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MATHEMATICAL SEDIMENT TRANSPORT MODEL OF HETEROGENEOUS TAILINGS*

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ABSTRACT

Tailings dams using the upstream hydraulic fill method may present deficiencies related to construction and safety. It is difficult to evaluate the variability of real deposits based on the field process. Therefore, a mathematical model was developed to describe and simulate the mechanism of formation of granular tailings dams based on equilibrium and continuity equations of the fluid and the heterogeneous sediments (quartz and iron). The model accounts for the main external variables (slurry concentration and flow rate) and internal variables related to the slurry mixture, such as particles sizes, grain density, and relative percentage of iron and quartz. Analytical and numerical results using the model were compared with real deposition profiles obtained in laboratory simulations and a good agreement was found. The model was also applied to predict the height and slope of a real tailings dam and the results matched very closely the observed ones.

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INTRODUCTION

Economical and constructive aspects determine the choice of the most appropriate hydraulic fill technologies. In this type of fill, the hydraulic deposition is the only activity linked to the construction procedure and the bulk material stored is not water but loose tailings, sometimes impervious and potentially liquefiable. In this case, it can be concluded that the hydraulic deposition parameters play an important role in the hydraulic fill formation. Flow characteristics impose different forms of sediment transport and generate different kinds of deposits. The flow mechanism can cause considerable changes in the density of the deposited material and in the stationary bed, affecting the geotechnical behavior of the hydraulic fill.

Different construction methods are available and have been experimented in many engineering projects (Klohn, 1981; Mittal & Morgenstern, 1975; Morgenstern, 1985; Morgenstern & Küpper, 1988; Vick, 1983). The upstream method is perhaps the oldest, simplest, and most economical one. However, this method may present some problems related to stability, piping, and liquefaction. There are some difficulties in establishing geotechnical parameters, mainly for previous layers, which are the foundation for the layer being currently deposited. Despite these shortcomings, safe structures can be built using the upstream method if their design and construction are based on geotechnical principles. Modern upstream construction tends to produce flatter downstream slopes and wider beaches (Küpper, 1991). Nevertheless, this kind of control generates more costs. The grain size distribution depends on the method of placement and, therefore, the resultant fill is, occasionally, controllable. This control is normally done by the discharge parameters, which strongly affect particle sorting. The hydraulic segregation plays an important role on the depositional density and consequently on the structure porosity. This review emphasizes the importance of the hydraulic deposition process analysis in order to improve the quality of tailings dams. Therefore, it is desirable to attempt establishing links between hydraulical and geotechnical aspects that can lead to advances in the safety of structures built using the upstream method.

The deposition mechanism in hydraulic fill constructions consists of the discharge of a solid-liquid mixture or pulp. The solid particles tend to deposit in accordance to their physical characteristics. During this process, the bed profile is altered according to the quantity of either deposited or eroded material. The fill area increases with time as slurry is supplied. The profile configuration presents different behavior due to the differences of concentration and flow rate values of the discharged pulp. In this context, the deposited material, under different flow conditions, develops very particular sedimentary structures, and consequently each type of these structures exhibits different geotechnical properties. Analyzing each depositional mechanism, it can be possible to predict the sedimentary structure and evaluate its geotechnical behavior. Additionally, it would be possible to determine an optimal condition that assures the maximum density values and consequently establishes the hydraulic deposition parameters that can produce safe structures in the field.

SEDIMENT TRANSPORT MODEL OF HETEROGENEOUS TAILINGS

Tailings dams built with the upstream method normally have deficient control during design and construction. These deficiencies, consequently, reflect in their geotechnical characteristics. However, many of these structures have been built satisfactorily, but also some failures have occurred due to geotechnical and construction problems. These problems have shown difficulties in establishing design parameters concerning hydraulic variables, which are essential for the hydraulic fill technique. It would be helpful to consider the similarity with compacted embankments, in which densities can be estimated by laboratory compaction tests. However, this situation rarely occurs with hydraulic fills, where technical specifications related to the hydraulic fill technique are deficient or simply do not exist. More often, it is common to use empirical methodologies and recommendations based on concentration and rate of discharge.

Küpper (1991) considers that the most significant characteristics that affect the design and performance of hydraulic fills are segregation, density, seismic strength, and drainage systems. In this context, there is a need for evaluating the geotechnical parameters of hydraulic fills during the design and construction phases. Therefore, it is important to have some design methodologies that could improve the density or establish a control of variables to produce more stable hydraulic fills. Since 1970, many researches related to hydraulic fill techniques used in tailings dams have been reported (Klohn, 1981; Mittal & Morgenstern, 1975; Morgenstern, 1985; Morgenstern & Küpper, 1988; Vick, 1983). Present experience has pointed out different parameters that influence the design and performance of hydraulic fills (Blight, 1994; Küpper, 1991; Ribeiro & Assis, 1999; Ribeiro, 2000).

Laboratory tests provide efficient tools to simulate tailings dam deposition via hydraulic fill techniques. In laboratory simulations, the deposition mechanisms can be observed closely in a more economical and controlled manner than on field scale. Considering the advantages of simulating the hydraulic parameters in laboratory, several hydraulic deposition simulation apparatus have been developed to study this phenomenon. There is a large number of tests reported in the literature involving flow of water and sediments. Considering geotechnical aspects, some hydraulic deposition simulation tests are presented in literature (Blight, Thomson, & Vorster, 1985; Boldt, 1988; De Groot, Heezen, Mastbergen, & Stefess, 1988; Fan & Masliyah, 1989; Ferreira, Peres, & Monteiro, 1980; Küpper, 1991; Winterwerp, De Groot, Mastbergen, & Verwoert, 1990).

In this article, the hydraulic deposition simulation test (HDST) is applied to the study of tailings dams built via hydraulic fill technique, focusing geotechnical aspects (Ribeiro, 2000). The material used in all simulation tests was the iron tailings from Morro Agudo Mine, located in Minas Gerais, Brazil. This mine belongs to Vale Company, which is one of the biggest mining industries in the world and responds for one-third of the global iron production. Additionally, a mathematical model was developed in order to evaluate the performance and results of the HDST linked to tailings dam's formation (Cavalcante, 2004; Cavalcante, Assis, & Farias, 2002, 2003). This model aims to simulate the hydraulic deposition of granular tailings, constituted by grains of quartz and iron with different specific gravities. In this type of hydraulic deposition, the conventional mechanism of sedimentation does not occur, since the grains are also subjected to a bed load process. Moreover, the hydraulic deposition process is controlled by the particles' weights and not only by their volume, since they have different mineralogy and consequently different specific gravities.

The model satisfies continuity and equilibrium equations of the fluid and particles (quartz and iron) and describes the morphological behavior of tailings dams. These field equations are (Cavalcante, 2004):

Equilibrium in the fluid:

$$\frac{\partial u}{\partial t}+u\cdot\frac{\partial u}{\partial x}+g\cdot\left(\frac{\partial a}{\partial x}+\frac{\partial z_b}{\partial x}\right)=-\frac{g\cdot u^2}{C^2\cdot a}; \tag{1}$$

Continuity of the fluid phase:

$$\frac{\partial a}{\partial t}+a\cdot\frac{\partial u}{\partial x}+u\cdot\frac{\partial a}{\partial x}=0; \tag{2}$$

Continuity of transported sediments:

$$\frac{\partial z_b}{\partial t}+\frac{\partial s}{\partial x}=0; \tag{3}$$

in which g and C are constants and $u = u(x, t)$, $a = a(x, t)$, $z_b = z_b(x, t)$, and $s = s(x, t)$ are the main unknowns to be determined as functions of the distance (x) from the discharge point and time (t). The constants are the gravity acceleration (g) and coefficient of Chèzy (C). The functions stand for the velocity of transport sediments (u), the film of water above the bed of the deposition profile (a), the height of deposited sediments (z_b) and the rate of transported sediments (s) composed of particles of two types (quartz and iron), as illustrated in Figure 1.

Since there are three equations and four unknowns, it is necessary to formulate another equation in order to solve the problem. In this article, this equation is a constitutive relation for the rate of sediment transport (s), which is computed

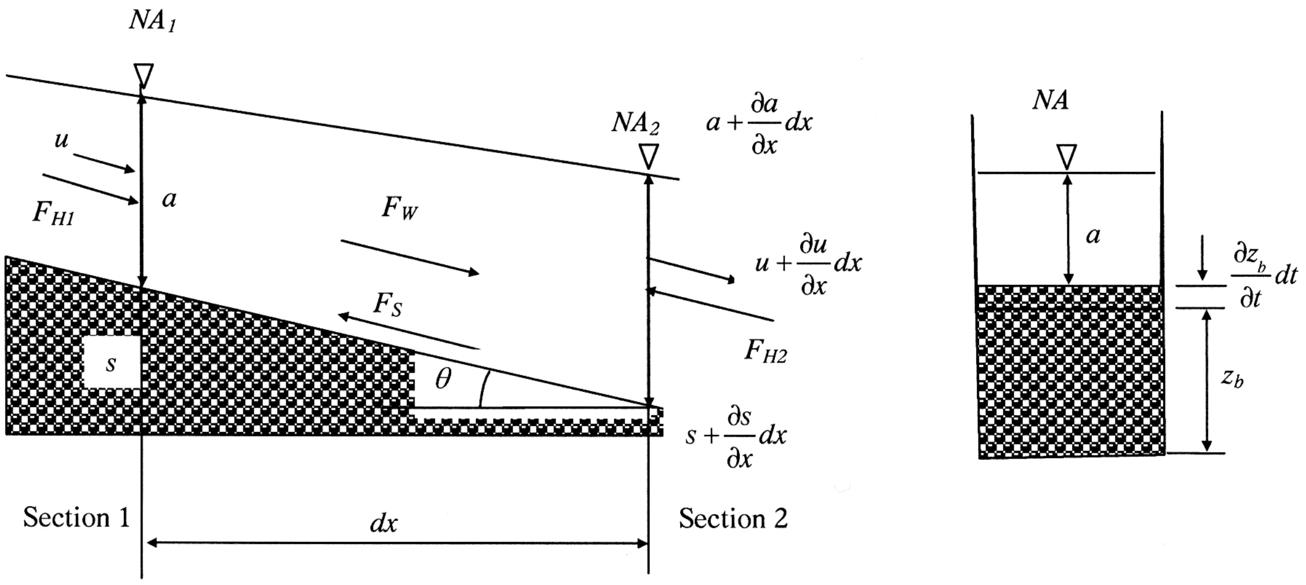

Figure 1. Details of an infinitesimal element in the upstream slope.

as a weighted average, as proposed in Cavalcante, Assis, and Farias (2002), Cavalcante, Assis, and Ribeiro (2003), and Cavalcante (2004):

$$s = (1 - C_w) \cdot \{Fe \cdot f_{Fe}\,(u, D_{50}, G_s) + (1 - Fe) \cdot f_{Qz}\,(u, D50, G_s)\}, \tag{4}$$

in which the weights relate to the concentration of solids in the slurry (C_w) and the relative amount of iron particles (Fe). The functions f_{Fe} and f_{Qz} give the rate of transport for each type of sediment.

In the model, the rate of transported particles for each type of grain can be described by a basic equation of bed load sediment transport (Engelund & Hansen, 1967; Meyer-Peter & Müller, 1948). In this article, the equation proposed by Meyer-Peter and Müller (1948) is assumed:

$$f_k = \frac{0.05 \cdot u^5}{(G_s - 1)^2 \cdot g^{0.5} \cdot D_{50} \cdot C^3}, \tag{5}$$

in which k indicates the type of particle (quartz, Qz or iron, Fe), G_s is the specific gravity of particles, D_{50} is the average diameter of particles and u is the average flow velocity.

The average flow velocity is calculated considering the percentage of iron Fe in the pulp (Cavalcante et al., 2002):

$$u = Fe \cdot u_{Fe} + (1 - Fe) \cdot u_{Qz}, \tag{6}$$

$$u_k = C \cdot \sqrt{a_0 \cdot i_0}, \tag{7}$$

$$C = 18 \cdot \log\left(\frac{12 \cdot a_0}{3 \cdot D_{90}}\right), \tag{8}$$

in which a_0 is the initial height of the film of water above the bed of the deposition profile, i_0 is the initial average beach slope and D_{90} is the sieve diameter of which 90% of the soil weight is finer.

ANALYTICAL SOLUTION

It can be shown that for moderate values of the Froude number, the water motion can be considered quasi-steady (De Vries, 1966). In this case, one can neglect $\partial u/\partial t$ from the equation of equilibrium in the fluid and $\partial a/\partial t$ from the equation of continuity of the fluid phase. Using this hypothesis and combining Equations (1), (2), and (3), the following hyperbolic equation can be written (Cavalcante, 2004; Cavalcante et al., 2002):

$$\frac{\partial z_b}{\partial t} - \frac{\aleph_0 \cdot a_0}{c_{b0}} \cdot \frac{\partial^2 z_b}{\partial x \partial t} - \aleph_0 \cdot \frac{\partial 2 z_b}{\partial x^2} = 0, \tag{9}$$

in which

$$\aleph_0 = \frac{3 \cdot c_{b0} \cdot u_0^2}{a_0^2 \cdot C^2}, \quad a_0 = 1 - F_R^2 \text{ and } F_R = \frac{u_0}{\sqrt{g \cdot a_0}}. \tag{10}$$

u_0 in Equation (10) corresponds to initial value of velocity of transport sediments, c_{b0} is a constant and F_R is the Froude number.

In the case of $\alpha_0 = 0$, the model reduces to a parabolic equation and for simplified initial and boundary conditions it was derived analytically, and solved applying Laplace transformation (Cavalcante, 2004; Cavalcante et al., 2002). At time ($t = 0$), the initial condition is:

$$z_b\,(x,0) = 0, \text{ for } x \geq 0 \text{ and } t = 0. \tag{11}$$

For a distance sufficiently far from the discharge point (x → ∞), the following boundary condition is assumed at any time t:

$$\lim_{x \to \infty} z_b\,(x,t) = 0. \tag{12}$$

The continuity of mass establishes that the initial volume of sediments (quartz and iron) deposited at point $x = 0$, during a certain test duration t, is equal to the total volume of sediments incorporated to the hydraulic fill in the same period of time. This can be written as:

$$\Delta s \cdot t = \int_0^\infty z_b \cdot dx. \tag{13}$$

The solution of the Partial Differential Equation system, Equations (1) to (4), for the initial and boundary conditions in Equations (11) to (13), is given by Cavalcante et al. (2002), Cavalcante, Assis, and Ribeiro (2003), and Cavalcante (2004):

$$z_b\,(x,t) = A \cdot \left\{ \sqrt{B \cdot t} \cdot \exp\left(-E \cdot \frac{x^2}{t} \right) - D \cdot x \cdot erfc\left(\sqrt{E \frac{x}{t}} \right) \right\}, \tag{14}$$

in which

$$A = \frac{z_b\,(0,t)}{\sqrt{Bt}}, B = \frac{60 \cdot s_0 \cdot i_0}{\pi \cdot a_0}, D = \frac{3 \cdot i_0}{a_0}, \text{and } E = \frac{9 \cdot i_0}{60 \cdot a_0 \cdot s_0} \tag{15a}$$

and erfc is the complimentary error function given by:

$$\text{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3 \cdot 1!} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!} + \ldots \right). \tag{15b}$$

The mathematical model is controlled by the Froude number. The flow characteristic can be chosen (i.e., turbulent or steady case). The mathematical model permits us to evaluate the profile of hydraulic deposition in different conditions for different materials. The authors hope that it can provide an insight about the mechanisms of hydraulic fill deposition, which can be extrapolated to real scale applications.

NUMERICAL SOLUTION

Cavalcante, Assis, and Farias (2003) and Cavalcante, Farias, and Assis (2007) presented a numerical solution for the system of Equations (1) to (4), based on the Finite Differences Method using advanced differences in time and central differences in the space domain. Taking x_0 as any number and Δx as a positive number; the mesh associated to x_0 and step Δx is composed by the following group of points:

$$x_i = x_0 \pm i \cdot \Delta x; \qquad i = 1,2,\ldots \tag{16}$$

Approximate values of $u(x, t)$, $a(x, t)$, $z_b(x, t)$, and $s(x, t)$ were calculated in the mesh points. In fact, the general idea of the Finite Differences Method is the discrete evaluation of the derivatives of $u(x, t)$, $a(x, t)$, $z_b(x, t)$, and $s(x, t)$ that appear in the differential equations of the sediment transport model of heterogeneous tailings.

The basic mathematical tool for evaluating approximate values of the derivatives is the series of Taylor. It gathers several information about the function in a point x, and uses them to evaluate the value of this function in the neighborhood of x, that is, in point $x + \Delta x$. Assuming that $u(x, t)$, $a(x, t)$, $z_b(x, t)$, and $s(x, t)$ have derivatives with respect to x, up to the order m, it can be expanded in the form of the following series:

$$\psi_{k+1}^{n} = \psi_{k}^{n} + \Delta x \cdot \frac{\partial \psi_{k}^{n}}{\partial x} + \frac{\Delta x^2}{2!} \cdot \frac{\partial^2 \psi_{k}^{n}}{\partial x^2} + \ldots + \frac{\Delta x^m}{m!} \cdot \frac{\partial^m \psi_{k}^{n}}{\partial x^m} + \ldots, \tag{17}$$

in which $\psi_{k}^{n} = \psi(x,t)$ and $\psi_{k+1}^{n} = \psi(x + \Delta x, t)$ assumes the values of $u(x, t)$, $a(x, t)$, $z_b(x, t)$, and $s(x, t)$.

Taking t_0 as any number and Δt as a positive number, the mesh associated to t_0 and Δt is given by the following group of points:

$$t_j = t_0 \pm j \cdot \Delta t; \qquad j = 1,2,\ldots . \tag{18}$$

Similarly, assuming that $u(x, t)$, $a(x, t)$, $z_b(x, t)$ and $s(x, t)$ have derivatives with respect to t, up to the order m, it can be expanded in the form of the following series:

$$\psi_k^{n+1} = \psi_k^n + \Delta t \cdot \frac{\partial \psi_k^n}{\partial t} + \frac{\Delta t^2}{2!} \cdot \frac{\partial^2 \psi_k^n}{\partial x^2} + \ldots + \frac{\Delta t^m}{m!} \cdot \frac{\partial^m \psi_k^n}{\partial x^m} + \ldots, \tag{19}$$

in which $\psi_k^n = \psi(x,t)$ and $\psi_k^{n+1} = \psi(x, t+\Delta t)$ assumes the values of $u(x, t)$, $a(x, t)$, $z_b(x, t)$, and $s(x, t)$.

Substituting the Equations of Finite Differences, Equations (17) and (19), in the equilibrium in the fluid equation, Equation (1), one can get:

$$u_k^{n+1} = u_k^n - \frac{g\Delta t}{C^2} \frac{(u_k^n)^2}{a_k^n} - \frac{\Delta t}{2\Delta x} \left\{ u_k^n [u_{k+1}^n - u_{k-1}^n] + g([a_{k+1}^n - a_{k-1}^n] + [z_{bk+1}^n - z_{bk-1}^n]) \right\}. \tag{20}$$

Substituting Equations (17) and (19) in the continuity of the fluid phase equation, Equation (2), one can obtain:

$$a_k^{n+1} = a_k^n - \frac{\Delta t}{2\Delta x} \{ a_k^n [u_{k+1}^n - u_{k-1}^n] + u_k^n [a_{k+1}^n - a_{k-1}^n] \}. \tag{21}$$

Substituting Equations (17) and (19) in the continuity of transported sediments equation, Equation (3), one can find:

$$z_{bk}^{n+1} = z_{bk}^n - \frac{\Delta t}{2\Delta x} [s_{k+1}^n - s_{k-1}^n]. \tag{22}$$

The relation for the rate of sediment transport, Equation (4), can be written in the form:

$$s_k^{n+1} = m_{Qz} [u_k^n]^{n_{Qz}} + m_{Fe} [u_k^n]^{n_{Fe}}, \tag{23}$$

in which m_{Qz}, m_{Fe}, n_{Qz}, and n_{Fe} are constants that depend on C_w, Fe, D_{50}, and G_s of iron and quartz particles.

EXPERIMENTAL SIMULATION OF THE BED LOAD TRANSPORT PROCESS AND MODEL VALIDATION

Hydraulic deposition simulation tests are developed mainly to provide experimental support for designing hydraulic fills. In this way it is important that the data obtained from these kinds of tests be useful to forecast the behavior of these structures in the field. The major challenge related to the simulation process in the laboratory is the data extrapolation to the field. Analyzing the parameters obtained from hydraulic deposition simulation tests, it would be possible to evaluate their relative importance and qualitatively forecast the behavior of hydraulic fills. Considering the importance of predicting the behavior of hydraulic fill structures in the field and afterwards analyzing the performance of different kinds of laboratory simulation tests, a hydraulic deposition simulation apparatus was developed at the University of Brasilia. A testing program was carried out aiming to study the behavior and performance of tailings dams built via the hydraulic fill technique, and evaluating the effects of each hydraulic deposition variable.

The apparatus consists of a depositional channel 6.0 m long, 0.4 m wide, and 1.0 m high. The channel was built using steel profiles and panels of tempered glass. This kind of wall allows the observation of the evolution of the deposition process during the entire test. Figure 2 shows a general view of hydraulic deposition simulation tests (HDST), developed at the University of Brasilia.

The experimental program consisted of discharging slurry at specific flow rates and concentration conditions, over a pre-deposited waste layer. This layer was very flat and had the objective of imposing flow conditions that are closer to those that happen in the field. After some time of flow and depending on the variables of discharge, a beach slope is formed.

At the end of each test, the final configuration of the hydraulically deposited beach and the densities of the deposit along the fill were measured. These results were used to validate the proposed model. A series of comparisons were made between the HDST results obtained by Ribeiro (2000) and those forecasted by the model. According to Ribeiro (2000), the main characteristics of the material used in the HDST are those presented in Tables 1 and 2. In Table 1, D_{50} is the average particle diameter, D_{90} is the diameter of which 90% of the particles are

Figure 2. General view of the HDST equipment developed at the University of Brasilia.

Table 1. Average Physical Characteristics of the Tailings Used in the Hydraulic Deposition Simulation Test (HDST) Carried Out by Ribeiro (2000)

	Quartz	Iron
D_{50} (mm)	0.265	0.240
D_{90} (mm)	0.645	0.640
G_s	2.65	5.50

Table 2. HDST Controlled Variables and Final Slope Inclination of the Hydraulically Deposited Beach (Ribeiro, 2000)

	HDST 1	HDST 9
C_w (%)	8.9	19.7
Fe (%)	23.0	23.0
Q (l/min)	4.8	9.4
I_m (%)	7.7	8.7

finer, and Gs is the relative density of the grain particles. Notice that the iron particles are more than twice as heavy as those made of quartz. In Table 2, *Fe* is the percentage of iron particles, C_w is the concentration of solid particles (quartz and iron) in the slurry, Q is the slurry flow rate, and I_m is the average (global) beach slope. Several tests were performed using different combinations of pulp parameters, but only the results of two tests (HDST 1 and HDST 9) are presented herein.

Figure 3 presents comparisons between the results obtained from tests HDST 1 and 9 (Ribeiro, 2000) and those obtained analytically using the proposed mathematical model. Axis x (abscissas) represents the distance from the discharge point and axis y (ordinates) is the normalized height of the deposited beach.

From Figure 3, one can notice that the analytical model is not able to describe the successive erosion and deposition processes, which are clearly observed in the HDST beaches. However, the model describes quite well the basic geometric characteristics of the deposited beaches, such as global slope inclination.

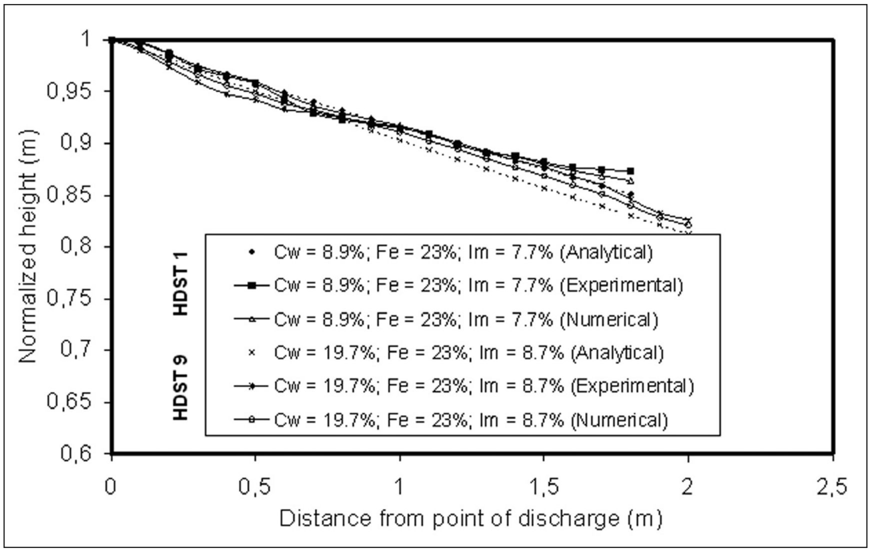

Figure 3. Comparison between HDST and model results.

Other results showed that the agreement is very good for both the homogeneous slurry and for the heterogeneous slurry mixture of iron and quartz (Cavalcante, Assis, & Ribeiro, 2003; Cavalcante et al., 2002, 2006). The model can also reproduce important aspects such as the distribution of deposited particle sizes along the beach profile, taking into account the slurry concentration, flow rate, and percentage of iron particles. It is worth mentioning that the model convergence occurs for a water film above the fill of 0.3 to 0.4 mm, indicating the bed load process.

PARAMETRIC ANALYSIS USING THE ANALYTICAL MODEL

After the experimental validation of the model, a parametric analysis including the main variables controlling the deposition process was carried out. These variables include grain size distribution, relative density, percentage of iron particles, and friction angle.

First, it was verified that the difference between the diameters and the relative density of the grains of iron and quartz particles are factors that affect the behavior of bed load transport process, resulting in different profiles of hydraulic deposition.

It was observed that the average beach slope depends both on particle sizes and relative density, as illustrated in Figure 4. When the mean diameters of the grains of quartz and iron are very similar ($D_{50Qz}/D_{50Fe} = 1.1$), the global inclination of the profiles of hydraulic deposition tends to increase for the larger values of the relative density of the pulp, because the heavier iron particles tend to deposit faster and closer to the discharge point. On the contrary, for the cases in which the ratio between the mean diameters of the grains of quartz and iron increases ($D_{50Qz}/D_{50Fe} = 4.4$), the global inclination of the profiles of hydraulic deposition tends to decrease for increasing relative densities of the tailings (Figure 4). This happens because the much bigger quartz particles will deposit first in this case and the smaller iron particles will be transported further away from the discharge point, thus creating a smoother beach.

It also was observed that the average beach slope depends both on particle sizes and percentage of iron particles in the pulp, as illustrated in Figure 5. When the particles are equidimensional ($D_{50Qz}/D_{50Fe} = 1.1$), the global inclination of the profiles of hydraulic deposition tends to increase for the larger values iron content in the pulp. Due to their higher density, the iron particles are almost two times heavier than quartz particles of the same size, and therefore the heavier iron particles tend to deposit faster and closer to the discharge point creating a

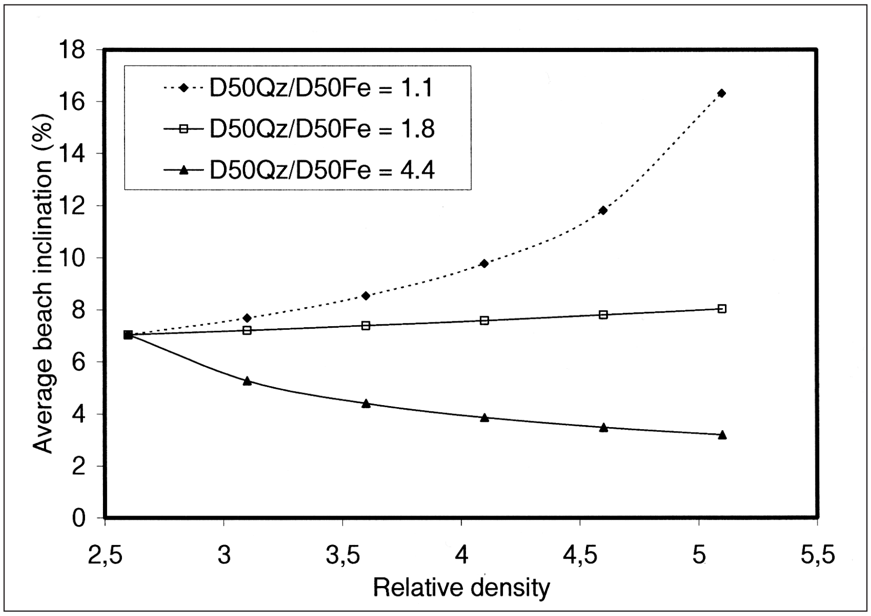

Figure 4. Average beach inclination versus relative density.

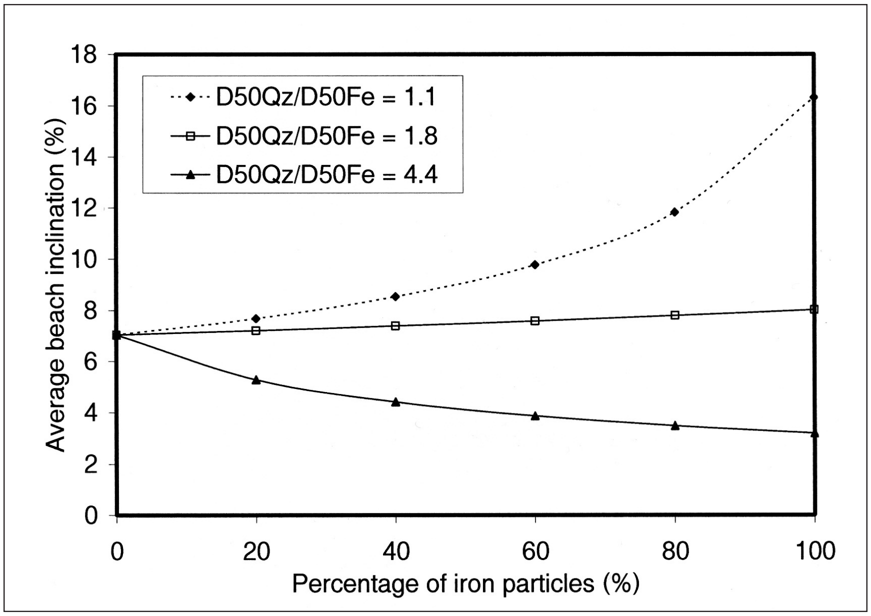

Figure 5. Average beach inclination versus percentage of iron particles.

steep beach. On the contrary, for the cases in which the ratio between the mean diameters of the grains of quartz and iron is very large ($D_{50Qz}/D_{50Fe} = 4.4$), the global inclination of the profiles of hydraulic deposition tends to decrease for increasing percentage of iron particles in the tailings (Figure 5). In this case, the larger quartz particles are around nine times heavier than the smaller ones made of iron. Therefore the much bigger quartz particles will deposit first in this case and the smaller iron grains will be transported further away generating a smooth profile.

In order to better understand the influence of the slurry concentration and composition on the average beach slope, a series of simulations was carried out considering different values of concentrations, iron content, and diameter ratio D_{50Qz}/D_{50Fe}. The final beach inclination is illustrated in Figure 6 versus relative density of the solid phase in the slurry, which varies from 2.65 (only quartz) to 5.5 (only iron) depending on the iron concentration in the pulp. The figure corroborates the previous observation about the influence of the iron content and particle size, thus making clear that the depositional profile is controlled by the weight of the particles. It can also be observed that, for any combination, the global inclination of the profiles of hydraulic deposition tends to increase for the larger values of the slurry concentration (Figure 6).

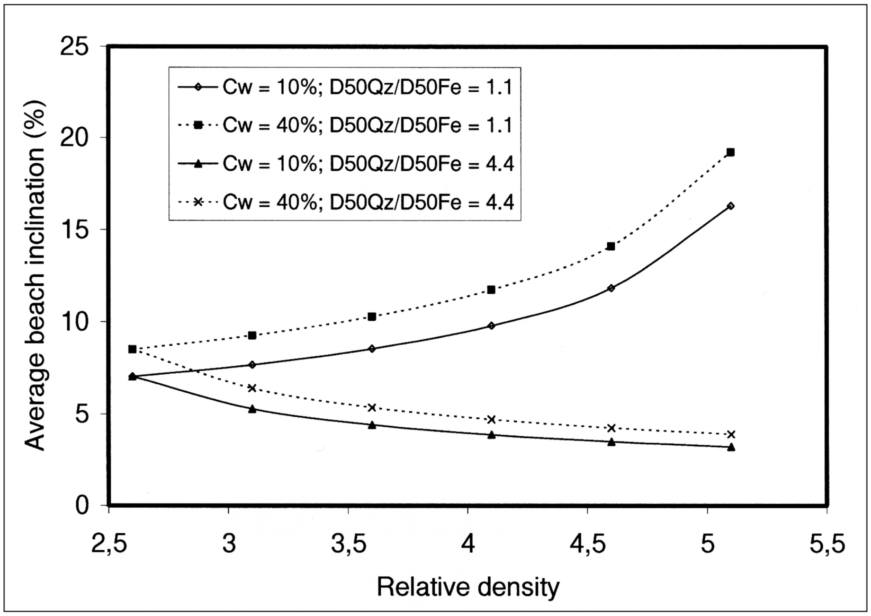

Figure 6. Average beach inclination versus relative density and concentration of solid particles (quartz and iron).

CASE-STUDY: MONJOLO DAM (VALE CO.)

After properly checked, the bed load transport model was used to simulate a real case of the construction of a tailings dam. The geotechnical data from the Monjolo Dam (Vale Co.) was used. The construction of this dam started in 1997 and a general view is shown in Figure 7. The dam accumulated tailings produced by spiral separators, and the grain size of the solids were in the range of fine to medium sand (diameter < 1.0 mm). According to Espósito and Assis (1997), the average grain density was 3.12 g/cm^3 and the chemical composition of the solids comprised 23% of iron (Fe), 65% of silicon dioxide or silica (SiO_2), and 0.4% of aluminium oxide (Al_2O_3). The tailings were transported and deposited hydraulically, and the fill was lifted using the upstream method.

The dam construction began with a start dike with a toe rockfill at elevation 800 m, then it was successively lifted by every 10 m using the deposited tailings. The fill should reach elevation 860 m in the 2000, with a height of 60 m and a total volume of 6 million cubic meters including the dam and the reservoir as illustrated in Figures 8 and 9. In the year 2002, the fill was expected to reach elevation 875 m, with 75 m in height and a total volume of 10 million cubic meters according to the designers (Geoconsultoria, 1997).

Figure 7. General view of Monjolo Dam (2000).

Table 3 presents a comparison between measured and computed heights of the dam in different times. The height simulated by the Mathematical Sediment Transport Model for the year 2000, 3 years after construction, was 59.6 m, which compares very well with the real observed height of 60 m. For year 2002, 5 years after construction, the model predicted a height of 76.9 m against a measured value of 75 m.

Figure 10 presents the predicted profile of hydraulic deposition for Monjolo Dam using the Mathematical Sediment Transport Model and deposition times of 3 and 5 years. Two points, illustrating the observed heights at the discharged point x = 0 in years 2000 and 2002, were also included in the figure for comparison. The beach profile observed in field has similar concavity as that obtained analytically. Besides, the inclination has the same order of magnitude (1 V: 40 H to 1 V: 50 H).

CONCLUSIONS

The behavior of tailings dams built by the upstream method is a subject of major importance for the mining companies and for the ambientalists, considering the social, environmental, and economic damages that an eventual failure may cause.

The hydraulic deposition mechanism becomes very complex because it involves a great number of variables. The bed load transport model proposed in

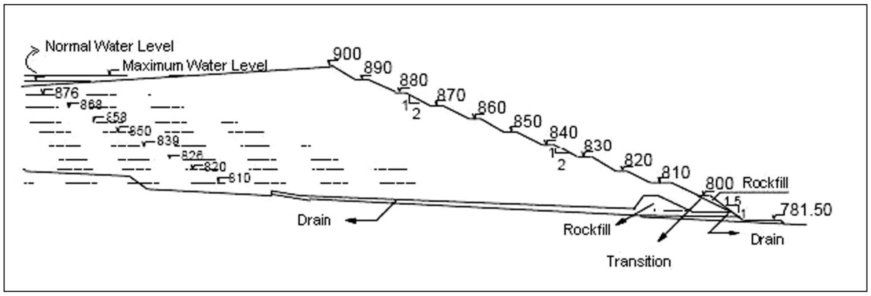

Figure 8. Beach profile, including designers prediction up to year 2002 (modified–Geoconsultoria, 1997).

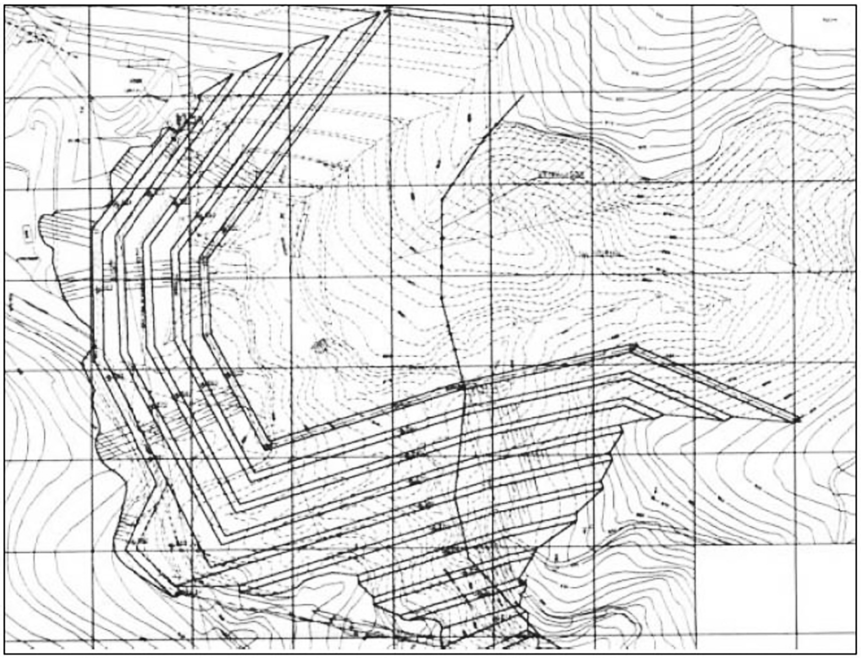

Figure 9. Final plan view of Monjolo Dam. (modified–Geoconsultoria, 1997).

Table 3. Comparison between Observed and Computed Fill High

	2000	2002
Time since start of construction (years)	3	5
Height of dam measured in field (m)	60.0	75.0
Height of dam simulated by the model (m)	59.6	76.9

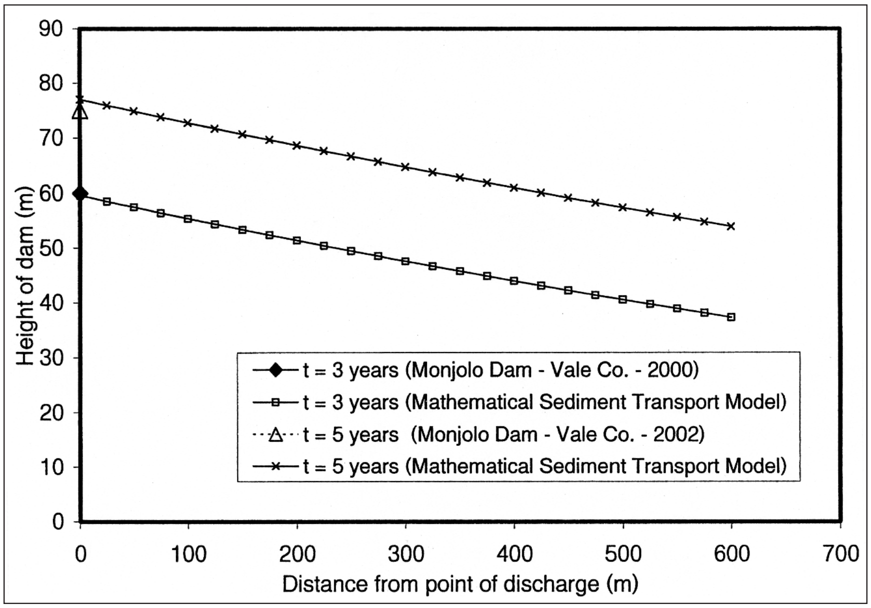

Figure 10. Predicted profile of hydraulic deposition for Monjolo Dam.

this article holds for heterogeneous granular tailings, formed by particles with different geotechnical properties, such as grain size and specific gravity. This type of tailings is very common in iron mines, since they are mainly composed by quartz and iron particles.

The mathematical equations that govern such kind of phenomenon were formulated considering the equilibrium in the fluid and the continuity of water and transported sediments, in addition to a constitutive relation for the rate of heterogeneous sediment transport. A scheme to solve the corresponding system of partial differential equations numerically, based on the Finite Differences Method, was shown. Also, an analytical solution was deduced for simple boundary conditions.

The mathematical model was validated in comparison to numerical results and the model predictions are consistent with measured results obtained from hydraulic deposition simulation tests carried out in laboratory, using both homogeneous and heterogeneous tailings.

The mathematical model proposed here allows investigating the influence of important process variables, such as the slurry concentration, the slurry composition (iron content), and the flow rate. It helps to gain further insight about the physical mechanism of hydraulic deposition, including the bed load process. The parametric analyses showed that the deposition mechanism is controlled by the weight of the particles, which is a function both of grain sizes and mineralogy (density). Heavier particles tend to deposit faster and the final inclination of the beach profiles depends on the iron content in the slurry. High solid concentration in the pulp produces steeper beaches.

The model was also applied to the prediction of the behavior of a real tailings deposition at Monjolo Dam (Vale Co.), in the state of Minas Gerais, Brazil. The predictions could match very closely the observed height of the deposit during 5 years from construction. Also, the beach slopes were also reproduced with reasonable accuracy.

Finally, the authors hope that the model may be helpful for designers, providing a better understanding of the geotechnical characteristics of tailings dams, deposited by hydraulic techniques.

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