum_{k=1,J} u_k}\right)$$

Here u_j is the importance weight associated with b_j, i.e., the importance of the criterion for which a given project has the jth largest performance score (see [10]). The above yields a weighted ordered weighted averaging (WOWA) operator [56-58]. The results of OWA aggregation using the linguistic quantifier "most" yield the following results

Alternative/approach	Overall score
A_1 – Full Busway	**0.215***
A_2 – Boggo Road	0.192
A_3 – Cornwall Street	0.137
A_4 – Kent Street	0.086

*Bold indicates better performance.

Here it is implicitly assumed that priorities derived in the AHP are normalized to sum to unity as in the conventional AHP.

GEOMETRIC ORDERED WEIGHTED AVERAGING OPERATOR

A geometric ordered weighted average (GOWA) [58-60] may be defined as

$$\text{GOWA}\ (\underline{\alpha}, \underline{a}) = \prod_{j=1,J} b_j^{\alpha_j}$$

where b_j is the jth largest element of the values $\underline{a} = [a_1, a_2, ..., a_J]$. GOWA operator weights, $\alpha = [\alpha_1, \alpha_2, ..., \alpha_J]$ are associated with the position of b_j and are such $a_j \in [0,1]$ and $\Sigma_{j=1,J}\alpha_j = 1$. Again the GOWA operator is bounded such that GOWA ([0, 0, ..., 1], $\underline{a}$) $\leq$ GOWA ($\underline{\alpha}$, $\underline{a}$) $\leq$ OWA ([1, 0,.... 0], $\underline{a}$). Thus from the definition of the GOWA operator, GOWA ((0, 0, ..., 1], $\underline{a}$) = $\min_{j=1,J} a_j$ and GOWA ([1, 0, ..., 0], $\underline{a}$). = $\max_{j=1,J} a_j$ [59].

Again, OWA weights may be obtained as $a_j = Q(j/J) - Q((j-1)/J)$ satisfying the conditions $\alpha_j \in [0,1]$ and $\Sigma_{j=1,J}\ \alpha_j = 1$. Using criterion weights w_j, the importance weight associated with b_j, positional weights, α_j, may be derived analogous to those for the OWA. Using the GOWA aggregation, with the quantifier, "most," defined as $Q(r) = r^2$, yields

Alternative/approach	Overall score
A_1 – Full Busway	0.105
A_2 – Boggo Road	**0.181***
A_3 – Cornwall Street	0.096
A_4 – Kent Street	0.070

*Bold indicates better performance.

Again it is implicitly assumed that priorities derived in the MAHP are normalized to sum to unity as in the MAHP. The results of the various aggregations are shown in Tables 1 and 2.

Table 1. AHP Scores and Quantifier-Guided OWA (QG_OWA) Scores

	AHP	QG_OWA "most"	QG_OWA "all" MIN	QG_OWA "not_none" MAX	QG_OWA "average" AVERAGE
A_1	**0.331***	**0.215**	0.054	0.595	**0.331**
A_2	0.264	0.192	**0.133**	**0.608**	0.264
A_3	0.251	0.137	0.052	0.650	0.251
A_4	0.153	0.086	0.045	0.402	0.153

*Bold indicates better performance.

Table 2. MAHP Scores and Quantifier-Guided GOWA (QG_GOWA) Scores

	MAHP	QG_GOWA "most"	QG_GOWA "all" MIN	QG_GOWA "not_none" MAX	QG_GOWA "average" AVERAGE
A_1	**0.243***	0.150	0.054	0.595	**0.243**
A_2	0.235	**0.181**	**0.133**	**0.608**	0.235
A_3	0.164	0.096	0.052	0.650	0.164
A_4	0.109	0.070	0.045	0.402	0.109

*Bold indicates better performance.

Here, A_1 (Full Busway approach) appears to be the better project for AHP and MAHP except for the extreme quantifiers "all" and "not_none" which effectively identify the "worst" and "best" performance across all criteria for each project. Most multiple objective decision problems would seek to account for the performance of "most" of the criteria. This quantifier identifies A_1 as "best" as does the conventional AHP and the less familiar MAHP (based on the Saaty response scale).

GENERALIZATION TO FOUR OR MORE LEVELS

Though this application involves only the simplest three level hierarchy, a four (or more) level hierarchy could use different quantifiers at different levels in the hierarchy. However, this is only applicable to the additive AHP as the multiplicative AHP never extends beyond a three level hierarchy. Consider, for example, the four-level hierarchy shown in Figure 2, consisting of one overall objective and two criteria C_1 and C_2.

Criterion C_1 is disaggregated into two sub-criteria (C_{11} and C_{12}) and criterion C_2 is disaggregated into three sub-criteria (C_{21}, C_{22}, and C_{23}) and two alternatives (A_1 and A_2) form the base of the hierarchy. Then different OWA operators may be used at different levels. The priorities would than be as illustrated in Figure 3 where, for example, "a" and "b" result from the pairwise comparison matrix of A_1 and A_2 with respect to sub-criterion, C_{11} and "γ" and "δ" are priorities resulting from the pairwise comparison matrix of sub-criteria C_{11} and C_{12} with respect to criterion C_1.

This structure follows the concatenation of priorities throughout a more general hierarchy given by Saaty [4]. However, using quantifier-guided aggregation yields the possibility for A_1, of $QG\_OWA_1(a, c; \gamma, \delta)$, $QG\_OWA_2(u, w, y; \zeta, \eta, \theta)$, and $QG\_OWA_3(QG\_OWA_1(a, c; \gamma, \delta), QG\_OWA_2(u, w, y; \zeta, \eta, \theta); \alpha, \beta)$. Here, for

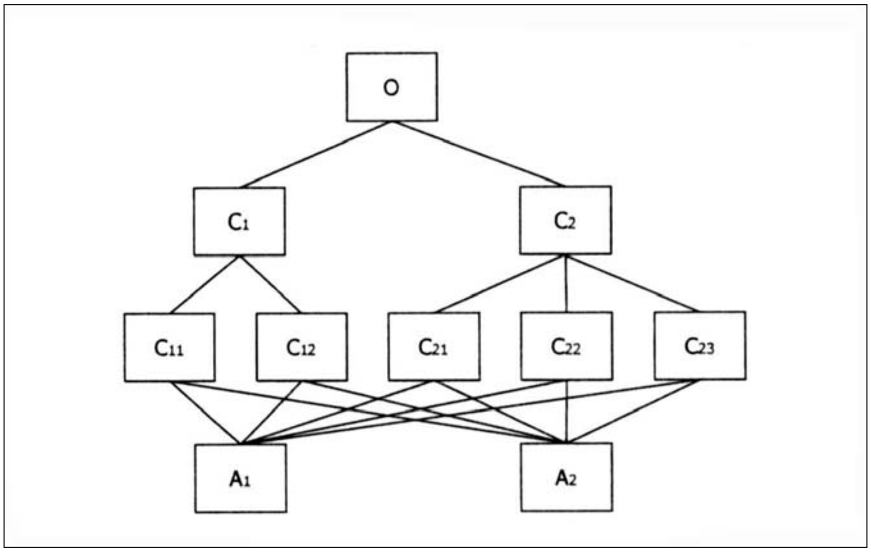

Figure 2. Four-level hierarchical structure.

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{ccccc} C_{11} & C_{12} & C_{21} & C_{22} & C_{23} \end{array} \\ \begin{bmatrix} a & c & u & w & y \\ b & d & v & x & z \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ C_{11} \\ C_{12} \\ C_{21} \\ C_{22} \\ C_{23} \end{array} \begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} \gamma & 0 \\ \delta & 0 \\ 0 & \zeta \\ 0 & \eta \\ 0 & \theta \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ C_1 \\ C_2 \end{array} \begin{array}{c} O \\ \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \end{array}$$

Figure 3. Priorities in a four-level hierarchical structure.

example, $QG\_OWA_1(a, c; \gamma, \delta)$ means a OWA(1) using priorities, "a" and "c" with weights (priorities) "γ" and "δ." $QG\_OWA_2(u, w, y; \zeta, \eta, \theta)$ means a quantifier-guided OWA(2) (possibly different from quantifier-guided OWA (1)) using priorities, "u," "w," and "y" with weights (priorities) "ζ", "η," and "θ." Finally, QG_OWA_3 (again, maybe a different quantifier) combines $QG\_OWA_1(a, c; \gamma, \delta)$ and $QG\_OWA_2(u, w, y; \zeta, \eta, \theta)$ with weights for criteria C_1 and C_2, respectively, "α" and "β." A similar concatenation, may be undertaken for A_2. However, this example will not be pursued further here.

CONCLUSION

More flexible aggregation of weights and performance score have been investigated in the AHP and MAHP. The OWA operator is guided by a linguistic quantifier involving the importance weights associated with each criterion. OWA operator weights are based on the "attitudinal character" of the decision-maker expressed in terms of the degree of "orness" and "andness" of the aggregation. It is claimed that quantifier guided OWA operators provide a more flexible range of aggregation possibilities available to the decision maker.

APPENDIX

	A_1	A_2	A_3	A_4	Priority
A_1	1	1/3	3	3	0.2300
A_2	3	1	7	7	0.6068
A_3	1/3	1/7	1	1	0.0816
A_4	1/3	1/7	1	1	0.0816

Criterion 1 — Link with trip generators (μ = 0.002928), CI = 0.002635)

	A_1	A_2	A_3	A_4	Priority
A_1	1	2	½	5	0.2836
A_2	½	1	¼	2	0.1335
A_3	2	4	1	7	0.05172
A_4	1/5	2	1/7	1	0.0657

Criterion 2 — Patronage growth potential (μ = 0.004519), CI = 0.004067)

	A_1	A_2	A_3	A_4	**Priority**
A_1	1	3	9	9	0.5857
A_2	1/3	1	7	7	0.3063
A_3	1/9	1/7	1	1	0.0540
A_4	1/9	1/7	1	1	0.0540

Criterion 3 — Value to transport network
(μ = 0.033721), CI = 0.030349)

	A_1	A_2	A_3	A_4	**Priority**
A_1	1	1/3	1/7	1/7	0.0541
A_2	3	1	1/3	1/3	0.1431
A_3	7	3	1	1	0.4014
A_4	7	3	1	1	0.4014

Criterion 4 — Local amenity and local impacts
(μ = 0.002928), CI = 0.002635)

	A_1	A_2	A_3	A_4	**Priority**
A_1	1	1	7	3	0.4014
A_2	1	1	7	3	0.4014
A_3	1/7	1/7	1	1/3	0.0541
A_4	1/3	1/3	3	1	0.1431

Criterion 5 — Support for complementary development
(μ = 0.002928), CI = 0.002635)

	A_1	A_2	A_3	A_4	Priority
A_1	1	½	1/6	3	0.5857
A_2	2	1	1/5	6	0.3063
A_3	6	5	1	9	0.0540
A_4	3	6	1/9	1	0.0540

Criterion 6 — Ease and success of implementation
(μ = 0.051861), CI = 0.046675)

	A_1	A_2	A_3	A_4	Priority
A_1	1	2	4	2	0.4444
A_2	½	1	2	1	0.2222
A_3	¼	½	1	½	0.1111
A_4	½	1	2	1	0.2222

Criterion 7 — Project cost
(μ = 0, CI = 0)

	A_1	A_2	A_3	A_4	Priority
A_1	1	3	3	7	0.5444
A_2	1/3	1	1	3	0.1934
A_3	1/3	1	1	3	0.1934
A_4	1/7	1/3	1/3	1	0.0688

Criterion 8 — Future opportunity
(μ = 0.002928), CI = 0.002635)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Priority
C_1	1	1	½	½	1	1	1	1	0.1
C_2	1	1	½	½	1	1	1	1	0.1
C_3	2	2	1	1	2	2	2	2	0.2
C_4	2	2	1	1	2	2	2	2	0.2
C_5	1	1	½	½	1	1	1	1	0.1
C_6	1	1	½	½	1	1	1	1	0.1
C_7	1	1	½	½	1	1	1	1	0.1
C_8	1	1	½	½	1	1	1	1	0.1

Pairwise comparison matrix of criteria ($\mu = 0.000$, CI = 0.000)

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