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Modeling the Public Welfare Systems: Part I

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ABSTRACT

The purpose of this study is to construct an analytical model of caseload dynamics to be employed in the projection of potential N.Y.S. Public Welfare cases. The term "welfare" is used in this study to mean Public Assistance. Public Assistance is disaggregated into five major categories. The model is composed of three major components. Markovian transition rates are determined for two of these components through an analysis of historical data compiled by the Department of Social Services. The components of the system are integrated into a set of difference equations to predict quarterly caseload distributions by category. In addition, the Markov Chain Model is used to analyze the static structure of caseload transitions within the welfare system. The model has been calibrated on three years of quarterly data from 1969 to 1971. The model was tested ex post facto for 1972. This validation procedure indicated mean absolute errors for individual categories and total welfare caseloads of 4. and .2 per cent, respectively. Since the closing part of the model introduced an appreciable portion of errors, further work is required. Part two extends the model to include a detailed methodology for forecasting the openings and closings portions of the model.

Introduction

Mathematical models have been used successfully in related fields of social welfare such as economics and demography. However, the application of mathematical models to Social Welfare Planning is a comparatively new development. A comprehensive survey of the all Welfare Departments in the U.S. has revealed that the models which have been developed in this field are mostly conceptional rather than analytical; although, the New York City Rand

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Institute [1], Denver Department of Welfare [2], State of Michigan Department of Social Services [3], and Illinois Department of Public Aid [4], have developed analytical models for caseload projections. The Rand Model, developed for New York City is the most elaborate and requires the establishment of new data files not normally available to most welfare departments. The remaining analytical models are of the regression type and deal with a single category primarily Aid to Dependent Children, (ADC).

Administrators, planners, and policymakers of Social Welfare Systems have found that their ad hoc procedures are no longer adequate to take into account the many variables impinging on their environment. In an attempt to overcome this difficulty, methodological approaches to analyze and predict the Social Welfare caseload are proffered. The present paper is concerned with the applicability of one model, the Markov Chain Model [5], to predict the Social Welfare Caseload in New York State. This paper is based on its initial version presented at the 8th Annual ACM Urban Symposion [6].

In addition, the Markov Chain Model is used to analyze the static structure of caseload transitions within and outside the Welfare System. The model has been calibrated on three years of quarterly data from 1969 to 1971. The model was tested ex post facto for 1972 and exhibited a .2 per cent average absolute value error on quarterly caseload totals.[1]

Public Assistance Categories

The study is concerned with Welfare Systems in New York State. The term "welfare" is used in this study to mean Financial Public Assistance. Financial Public Assistance is disaggregated into its five major categories; Old Age Assistance (OAA), Aid to the Blind (AB), Aid to the Disabled (AD), Aid to Dependent Children (ADC) and Home Relief (HR).

Under (OAA), assistance is provided to financially needy applicants who are over sixty-five years of age. The (AD) category is limited to those between the ages of eighteen and sixty-four, with disabilities, while there is no age limit on (AB). A needy person is defined as one whose income and resources are below the State's eligibility requirements. The (ADC) Program provides financial aid for children who are deprived of parental support and care either through the continued absence or incapacity of a parent or the unemployment of a father. Families with an unemployed father (ADC-U) considered under ADC, are eligible for assistance only if: 1) the unemployment is in excess of 30 days duration, 2) unemployment insurance benefits are being received, 3) a "substantial attachment" to the labor force exists and 4) the employable person conforms to State requirements pursuant to securing employment. Home Relief (HR) is granted to individuals and families with income and resources below

[1] The terms category and program are used interchangeably.

State Assistance standards, but who do not meet the technical eligibility requirements for Federal participation under the remaining categories.

The process of caseload assignment is initiated by a prospective welfare recipient's application which may be rejected due to misrepresentation, incompleted data or financial ineligibility. Acceptable clients are then assigned to one of the five welfare categories, followed by review and authorization before being placed on the active caseload roles.

Thus, the assignment process may be classified into the following four major operations:

1. *Application:* This is the collection, validiation, and recording of the client-supplied data.
2. *Budget Computation:* This is the process to determine the client financial eligibility by manipulating the income and resources data which is submitted in the application.
3. *Categorical Eligibility:* This is the evaluation of the client's social and family data, combining it with the information developed in the budget computation process, and comparing the results against the various program criteria. The resulting decisions allow the proper assignment of program to the client.
4. *Authorization:* The final decision detailing the client's classification as to program areas and specification of the various grants which the client is entitled to. It is followed by the placement of the welfare recipient on the active caseload roles.

The active caseload is defined as the total number of cases at a particular point in time which are authorized for assistance and recorded in the Department of Social Services records. It is implicitly assumed that the above program definitions have remained constant over the time period of the analysis. In cases where program definitions have varied appropriate modifications are made to conform to the five major categories designated above.

The Markov Chain Model

The model is composed of three major components which represent the behavior of welfare case movements over a particular period of time. The components are:

Transfer Cases: The cases which are transferred from one public assistance category to another.

Closed Cases: The cases which are removed (closed) from the rolls of any category.

Opened Cases: New (opened) cases from outside the welfare system.

These three components represent the movement of cases between six states, i.e., the five states (1 to 5) corresponding to the five categories within the welfare system, and one state (state 6) corresponding to potential welfare cases outside the system. A Markov assumption that transitions from a given state are a direct function of the number of cases in the state are made for the five categories in the welfare system. Thus, of the 36 possible transition rates, only the 30 dependent on the number of cases within the welfare system are considered. Opened cases are obtained exogenously in lieu of the transition rate approach and are a function of the number of applications and caseload assignment process. Let the transition rate, q_{ij}^t represent the fraction of cases in state i at the start of period t, that moved to state j during the interval of time (t, t + 1). If p_j^t represents the total number of cases in state j at the start of period t, and n_j^t represents the number of new cases opened in category j then,

$$p_j^{t+1} = \sum_{i=1}^{5} q_{ij}^t \; p_i^t + n_j^t \quad , \; j = 1, \ldots 5 \tag{1}$$

represents the number of cases in category j at the start of period t + 1. Written compactly in matrix form:

$$P^{t+1} = P^t \, Q^t + N^t \tag{2}$$

Where

P^t = the (1 X 5) vector of the total number of cases within the welfare system at the beginning of period t.

Q^t = the (5 X 5) matrix of the transition coefficients between major categories during (t, t + 1).

N^t = the (1 X 5) vector of cases opened from outside the welfare system during (t, t + 1).

To determine the rate of leaving the welfare system from category i let;

$$q_{i6}^t = 1 - \sum_{j=1}^{5} q_{ij}^t \quad , \; i = 1, \ldots 5 \tag{3}$$

Under the Markov hypothesis, s_i^t, the number of cases leaving the welfare system from category i (Closing cases) may be determined as:

$$s_i^t = p_i^t \, q_{i6}^t \quad , \; i = 1, \ldots 5 \tag{4}$$

Written compactly in matrix form:

$$S^t = P^t \, \bar{Q}^t$$

Where $\bar{Q}^t$ = is a (5 X 5) matrix with q_{i6}^t in the ith diagonal position and zeros elsewhere.

S^t = is a (1 X 5) vector of cases leaving the welfare system.

Calibration of the Model

Markovian transfer and closing rates for 12 periods are determined for each of the five categories through an analysis of historical data compiled quarterly by the Department of Social Services for the period 1969 up to the end of 1971 [7]. Knowing, 1) the total number of active cases of each category at the end of last quarter, 2) the cases transferred to other categories in the present quarter, and 3) closed cases during this quarter; the total number of remaining cases in each category may be computed at the end of the present quarter. The totals determined in this manner for each category at the end of each quarter were balanced with the totals obtained from historical data [5] at the beginning of the next quarter. The transition rates between categories and closed cases are computed by dividing the number of cases transferred from category i to category j by the total number of cases in i at the beginning of the quarter. Analysis of the data over the 12 quarters indicates that the transition rates between categories are relatively stable. Also, it does not appear that there is seasonality in the caseload.

The mean[2] and standard deviation of the refined quarterly rates are computed and presented in Table 1.

Table 1. Mean and Standard Deviations of Transition Rates

To j / *From i*	*1 OAA*	*2 AB*	*3 AD*	*4 HR*	*5 ADC*	*6 Outside*
1. OAA	.942 (.005)	–	–	–	–	.058 (.005)
2. AB	.003 (.001)	.941 (.007)	.001 (.000)	.003 (.001)	.001 (.000)	.051 (.006)
3. AD	.007 (.002)	–	.845 (.020)	.013 (.003)	.002 (.000)	.133 (.018)
4. HR	.007 (.003)	–	.044 (.013)	.701 (.045)	.028 (.015)	.220 (.034)
5. ADC	–	–	.001 (.000)	.008 (.003)	.902 (.005)	.089 (.008)

The analysis of the above matrix reveals the special structure of caseload transfers within the system. This structure may be readily seen by placing an X in the positions containing significant rates (those $\geqslant .01$) and zero elsewhere. The resulting matrix is shown in Table 2. Using the matrix in Table 2 to define

[2] The actual mean values were adjusted to allow the row sums to add to unity.

Table 2. Matrix of Significant Transition Rates

	OAA	*AB*	*AD*	*HR*	*ADC*
OAA	X	O	O	O	O
AB	O	X	O	O	O
AD	O	O	X	X	O
HR	O	O	X	X	X
ADC	O	O	O	O	X

the major flows between welfare categories one may decompose the public assistance system into the three independent groups shown in Figure 1.

An analysis of the above structure indicates that both the OAA and AB categories are independent of inter-case movement. Therefore, we may say that the major increase in their caseloads is attributed mostly to newly opened cases rather than to transfers from other categories. There is significant interchange between AD and HR while ADC receives most of its transfers from the HR category and does not contribute to any other category in the system.

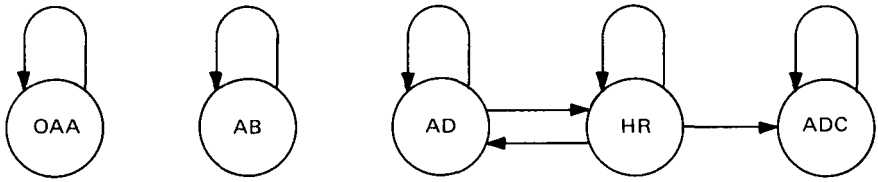

Figure 1. The structure of inter-category transitions.

Projection of Caseloads

Projections on a quarterly basis for one year are considered to be useful for planning and evaluation purposes in the welfare system. As the Markov Model is calibrated for the years 1969 to 1971, these years cannot be used to test the model as a forecasting tool.[3] Due to the availability of observed caseload data for 1972 this year was selected for an ex post facto test of the model as a projection tool. The model was tested under the assumption of stationary intercategory and closing transition rates. The actual number of openings were used to determine the effect on the forecast error introduced by the Markov assumption alone. These Opening data were obtained from the Department of Social Services Summary of Reasons for Opening and Closing [5] and are reproduced in Table 3.

The caseload projections by category, using equation (1) with constant transition rates obtained from Table 1, are shown in Table 4. It is noted that the

[3] Quarterly data do not exist in the Department of Social Services prior to 1969.

Table 3. Quarterly Summary of Opened Cases by Public Assistance Categories 1972

Category \ Quarter	1	2	3	4
OAA	7462	6555	5720	5021
AB	277	235	204	180
AD	22932	20000	17237	15337
HR	45749	43788	39292	32898
ADC	32886	28740	30982	27922

Table 4. Quarterly 1972 Projections and Error Analysis of Caseload by Category

					Per cent Error	
Category	Quarter	Estimated	Observed	Difference	Caseload	Closing
OAA	1	114469	112308	+2161	+2.	-17
	2	116153	112547	+3606	+3.	-11
	3	116968	110704	+6264	+6.	-20
	4	117049	108499	+8550	+8.	-18
AB	1	4051	4085	-34	-.8	-5
	2	4046	4164	-118	-3	+4
	3	4011	4213	-202	-5	-.9
	4	3954	4228	-274	-6	-9
AD	1	138626	139513	-887	-.6	+2
	2	142424	144820	-2396	-1.6	+6
	3	143106	148065	-4959	-3	+16
	4	141831	151789	-9958	-6.5	+30
HR	1	112176	111926	+250	+.2	-9
	2	117556	111162	+6394	+5	-9
	3	118659	109281	+8378	+8	+11
	4	115384	103529	+11855	+11	-3
ADC	1	345530	348646	-3116	-.9	-19
	2	343830	349745	-5915	-1.7	-13
	3	344697	353654	-8957	-2.5	-6
	4	339129	350832	-11703	-3	-4
TOTAL	1	714852	716478	-1626	-.2	-5
	2	724009	722438	+1571	+.2	-1
	3	727441	725917	+1524	+.2	+3
	4	717147	718877	-1730	-.2	+.8

consistent direction of the error for given categories is primarily due to the errors in the closing projections shown in column 7 of Table 4. Although the closing projection errors were quite large (up to 30 per cent in some cases) the caseload errors by category are close to ± 10 per cent with an average absolute value error of 4 per cent. It is also of interest to observe that the total caseload projection (the sum of all categories also shown in Table 4) exhibited an error of ± .2 per cent. To this, of course, must be added any error introduced by opening forecasts which were not a part of this analysis.

Admittedly, the assumption of stationary Markov closing rates introduces a major portion of the forecasting error and must be modified to include other factors. In addition, in 1972 there were many policy changes that appear to have shifted the trend of the caseload from its traditional pattern, such as the increase of Social Security Benefits by 20 per cent.

Conclusion

An evaluation of the applicability and use of a Markov Chain Model for structural analysis and projection is attempted. An analysis of the welfare caseload structure indicates that the caseload categories may be grouped into the three independent subgroups of (OAA), (AB), (AD,HR,ADC). The model is calibrated by using 1969-71 data and validated as a forecasting tool against 1972 data.

The validation procedure indicated mean absolute errors of 4 and .2 per cent for individual categories and total welfare caseloads, respectively. It is concluded, however, that the Markov assumption on closed cases introduced a major portion of the error. As such, the closing component of the model must be predicted through factors other than caseload population alone. This modification is presented in the second part of this study. In addition, part two provides a mathematical model to forecast the number of new openings by category on the basis of admission criteria variables used by the New York State Department of Social Services. These variables reflect the department policy changes in terms of financial and categorical eligibility determination criteria.

In addition, the effect of the outside influencing factors on the caseload behavior are included such as population growth, unemployment, working days in month, business index, average weekly earnings, average weekly hours, etc. It is anticipated that future extensions of the model will be helpful for Public Welfare planning, management and policy evaluation.

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