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ENVIRONMENTAL PROJECT EVALUATION USING INTUITIONISTIC FUZZY INFORMATION

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ABSTRACT

An intuitionistic fuzzy set (IFS) is a generalization of a fuzzy set characterized by a truth membership function and a false membership function. The former is a lower bound on the grade of membership of the evidence in favor of a particular element belonging to the set and the latter is a lower bound on the negation of that element belonging to the set, derived from evidence against that element belonging to the set. A similar concept is a vague set, though vague sets have been shown to be identical to IFSs. In the context of project evaluation, an IFS may be used to represent the degree to which a project satisfies a criterion or factor and the degree to which it does not. Aggregation of such IFSs has been considered in recent years to identify a best project in terms of several factors. A particular desirable way to aggregate IFSs is in terms of an ordered weighted average (OWA) which can be expressed in different forms, such as arithmetic and geometric. In an OWA, weights are applied to the position of an element in the aggregation. In addition, hybrid OWA operators may be developed to not only weight the position of elements in the aggregation but the element itself. A simple example based on a hypothetical but realistic example by Horsak and Damico [4] is given which involves the location of a hazardous waste disposal facility (PCB-contaminated transformer fluids) at one of three sites based on 10 factors.

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INTRODUCTION

Ordered weighted averaging operators have been considered previously in this journal [1-3]. These have been elaborated in the context of decisions relating to major projects (e.g., the siting of hazardous waste storage facilities [4, 5]) based on available data and information that are vague, imprecise, and uncertain by nature. The nature of vagueness, imprecision, and uncertainty is fuzzy rather than random, especially when subjective assessments are involved in the decision-making process. Fuzziness derives from the lack of precise boundaries in some of the sets of data and information considered in a given situation. Fuzzy set theory [6] offers a possibility of handling these sorts of data and information which involve the subjectivity characteristic of the human decision-making process.

Following [4, 5], the basic structure for decisions relating to projects with multiple (ecological, social, economic, aesthetic, etc.) consequences is an outcome matrix which shows the satisfaction of project p_i with respect to factor/impact F_j. $P = \{p_1, p_2, ..., p_I\}$ is a set of I mutually exclusive projects and $F = \{F_1, F_2, ..., F_J\}$ is a set consisting of J factors/impacts. Commonly in the decision process, weights $\underline{w} = [w_1, w_2, ..., w_J]$ are introduced to represent the differential importance (salience, significance) of factors/impacts.

In terms of fuzzy set theory, each factor, F_j, may be construed as a fuzzy set of the set of projects represented as $F_j = \{F_j(p_1|p_1, F_j(p_2)|p_2, ..., F_j(p_I)|p_I\}$, where $F_j(p)$ indicates the degree to which project $p \in P$ satisfies factor/impact F_j. Note that the satisfaction of a given project (denoted either as p or p_i) is represented as $\underline{F}(p) = [F_1(p), F_2(p), ..., F_J(p)]$, $p \in P$. Each column of the project/factor matrix (Table 1) is a fuzzy set.

Project evaluation typically involves the identification of a "best" project which satisfies as much as possible each factor/impact. "Satisfies" implies lower values of negative factors/impacts (e.g., cost, ecological impact) and higher values of positive factors/impacts (e.g., accident reduction, aesthetic impact, savings in travel time). Rarely will any real project completely satisfy all factors/impacts and will be characterized by variable achievement across factors/impacts. For brevity, the term "factor" will be used, where possible, to include also impacts [1-3].

Table 1. Fuzzy Project/Factor Matrix

	F_1	F_2	…	F_J
p_1	$F_1(p_1)$	$F_2(p_1)$	…	$F_J(p_1)$
p_2	$F_2(p_2)$	$F_2(p_2)$	…	$F_J(p_2)$
⋮	⋮	⋮		⋮
p_I	$F_1(p_I)$	$F_2(p_I)$	…	$F_J(p_I)$

EXAMPLE OF ENVIRONMENTAL PROJECT EVALUATION

Consider an example adapted from Horsak and Damico [4] (also considered by Anandalingam and Westfall [5] and see Smith [1-3]) involving the location of a hazardous waste disposal facility with three possible sites assessed against 10 factors: *air quality* (dispersive capabilities of site/plant and degree to which waste emissions could concentrate onsite and offsite, F_1); *surface water quality* (potential for surface water degradation due to spills associated with handling storage and waste, F_2); *groundwater quality* (potential for groundwater degradation due to spills associated with handling and storage of waste, including leaching into aquifer, F_3); *impact on ecology* (potential impact on ecological resources of an area due to routine operations or emergency conditions, F_4); *impact on aesthetics* (visual impacts of hazardous waste management operations, including handling, storage, and disposal, F_5); *impact on population* (potential long-term exposure to emissions due to routine operations or emergencies, F_6); *impact on surrounding land use* (compatibility of surrounding land use with the hazardous waste operation, F_7); *possibility of emergency response* (ability of a response team to combat an emergency associated with a spill or other exposure, F_8); *distance from sources of waste* (distance through which the waste should travel to get to the site, F_9); and *political opposition* (political or other organized intervention or opposition to the hazardous waste operation, F_{10}). Factors are fuzzy subsets of the projects (sites), for example, $F_1 = \{0.9|p_1, 0.7|p_2, 0.3|p_3\}$ for *air quality* (F_1). The project/factor matrix is given in Table 2.

Note that F_3 (*groundwater quality*) could be excluded as it fails to discriminate between sites, though it is retained here. It is clear that site 1 (p_1) is a strong competitor for the overall "best" site [4]. Further assume factor weights (based on [4]) as follows w = [0.161, 0.156, 0.148, 0.115, 0.111, 0.106, 0.074, 0.052, 0.046, 0.031], normalized such that $\sum_{j=1}^{J} w_j = 1$.

Horsak and Damico [4] used *weighted conjunctive aggregation* to select a "best" site and identified a preference order, $p_1 \succ p_2 \succ p_3$. The weighted fuzzy decision $D = \{D(p_1)|p_1, D(p_2)|p_2, ..., D(p_I)|p_I\}$ is $D = \cap_{j=1,J} F_j^{w_j}$ with membership grade $D(p) = \min_{j=1,J} F_j(p)^{w_j}$ for $p \in P$ (see [4]).

Table 2. Project/Factor Matrix for Siting Hazardous Waste Facility

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
p_1	0.9	0.8	1.0	0.9	0.8	1.0	0.8	0.8	1.0	0.5
p_2	0.7	0.9	1.0	0.9	0.9	0.5	0.6	0.5	0.6	1.0
p_3	0.3	0.2	1.0	0.2	1.0	1.0	0.2	0.2	0.3	0.3

INTUITIONISTIC FUZZY SETS

An intuitively straightforward extension of a fuzzy set is an *intuitionistic fuzzy set* (IFS). Together with *interval-valued fuzzy sets* (IVFS), IFSs were conceived to alleviate some of the drawbacks of Zadeh's fuzzy set concept [6]. Antanassov developed intuitionistic set theory [7, 8].

IFS theory basically defies the assertion implicit in the concept of a fuzzy set that if an element u *belongs* to a given degree (say $\mu(u)$, $u \in U$) to a fuzzy set A, then it naturally follows that u should *not belong* to A to the extent $1 - \mu(u)$. IFSs assign to each element of the universe both a degree of membership $\mu(u)$ and one of non-membership $\nu(u)$ such that $0 \leq \mu(u) + \nu(u) \leq 1$, thus relaxing the enforced duality (i.e., $\nu(u) = 1 - \mu(u)$) of fuzzy set theory. However, it is clear that when all elements of the universe are such that $\mu(u) + \nu(u) = 1$, the traditional fuzzy set concept [6] is recovered. IFSs owe their name to the fact that the identity $\mu(u) + \nu(u) = 1$ is weakened into an inequality. The *law of the excluded middle* (either "A" or "not A"), one of the defining properties of classical systems of logic, is denied at the element level, since $\mu(u) + \nu(u) < 1$ is possible. This is one of the main ideas of intuitionism [8].

IVFS theory emerged from the observation that in many situations, no objective procedure is available to select the crisp membership degrees of elements in a fuzzy set. To alleviate this problem, the construction of an interval $[\mu_1(u), \mu_2(u)]$ to which the actual membership degree is assumed to belong was suggested [9]. A related approach, second-order fuzzy set theory, also introduced by Zadeh goes one step further by allowing the membership degrees themselves to be fuzzy sets in the unit interval [10].

Both approaches, IFS and IVFS theory, complement fuzzy set theory, in that both are able to model vagueness, with an ability to model uncertainty as well [8] (in [8], vagueness and uncertainty are juxtaposed as two elements of imprecision). IVFSs reflect this uncertainty by the length of the interval membership degree $[\mu_1(u), \mu_2(u)]$. In IFS theory, for every membership degree $[\mu(u), \nu(u)]$, the value $\pi(u) = 1 - \mu(u) + \nu(u)$ denotes a measure of *non-determinacy, undecidedness, or hesitancy* [8]. The concept of a *vague set* was also introduced by Gau and Buehrer [11], thought Bustince and Burillo [12] showed vague sets to be IFSs.

Let U be the universe of discourse, $U = \{u_1\ u_2, ..., u_n\}$ where u_i is a generic element of U. An IFS (or vague set) $\tilde{A}$ in U is characterized by a truth-membership function $t_{\tilde{A}}$ and a false-membership function $f_{\tilde{A}}$ where $t_{\tilde{A}}: U \rightarrow [0, 1]$ and $f_{\tilde{A}}: U \rightarrow [0, 1]$. $t_{\tilde{A}}(u_i)$ is a lower bound on the grade of membership of u_i derived from the evidence for u_i and $f_{\tilde{A}}(u_i)$ is a lower bound on the negation of u_i derived from the evidence against u_i, and $0 \leq t_{\tilde{A}}(u_i) + f_{\tilde{A}}(u_i) \leq 1$. The grade of membership of u_i in the IFS (or vague set) $\tilde{A}$ is bounded to a subinterval $[t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]$ of [0, 1]. Gau and Buehrer [11] refer to $[t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]$ as a *vague value* (equivalently, an *intuitionistic fuzzy value*) which indicates that the exact grade of membership

$\mu_{\tilde{A}}(u_i)$ of u_i may be unknown, but it is bounded by $t_{\tilde{A}}(u_i) \le \mu_{\tilde{A}}(u_i) \le 1 - f_{\tilde{A}}(u_i)$, where $0 \le t_{\tilde{A}}(u_i) + f_{\tilde{A}}(u_i) \le 1$ (see Figure 1).

When the universe of discourse U is discrete, an IFS or vague set Ã can be written as

$$\tilde{A} = \sum_{i=1}^{n} [t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)] / u_i$$

For example, let U be the universe of discourse, U = {6, 7, 8, 9, 10}. A vague set "LARGE" of U may be defined as

$$\text{LARGE} = [0.1,0.2]/6 + [0.3,0.5]/7 + [0.6,0.8]/8 + [0.9,1]/9 + [1,1]/10$$

An intuitionistic fuzzy value $\tilde{a} = [0.5,0.7]$ may be interpreted as five votes "for," three "against," and two "absentions" [11].

Obtaining an IFS requires both $\mu(u)$ and $\nu(u)$. Both need to satisfy $0 \le \mu(u) + \nu(u) \le 1$ and $\pi(u) = 1 - \mu(u) - \nu(u)$. If information on consistency is available, it may be useful to express it as a linguistic variable and then convert the linguistic value of this variable to a numerical value for consistency as shown in Table 3 [13].

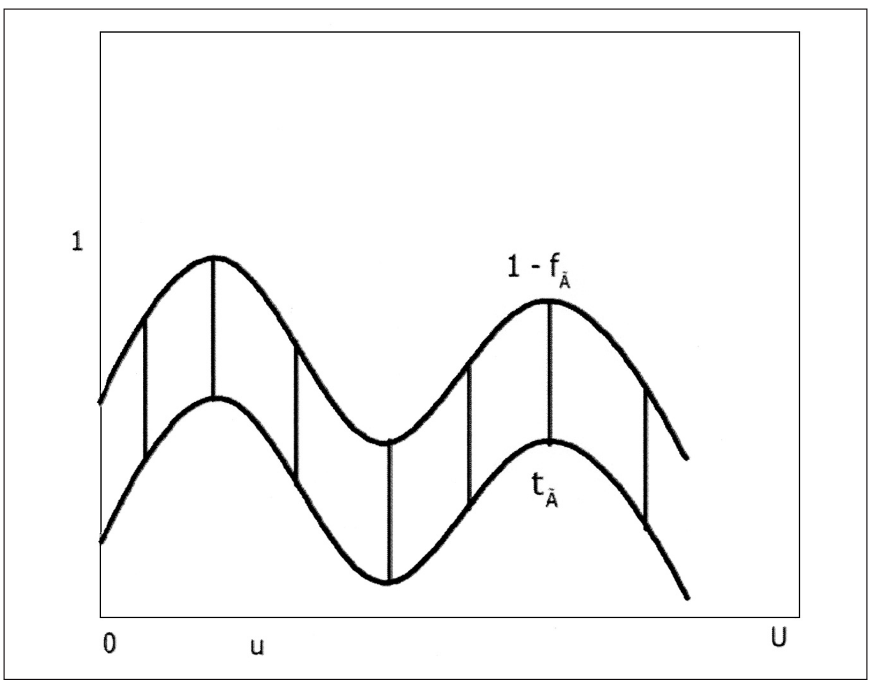

Figure 1. Graphic representation of an intuitionistic fuzzy set, Ã.

Table 3. Linguistic and Numeric Values for Linguistic Variable, Consistency

Consistency	$\pi(u)$
No or very low consistency	0.8–1.0
Low consistency	0.6–0.8
Moderate consistency	0.4–0.6
High consistency	0.2–0.4
Very high or total consistency	0.0–0.2

Hersh [13] suggested a fuzzification technique to scale a conventional fuzzy membership function, $\mu°(u)$. Given $\mu°(u)$, then $\mu(u)$ and $\nu(u)$ in the IFS are obtained as follows:

$$\mu(u) = \mu°(u)(1 - \pi(u))$$

$$\nu(u) = 1 - \mu(u) - \pi(u)$$

$$= [1 - \mu°(u)][1 - \pi(u)]$$

In terms of the fuzzy membership values for the above example relating to the location of a hazardous waste disposal facility with three possible sites assessed against 10 factors, $\mu(u)$ and $\nu(u)$ are as shown in Table 4 (using a consistency of $\pi(u) = 0.2$ (high consistency)).

In terms of intuitionistic fuzzy set theory, each factor F_j, may be construed as an intuitionistic fuzzy set of the set of projects represented as $\widetilde{F}_j = \{\widetilde{F}_j(p_1)|p_1, \widetilde{F}_j(p_2)|p_2, ..., \widetilde{F}_j(p_1)|p_I\}$, where $\widetilde{F}_j(p)$ indicates the intuitionistic fuzzy value expressing the degree and the non-degree to which project $p \in P$ satisfies factor/impact F_j. Note that the satisfaction of a given project (denoted either as p or p_i) is represented as $\widetilde{F} = \{\widetilde{F}_1(p)|p, \widetilde{F}_2(p)|p, ..., \widetilde{F}_j(p)|p\}$, $p \in P$. Each column of the project/factor matrix (Table 5) is an intuitionistic fuzzy set.

In Table 5, $\widetilde{F}_j(p_i) = [t_{\widetilde{F}j}(p_i), 1 - f_{\widetilde{F}j}(p_i)]$ (i = 1, 2, ..., I; j = 1, 2, ..., J) is an intuitionistic fuzzy value for factor F_j with respect to project p_i.

AGGREGATION OF INTUITIONISTIC FUZZY SETS

Chen and Tan [14] developed a method for aggregating intuitionistic fuzzy sets by *conjunctive* aggregation involving the minimum operator and/or *disjunctive* aggregation involving the maximum operator. Let $\widetilde{b} = [t_{\widetilde{b}}, 1 - f_{\widetilde{b}}]$ and $\tilde{a} = [t_{\tilde{a}}, 1 - f_{\tilde{a}}]$ be two IFSs. Then, the minimum $\widetilde{c} = [t_{\widetilde{c}}, 1 - f_{\widetilde{c}}]$ is given as $t_{\widetilde{c}} = \min\{t_{\tilde{a}}, t_{\widetilde{b}}\}$ and $1 - f_{\widetilde{c}} = \min\{1 - f_{\tilde{a}}, 1 - f_{\widetilde{b}}\}$. Similarly, the maximum $\widetilde{d} = [t_{\tilde{a}}, 1 - f_{\widetilde{d}}]$ is given as $t_{\widetilde{c}} =$

Table 4. $\mu(x)$ (i.e., $t_{\tilde{F}j}(p_i)$) and $\upsilon(x)$ (i.e., $f_{\tilde{F}}(p_i)$) Based on Project/Factor Matrix in Table 2

$\mu(x)$	$\tilde{F}_1$	$\tilde{F}_2$	$\tilde{F}_3$	$\tilde{F}_4$	$\tilde{F}_5$	$\tilde{F}_6$	$\tilde{F}_7$	$\tilde{F}_8$	$\tilde{F}_9$	$\tilde{F}_{10}$
p_1	0.72	0.64	0.8	0.72	0.64	0.8	0.64	0.64	0.8	0.4
p_2	0.56	0.72	0.8	0.72	0.72	0.4	0.48	0.4	0.48	0.8
p_3	0.24	0.16	0.8	0.16	0.8	0.8	0.16	0.16	0.24	0.24
$\upsilon(x)$	$\tilde{F}_1$	$\tilde{F}_2$	$\tilde{F}_3$	$\tilde{F}_4$	$\tilde{F}_5$	$\tilde{F}_6$	$\tilde{F}_7$	$\tilde{F}_8$	$\tilde{F}_9$	$\tilde{F}_{10}$
p_1	0.08	0.16	0	0.08	0.16	0	0.16	0.16	0	0.4
p_2	0.24	0.08	0	0.08	0.08	0.4	0.32	0.4	0.32	0
p_3	0.56	0.64	0	0.64	0	0	0.64	0.64	0.56	0.56

Table 5. Intuitionistic Fuzzy Project/Factor Matrix

	$\tilde{F}_1$	$\tilde{F}_2$	…	$\tilde{F}_J$
p_1	$\tilde{F}_1(p_1)$	$\tilde{F}_2(p_1)$	…	$\tilde{F}_J(p_1)$
p_2	$\tilde{F}_1(p_2)$	$\tilde{F}_2(p_2)$	…	$\tilde{F}_J(p_2)$
⋮	⋮	⋮		⋮
p_I	$\tilde{F}_1(p_I)$	$\tilde{F}_2(p_I)$	…	$\tilde{F}_J(p_I)$

$\max\{t_{\tilde{a}}, t_{\tilde{b}}\}$ and $1 - f_{\tilde{c}} = \max\{1 - f_{\tilde{a}}, 1 - f_{\tilde{b}}\}$. Thus, $\tilde{D}(p) = \tilde{F}_1(p)\ \tilde{F}_2(p) \cap ... \cap \tilde{F}_J(p)$, $p \in P$ where $\cap$ denotes conjunction, and

$$[t(p), 1 - f(p)] = [\min_{j=1,2,...,J} [t_{\tilde{F}j}(p)], \min_{j=1,2,...,J} [1 - f_{\tilde{F}j}(p)]], p \in P.$$

Chen and Tan [14] used a *scoring function* (see below) to rank alternatives. Hong and Choi [15] proposed some modifications to this method introducing the *accuracy function* and Ye [16] and Lin et al. [17] improved on these functions.

Let $\tilde{a}_j = [t_{\tilde{a}_j}, 1 - f_{\tilde{a}_j}]$ $(j = 1, 2, ..., J)$ be a collection of intuitionistic fuzzy values. Then the *intuitionistic fuzzy weighted average* (IFWA) operator is an intuitionistic fuzzy value [18] defined as follows:

$$IFWA_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = w_1\tilde{a}_1 \oplus w_2\tilde{a}_2 \oplus ... \oplus w_j\tilde{a}_J$$

$$IFWA_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \left[1 - \prod_{j=1}^{J}(1 - t_{\tilde{a}_j})^{w_j}, 1 - \prod_{j=1}^{J}(f_{\tilde{a}_j})^{w_j}\right]$$

Here $w = [w_1, w_2 ..., w_J]$ are the weights of the intuitionistic fuzzy values, $\tilde{a}_j$, $w_j \in [0,1]$ and $\sum_{j=1}^{J} w_j = 1$.

This operator is based on the following IFS operations [18]. Let $\tilde{a} = [t_{\tilde{a}}, 1 - f_{\tilde{a}}]$, $\tilde{b} = [t_{\tilde{b}}, 1 - f_{\tilde{b}}]$ be two IFSs. Then $\tilde{a} \oplus \tilde{b} = t_{\tilde{a}} + t_{\tilde{b}} - t_{\tilde{a}} t_{\tilde{b}}, 1 - f_{\tilde{a}} f_{\tilde{b}}] = [1 - (1 - t_{\tilde{a}})(1 - t_{\tilde{b}}), 1 - f_{\tilde{a}} f_{\tilde{b}})$ and $\lambda\tilde{a} = [1 - (1 - t_{\tilde{a}})^{\lambda}, 1 - f_{\tilde{a}}^{\lambda}]$, $\lambda > 0$.

Let $\tilde{a}_j = [t_{\tilde{a}_j}, 1 - f_{\tilde{a}_j}]$ $(j = 1, 2, ..., J)$ be a collection of intuitionistic fuzzy values. The *intuitionistic fuzzy ordered weighted averaging* (IFOWA) operator is an intuitionistic fuzzy value [18] defined as follows:

$$\text{IFOWA}_{\omega}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \omega_1 \tilde{a}_{\sigma(1)} \oplus \omega_2 \tilde{a}_{\sigma(2)} \oplus ... \oplus \omega_J \tilde{a}_{\sigma(J)}$$

$$\text{IFOWA}_{\omega}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_j) = \left[1 - \prod_{j=1}^{J} (1 - t_{\tilde{a}_{\sigma(j)}})^{\omega_j}, 1 - \prod_{j=1}^{J} (f_{\tilde{a}_{\sigma(j)}})^{\omega_j} \right]$$

Here $\omega = [\omega_1\ \omega_2, ..., \omega_J]$ are the (positional) weights of the IFOWA operator, $\omega_j \in [0,1]$ and $\sum_{j=1}^{J} w_j = 1$. $[\sigma(1), \sigma(2), ..., \sigma(J)]$ is a permutation of $[1, 2, ..., J]$ such that $\tilde{a}_{\sigma(j-1)} \geq \tilde{a}_{\sigma(j)}$. A scoring function [14] (see below) is used to rank-order the intuitionistic fuzzy values. Note that, if $\omega = [\omega_1, \omega_2 ..., \omega_J] = [1/J, 1/J, ..., 1/J]$, then the IFOWA becomes a intuitionistic fuzzy average (IFWA).

Xu [18] also introduces a *intuitionistic fuzzy hybrid averaging* (IFHA) operator which weights both each intuitionistic value and its ordered position. The IFHA operator is an intuitionistic fuzzy value defined as follows:

$$\text{IFHA}_{\omega,w}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \omega_1 \tilde{\alpha}_{\sigma(1)} \oplus \omega_2 \tilde{a}_{\sigma(2)} \oplus ... \oplus \omega_J \tilde{a}_{\sigma(J)}$$

$$\text{IFHA}_{\omega,w}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \left[1 - \prod_{j=1}^{J} (1 - t_{\tilde{\alpha}_{\sigma(j)}})^{\omega_j}, 1 - \prod_{j=1}^{J} (f_{\tilde{\alpha}_{\sigma(j)}})^{\omega_j} \right]$$

where $\tilde{\alpha}_j = J w_j \tilde{a}_j$ $(j = 1, 2, ..., J)$. Here, w_j $(j = 1, 2, ..., J)$ is the weight of intuitionistic fuzzy value $\tilde{a}_j$ $(j = 1, 2, ..., J)$. Clearly, the IFHA operator generalizes both the IFOWA and IFWA operators and reflects the importance of both the given intuitionistic fuzzy values and the ordered position of these values.

Let $\tilde{a} = [t_{\tilde{a}}, 1 - f_{\tilde{a}}]$ be an intuitionistic fuzzy value, then a *scoring function* is defined as $S(\tilde{a}) = t_{\tilde{a}} - f_{\tilde{a}}$ [14]. Also, $H(\tilde{a}) = t_{\tilde{a}} + f_{\tilde{a}}$ is defined as an *accuracy function* [14]. Given an intuitionistic fuzzy value, $\tilde{a} = [t_{\tilde{a}}, 1 - f_{\tilde{a}}]$, then the hesitancy degree $\pi(\tilde{a}) = 1 - t_{\tilde{a}} - f_{\tilde{a}}$ and the accuracy degree are such that $\pi(\tilde{a}) + H(\tilde{a}) = 1$. That is, the higher the degree of accuracy, the lower the degree of hesitancy [13].

Xu and Yager [19] introduce the *intuitionistic fuzzy geometric weighted averaging* (IFWGA) operator. Let $\tilde{a}_j = [t_{\tilde{a}}, 1 - f_{\tilde{a}_j}]$ (j = 1, 2, ..., J) be a collection of intuitionistic fuzzy values. Then the IFWGA operator is an intuitionistic fuzzy value defined as follows:

$$\text{IFWGA}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \tilde{a}_1^{w_1} \otimes \tilde{a}_2^{w_2} \otimes ... \otimes \tilde{a}_J^{w_J}$$

$$\text{IFWGA}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \left[1 - \prod_{j=1}^{J} (t_{\tilde{a}_j})^{w_j},\ 1 - \prod_{j=1}^{J} (1 - f_{\tilde{a}_j})^{w_j}\right]$$

Here $w = [w_1, w_2, ..., w_J]$ are the weights of the intuitionistic fuzzy values, $\tilde{a}_j$, such that $w_j \in [0,1]$ and $\sum_{j=1}^{J} w_j = 1$.

This operator is based on the following IFS operations [19]. Let $\tilde{a} = [t_{\tilde{a}}, 1 - f_{\tilde{a}}]$, $\tilde{b} = [t_{\tilde{b}}, 1 - f_{\tilde{b}}]$ be two IFSs. Then $\tilde{a} \otimes \tilde{b} = [t_{\tilde{a}} t_{\tilde{b}}, (1 - f_{\tilde{a}})(1 - f_{\tilde{b}})]$, and $\tilde{a}^{\lambda} = [(t_{\tilde{a}})^{\lambda}, (1 - f_{\tilde{a}})^{\lambda}]$, $\lambda > 0$.

Let $\tilde{a}_j = [t_{\tilde{a}_j}, 1 - f_{\tilde{a}_j}]$ (j = 1, 2, ..., J) be a collection of intuitionistic fuzzy values. The *intuitionistic fuzzy ordered weighted geometric averaging* (IFOWGA) operator is an intuitionistic fuzzy value defined as follows [18]:

$$\text{IFOWGA}_\omega(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \tilde{a}_{\sigma(1)}^{\omega_1} \otimes \tilde{a}_{\sigma(2)}^{\omega_2} \otimes ... \otimes \tilde{a}_{\sigma(J)}^{\omega_J}$$

$$\text{IFOWGA}_\omega(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \left[1 - \prod_{j=1}^{J} (1 - t_{\tilde{a}_{\sigma(j)}})^{\omega_j},\ 1 - \prod_{j=1}^{J} (f_{\tilde{a}_{\sigma(j)}})^{\omega_j}\right]$$

where $\omega = [\omega_1, \omega_2, ..., \omega_J]$ are the (positional) weights of the IFOWGA operator, $\omega_j \in [0,1]$ and $\sum_{j=1}^{J} \omega_j = 1$. $[\sigma(1), \sigma(2), ..., \sigma(J)]$ is a permutation of [1, 2, ..., J] such that $\tilde{a}_{\sigma(j-1)} \geq \tilde{a}_{\sigma(j)}$. If $\omega = [\omega_1, \omega_2, ..., \omega_J] = [1/J, 1/J, ..., 1/J]$, the IFOWGA becomes a IFWGA.

Xu and Yager [19] also introduce an *intuitionistic fuzzy hybrid geometric averaging* (IFHGA) operator which weights both each intuitionistic value and its ordered position. The IFHGA operator is an intuitionistic fuzzy value defined as follows:

$$\text{IFHGA}_{\omega,w}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \tilde{\alpha}_{\sigma(1)}^{\omega_1} \otimes \tilde{\alpha}_{\sigma(2)}^{\omega_2} \otimes ... \otimes \tilde{\alpha}_{\sigma(J)}^{\omega_J}$$

$$\text{IFHGA}_{\omega,w}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \left[1 - \prod_{j=1}^{J} (t_{\tilde{\alpha}_{\sigma(j)}})^{\omega_j},\ - \prod_{j=1}^{J} (1 - f_{\tilde{\alpha}_{\sigma(j)}})^{\omega_j}\right]$$

where $\tilde{\alpha}_j = J w_j \tilde{a}_j$ (j = 1, 2, ..., J). Here, w_j (j = 1, 2, ..., J) is the weight of intuitionistic fuzzy value $\tilde{a}_j$ (j = 1, 2, ..., J).

Wang et al. [20] provide a new scoring function based on the scoring and accuracy functions [14]. Let $\tilde{a} = [t_{\tilde{a}}, 1 - f_{\tilde{a}}]$, then

$$S(\tilde{a}) = t_{\tilde{a}} - f_{\tilde{a}} - \frac{1 - t_{\tilde{a}} - f_{\tilde{a}}}{2}$$

$$= \frac{3}{2} t_{\tilde{a}} - \frac{1}{2} f_{\tilde{a}} - \frac{1}{2}$$

This function may be used to rank the intuitionistic values from the various weighted or order weighted operators above. Here, a normalized scoring function (based on [20]) may be developed by taking into account that $\min\{S(\tilde{a})\} = -1$ and $\max\{S(\tilde{a})\} = 1$ (since $S([1,0]) = 1$ and $S([0,1]) = -1$).

$$\hat{S}(\tilde{a}) = \frac{S(\tilde{a}) - \min\{S(\tilde{a})\}}{\max\{S(\tilde{a})\} - \min\{S(\tilde{a})\}}$$

$$= \frac{S(\tilde{a}) + 1}{2}$$

APPLICATION IN SITING A HAZARDOUS WASTE FACILITY

Table 6 shows the weighted intuitionistic fuzzy values based on Table 4 for the hybrid intuitionistic operator, IFHA, using factor weights, w = [0.161, 0.156, 0.148, 0.115, 0.111, 0.106, 0.074, 0.052, 0.046, 0.031] and

$$\tilde{a}_j(p_i) = [t_{\tilde{a}_j}(p_i), 1 - f_{\tilde{a}_j}(p_i)] = [1 - (1 - t_{\tilde{F}_j}(p_i))^{Jw_j}, 1 - (f_{\tilde{F}_j}(p_i))^{Jw_j}]$$

The results of the hybrid intuitionistic operator for each project:

$$IFHA_{\omega,w}(\tilde{a}_1(p_1), \tilde{a}_2(p_i), ..., \tilde{a}_J(p_i)) = \left[1 - \prod_{j=1}^{J} (1 - t_{\tilde{\alpha}_{\sigma(j)}})^{\omega_j}, 1 - \prod_{j=1}^{J} (f_{\tilde{\alpha}_{\sigma(j)}})^{\omega_j}\right]$$

where $\tilde{\alpha}_j(p_i) = Jw_j\tilde{F}_j(p_i)$ (j = 1, 2, ..., J) are for p_1, p_2, p_3, [0.714, 1], [0.656, 1], and [0.514, 1], respectively. Here, positional weights are ω = [1/J, 1/J, ..., 1/J] = [0.1, 0.1, ..., 0.1]. The normalized score function yields 0.785, 0.742, and 0.636 for p_1, p_2, p_3, respectively.

Using positional weights, ω = [0.01, 0.03, 0.05, 0.07, 0.09, 0.11, 0.13, 0.15, 0.17, 0.19] derived from a quantifier "most," $Q(r) = r^2$ (see [2]), where $\omega_j = Q(j/J) - Q((j - 1)/J)$, (j = 1, 2, ..., J) yields, respectively, for p_1, p_2, p_3, [0.581, 1], [0.490, 1], and [0.265, 1]. The normalized scoring function yields 0.685, 0.618, and 0.449 for p_1, p_2, p_3, respectively. Again, all results indicate a preference order, $p_1 \succ p_2 \succ p_3$ as shown by using a *weighted conjunctive aggregation* [4].

Table 6. Weighted Intuitionistic Fuzzy Values for IFHA Operator for Siting Hazardous Waste Facility

p_1	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_3$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$	$\tilde{\alpha}_6$	$\tilde{\alpha}_7$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_{10}$
$t_{\tilde{\alpha}_j}(p_1)$	0.871	0.797	0.908	0.769	0.678	0.818	0.53	0.412	0.523	0.146
$1 - f_{\tilde{\alpha}_j}(p_1)$	0.983	0.943	1	0.945	0.869	1	0.742	0.614	1	0.247
p_2	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_3$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$	$\tilde{\alpha}_6$	$\tilde{\alpha}_7$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_{10}$
$t_{\tilde{\alpha}_j}(p_2)$	0.733	0.863	0.908	0.769	0.757	0.418	0.384	0.233	0.26	0.393
$1 - f_{\tilde{\alpha}_j}(p_2)$	0.9	0.981	1	0.945	0.939	0.621	0.57	0.379	0.408	1
p_3	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_3$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$	$\tilde{\alpha}_6$	$\tilde{\alpha}_7$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_{10}$
$t_{\tilde{\alpha}_j}(p_3)$	0.357	0.238	0.908	0.182	0.832	0.818	0.121	0.087	0.119	0.082
$1 - f_{\tilde{\alpha}_j}(p_3)$	0.607	0.502	1	0.401	1	1	0.281	0.207	0.234	0.165

Table 7 shows the weighted intuitionistic fuzzy values based on Table 4 for the hybrid geometric operator, IFHGA, using factor weights, w = [0.161, 0.156, 0.148, 0.115, 0.111, 0.106, 0.074, 0.052, 0.046, 0.031] and

$$\tilde{\alpha}_j(p_i) = [t_{\tilde{\alpha}_j}(p_i), 1 - f_{\tilde{\alpha}_j}(p_i)] = [(t_{\tilde{F}_j}(p_i))^{Jw_j}, (1 - (f_{\tilde{F}_j}(p_i))^{Jw_j}]$$

The results of the hybrid intuitionistic geometric operator for each project:

$$\text{IFHGA}_{\omega,w}(\tilde{a}_1(p_1), \tilde{a}_2(p_i), ..., \tilde{a}_J(p_i)) = \left[\prod_{j=1}^{J}(t_{\tilde{a}_{\sigma(j)}})^{\omega_j}, \prod_{j=1}^{J}(1 - f_{\tilde{a}_{\sigma(j)}})^{\omega_j}\right]$$

where $\tilde{\alpha}_j(p_i) = Jw_j\tilde{F}_j(p_i)$ (j = 1, 2, ..., J) are for p_1, p_2, p_3, [0.697, 0.898], [0.612, 0.816], and [0.371, 0.548], respectively. Here, ω = [1/J, 1/J, ..., 1/J] = [0.1, 0.1, ..., 0.1]. The score function yields 0.747, 0.663, and 0.375 for p_1, p_2, p_3, respectively.

Using weights, ω = [0.01, 0.03, 0.05, 0.07, 0.09, 0.11, 0.13, 0.15, 0.17, 0.19] again derived from a quantifier "most," $Q(r) = r^2$, where $\omega_j = Q(j/J) - Q((j-1)/J)$, j = 1, 2, ..., J) yields, respectively, for p_1, p_2, p_3, [0.639, 0.849], [0.533, 0.743], and [0.193, 0.402]. The normalized score function yields 0.694, 0.586, and 0.245 for p_1, p_2, p_3, respectively. Clearly both indicate a preference order, $p_1 \succ p_2 \succ p_3$, as shown by using a *weighted conjunctive aggregation* [4].

Table 7. Weighted Intuitionistic Fuzzy Values for IFHGA Operator for Siting Hazardous Waste Facility

p_1	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_3$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$	$\tilde{\alpha}_6$	$\tilde{\alpha}_7$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_{10}$
$t_{\tilde{\alpha}_j}(p_1)$	0.589	0.498	0.719	0.685	0.609	0.789	0.719	0.793	0.902	0.753
$1 - f_{\tilde{\alpha}_j}(p_2)$	0.874	0.762	1	0.909	0.824	1	0.879	0.913	1	0.0854
p_2	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_3$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$	$\tilde{\alpha}_6$	$\tilde{\alpha}_7$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_{10}$
$t_{\tilde{\alpha}_j}(p_2)$	0.393	0.599	0.719	0.685	0.694	0.379	0.581	0.621	0.713	0.933
$1 - f_{\tilde{\alpha}_j}(p_2)$	0.643	0.878	1	0.909	0.912	0.582	0.752	0.767	0.837	1
p_3	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_3$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$	$\tilde{\alpha}_6$	$\tilde{\alpha}_7$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_{10}$
$t_{\tilde{\alpha}_j}(p_3)$	0.1	0.057	0.719	0.122	0.781	0.789	0.258	0.386	0.519	0.642
$1 - f_{\tilde{\alpha}_j}(p_3)$	0.267	0.203	1	0.309	1	1	0.47	0.588	0.685	0.775

CONCLUSION

In the context of project evaluation, an IFS may be used to represent the degree to which a project satisfies a criterion and the degree to which it does not. Aggregation of such IFSs has been considered in recent years to identify a best project in terms of several criteria. A particular desirable way to aggregate IFS is in terms of an ordered weighted average (OWA) which can be expressed in different forms such as arithmetic and geometric. In an OWA operator, weights are applied to the position of an element in the aggregation. In addition, hybrid OWA operators may be developed to not only weight the position of elements in the aggregation but the element itself.

A simple example drawn based on a hypothetical but realistic example by Horsak and Damico is given which involves the location of a hazardous waste disposal facility (PCB-contaminated transformer fluids) at one of three sites based on 10 factors.

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