J. ENVIRONMENTAL SYSTEMS, Vol. 5(4), 1975

SAMPLING FOR GROUP UTILITY*

GREGORY B. BAECHER
Department of Civil Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts

ABSTRACT

A sampling theory approach is developed for estimating group utility functions for inclusion in decision-analytic approaches to public plan evaluation. This approach is based on Bayesian sampling theory and leads to estimates of group utility accounting for sampling and measurement error. The results of the estimation may be directly incorporated in decision analysis. The strength of this approach is that it leads to more rigorously based estimates of interest group utility functions than commonly used surrogates, and can be analytically balanced with other forms of preference information such as market data.

Introduction

Project evaluation in urban and regional planning is a process in which impacts generated by proposed alternative designs are predicted and the aggregate desirability of those impacts relative to societal values are measured. The hoped for result is a judgment of which competing alternative, by this criterion, is "best." While the prediction of impacts is a major part of evaluation, the central issue is the assessment of impact desirability. The manner in which desirabilities are ascertained determines to a great extent the results of the analysis.

Impact desirabilities have been traditionally approached by inference from economic (i.e., market) data and from the results of

* The work reported here was undertaken while the author was in residence at the International Institute for Applied Systems Analysis, Laxenburg, Austria. The views expressed are those of the author, and do not necessarily reflect those of IIASA.

doi: 10.2190/KRKE-HN4M-GEG3-TTJH
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opinion surveys. However, with the introduction of more recent evaluation methodologies (e.g., utility theory), new emphasis is being given to the assessment procedures. In particular, emphasis is being given to procedures which are more direct than market data, yet which yield more quantitative results than traditional opinion surveys. One of these is the technique of utility assessment which has grown out of statistical decision theory [1].

In applying direct methods of assessment, the question of differing and disaggregated perceptions of impact desirability must be squarely confronted. Usually, this takes the form of assessing utility functions (i.e., preference structures) for each of several "interest groups," and inputting these differing structures into an analysis to obtain starting points for more traditional political decision making. To this point, however, rigorous approaches to assessing these interest group utility functions have not been extensively explored.

Individual utility assessment is a time consuming process of game playing and feedback from analyst to subject. Interviewing most or even many individuals within an interest group is, therefore, simply not possible. However, by approaching group assessment as a question of sampling and Bayesian inference, a group function may be estimated from a finite number of individual assessments in much the same way that other sampling inferences are made. By structuring the approach in Bayesian terms, probability functions on the parameters of group utility functions may be obtained, which may be subsequently incorporated directly into the decision-analytic formulation of evaluation. A very significant further capacity of this approach is that preference data from other sources (e.g., market data) may be analytically combined with direct individual assessments to yield a combined inference. Such an analytical combination of different sets and types of data may contribute to a lessening of the arguments over the appropriateness of different measures of impact desirability.

Utility Theory Approach to Evaluation

The utility theory approach to evaluation is based on the theory of measurable utility of von Neumann and Morgenstern, and recently the approach has been applied to plan evaluation problems with growing frequency [2, 3].

In essence, the utility theory approach structures evaluation as shown in Figure 1. Several objectives are specified against which impacts are considered to be important (e.g., cost, environmental degradation, social disruption), and indices, called attributes, are

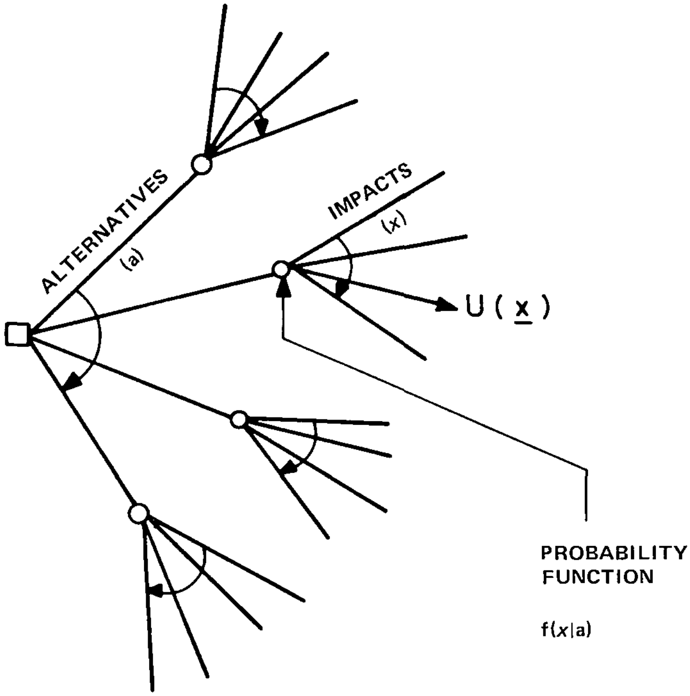

Figure 1.

selected on which to scale impact predictions against each objective. Impact predictions are made in the form of probability density functions (pdf) over the set of attributes, x, conditioned on the alternative chosen. A utility function is defined over the set of attributes, u(x), which serves as an objective-function. The criterion of optimality is maximum *expected* utility over the probability density function of impacts measured on the set of attributes. Because of the hierarchal nature of this evaluation, the analysis is left unchanged if a node on the "decision tree" is replaced by the expected utility of all branches leading from it. Thus, if at any level in the tree a further branching of uncertainty emanates, these branches may be replaced by their expectation in utility. This allows parametric uncertainties to be included in the analysis in an exceedingly simple way, by taking the expectation of utility over those uncertainties.

The plan alternative which leads to the maximum expected utility, and thus the "best" plan, obviously, may change if different individuals' or groups' preferences are used as the objective function. Thus, one normally assesses utility functions for several groups and performs the analysis using each function to arrive at a small number of alternatives each of which is preferred by one of the groups. Most often these group utility functions have not been assessed directly, but rather surrogates for them have been used. In Gros' analysis of power plant siting, for example, he assessed utility functions for "knowledgeable observers" of each group—this may have been a spokesman for the group, an influential member, or the like—and used these functions as approximations to the group functions [4]. Clearly, however, a more rigorous estimating procedure would be preferable.

Sampling Approach to Assessment

A sampling approach to assessment may be developed over single attributes of impact if three assumptions are made:

First, it will be assumed that each individual within the interest group has a "similar" utility function over the impact being treated. By "similar" we mean that an analytical expression of the same form, with only differing parameters, may be used to approximate each individual's function. For example, if the utility function

$$u(x) = -e^{bx} \quad (1)$$

may be used as an approximation for one individual's utility function, then it may be used as an approximation for the others.

Second, changes in the utility of each individual in the group are given equal weight. That is, changes in utility for each individual are considered to be equally important. This makes no assumption on weights given individuals in *different* groups, however.

Third, all members of an interest group are impacted precisely the same by the real outcome (i.e., impact) of a plan alternative; the level of impact as measured on the selected attribute is the same for each individual. This mitigates questions of equity in impact distribution within the group.

Assumption #1 in a sense defines what is meant here by an "interest group"; this is the only assumption we make about group structure. We define an interest group to be that collection of people with similarly shaped utility functions over the impact in question (Figure 2). According to this usage, those individuals whose utility functions are labeled A in Figure 2 would be classified as one interest group, while those whose functions are labeled B would be classified as another.

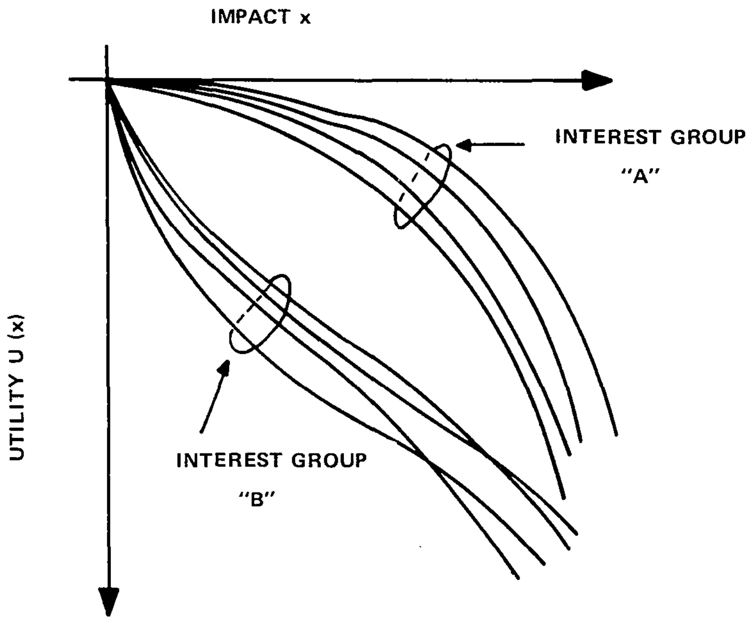

Figure 2.

We will not argue with the proposition that this assumption is naive. In reality "interest groups" are coalitions, and are not necessarily entities within which preferences are similar. Individuals join into coalitions to achieve ends, and not because their entire structures of preference are similar: they favor the same decision alternative, but not necessarily for the same reasons. Nevertheless, the homogeneity assumption seems a good place to begin an analytical treatment of the group preference problem, and might be weakened in future analyses.

Given these assumptions, Keeney and Kirkwood show that the proper group utility function is of the additive form

$$U(x) = \sum_i w_i u_i (x|b), \tag{2}$$

in which $u_i(x)$ is the utility function of the i^{th} individual and w_i is the weight given to changes in his utility [5]. By assumption 2,

$$w_i = w_j \quad \text{for all i, j}, \tag{3}$$

and thus w_i becomes a normalizing constant. The term b is the set of parameters of the analytical model of the utility function.

If the size of the group is assumed large and the distribution of preference across the group is assumed represented by a probability density function on the parameters b, denoted $f(b)$, then equation (2) becomes

$$U(x) = \int_b u(x|b)\, f(b)\, db. \tag{4}$$

Estimating group utility, U(x), becomes partially a sampling problem and the probability density function $f(b)$ is not known with certainty. Allowing the pdf of b to be expressed in some analytical form with parameters a transforms the problem into one of estimating a from the utility functions of that finite number of individuals whose preferences have been assessed.

If utility functions of a sample of n individuals within the group are assessed, and if some prior pdf on the parameters a, $f^{o}(a)$ is assumed (which may be uniform), the posterior pdf of a is

$$f'(a|\text{data}) \propto f^{o}(a)\, L(\text{data}|a). \tag{5}$$

Assuming simple random sampling ("exchangeability"), the posterior distribution becomes

$$f'(a|\text{data}) \propto f^{o}(a) \prod_i L(b_i|a), \tag{6}$$

$$\propto f^{o}(a) \prod_i f(b_i|a), \tag{7}$$

in which b_i are the parameters of the i^{th} individual's utility function.

Combining equations (4) and (7), the expected group utility function over sampling error is

$$U(x) = \int_b \int_a u(x|b,a)\, f(b|a)\, f'(a|\text{data})\, db\, da, \tag{8}$$

which may be incorporated directly within the decision theory framework.

If in addition to sampling error we assume *measurement error*, that is, error in the value of b_i for each individual, equation (7) would have to be expanded by an additional term leading to a more diffuse posterior pdf on a. Measurement error will be taken up in Section 5.

No Measurement Error

Consider the case of water pollution impact from a major facility; let the attribute of pollution be BOD, a scalar, and let individuals' utility functions be approximated by the analytical form

$$u(x|b_i) = -e^{b_i x}, \tag{9}$$

where x = BOD. This form is shown in Figure 3.

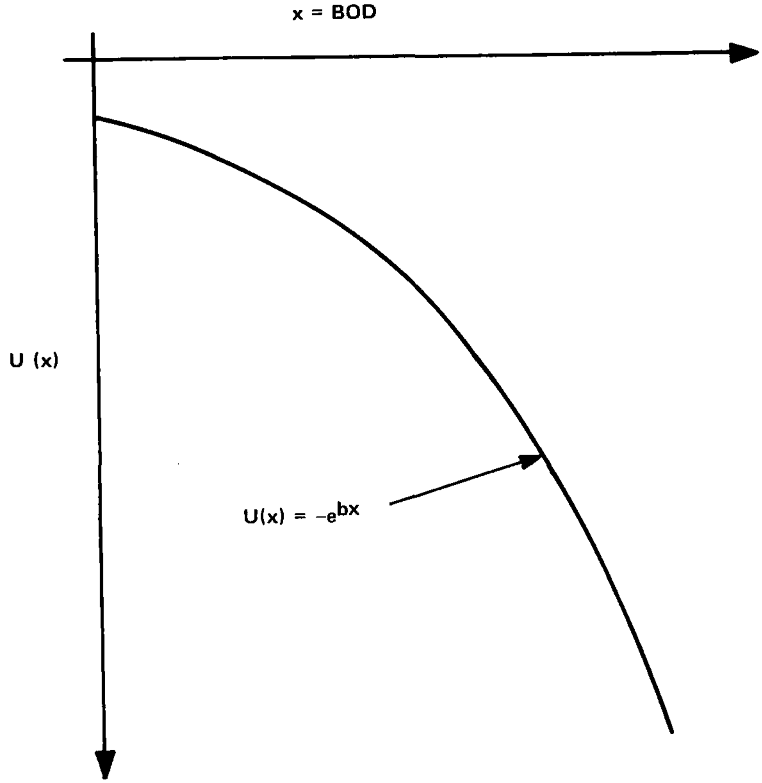

Figure 3.

Let the distribution of b_i within the group be assumed normally distributed. In this case the parameters of the pdf of b are the mean and standard deviation, or

$$a = (\text{mean, standard deviation}), \tag{10}$$

$$= (\mu,\sigma), \tag{11}$$

and equation (7) becomes

$$f'(\mu,\sigma|\text{data}) \propto f^{o}(\mu,\sigma) \underset{i}{\pi} N(b_i|\mu,\sigma). \tag{12}$$

Taking the prior distribution on a to be diffuse,

$$f^{o}(\mu,\sigma) \propto \sigma^{-1}, \tag{13}$$

the posterior distribution is of the multivariate student t form [6],

$$f'(\mu,\sigma|\text{data}) \propto \exp \left\{ -(2\sigma^2)^{-1} (\upsilon S^2 + n(b - \bar{b})^2) \right\}, \tag{14}$$

in which $\bar{b}$ is the sample average, $\upsilon = n - 1$,

$$S^2 = \frac{1}{\upsilon} \sum_i (b_i - \bar{b})^2, \tag{15}$$

and

$$k = (n/2\pi)^{1/2} \; (\tfrac{1}{2}\Gamma[\upsilon/2])^{-1} \; (\upsilon s^2/2)^{\upsilon/2}. \tag{16}$$

Substituting in equation (8),

$$U(x) = \int_b \int_{\hat{b}} \int_\sigma -\exp(bx) \, [2\pi\sigma^2]^{-1} \exp[-\tfrac{1}{2}(b - \hat{b})^2/\sigma^2]\, k \quad \exp[-(2\sigma^2)^{-1} (\upsilon s^2 + n(\hat{b} - \bar{b})^2)] \; db \, d\hat{b} \, d\sigma. \tag{17}$$

This analysis has been applied to the sample data shown in Appendix A, and the resulting expected group utility function solved for numerically (Figure 4).

With Measurement Error

Utility assessment data as collected consists of a set of points corresponding to different levels of the impact attribute (Figure 5), and from these points a value of b_i is inferred. Typically, about four to six points are assessed. Therefore, there are two components of measurement error, error in the true value of individual utility for each assessed point and error in the value of b_i which is inferred from those points.

Error of the first kind results from bias and random errors generated by the procedure of questioning during assessment, by the subject's consistency in his answers, and by the time and care which are exerted in assessment. The magnitude of these errors are the subject of debate, and procedures for determining them have yet to be adequately developed [7]. In the present analysis we will ignore such error.

The second kind of error results from the procedure adopted for fitting a "best" curve through the data. This error can be established through a regression scheme. Transforming the utility expression of equation (1) into a linear form

$$\ln u_i(x) = b_i x + e, \tag{18}$$

in which e is a random error term assumed distributed as $N(O,\sigma_i^2)$, points on the individual utility curve can be fit using normal Bayesian regression theory to obtain a probability distribution on b_i describing the second kind of error (Figure 6). Assuming the prior distribution on (b_i,σ_i) to be diffuse (i.e., $\propto \sigma_i^{-1}$),

$$f'(b_i,\sigma_i|\text{data}) \propto f^\circ(b_i,\sigma_i) \; L(\text{data}|b_i,\sigma_i) \tag{19}$$

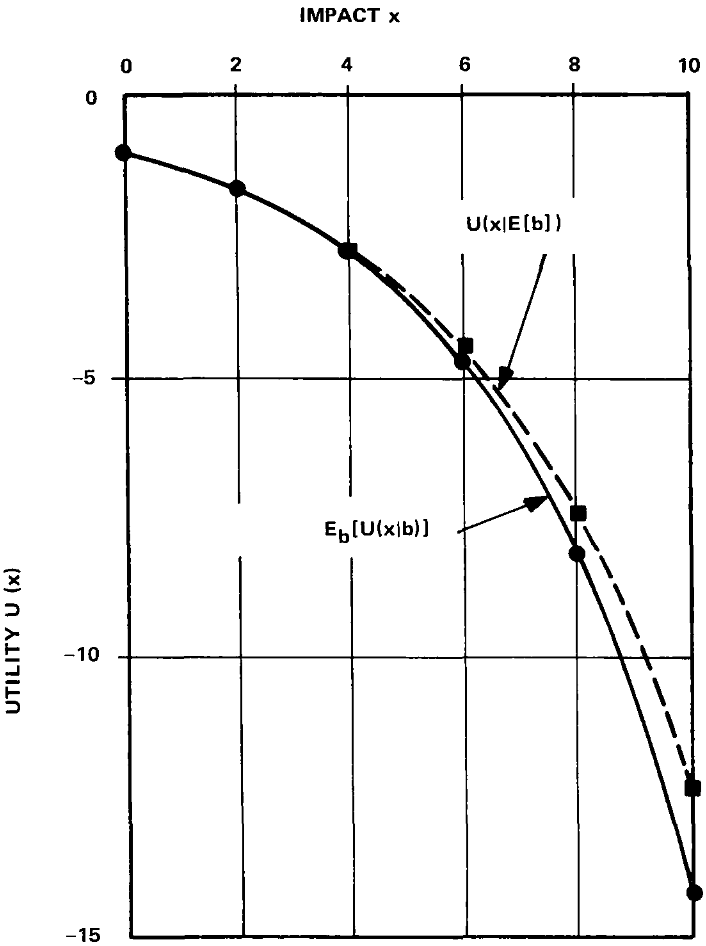

Figure 4.

$$\propto (\sigma_i^{-1}) \prod_i N(\text{data}|b_i,\sigma_i) \tag{20}$$

$$\propto \sigma_i^{-(n+1)} \exp\left\{-\frac{n}{2\sigma^2}\sum_j (y_j - b_i x_j)^2\right\}, \tag{21}$$

in which y_j are the assessment points. Integrating to obtain the marginal distribution on b_i yields $f'(b_i|\text{data})$ distributed as the univariate t distribution [8].

The uncertainty in the parameters a of the group distribution including measurement error becomes

$$f'(a|\text{data}) \propto f^o(a) \prod_i L[f'(b_i|\text{data})\,|a], \tag{22}$$

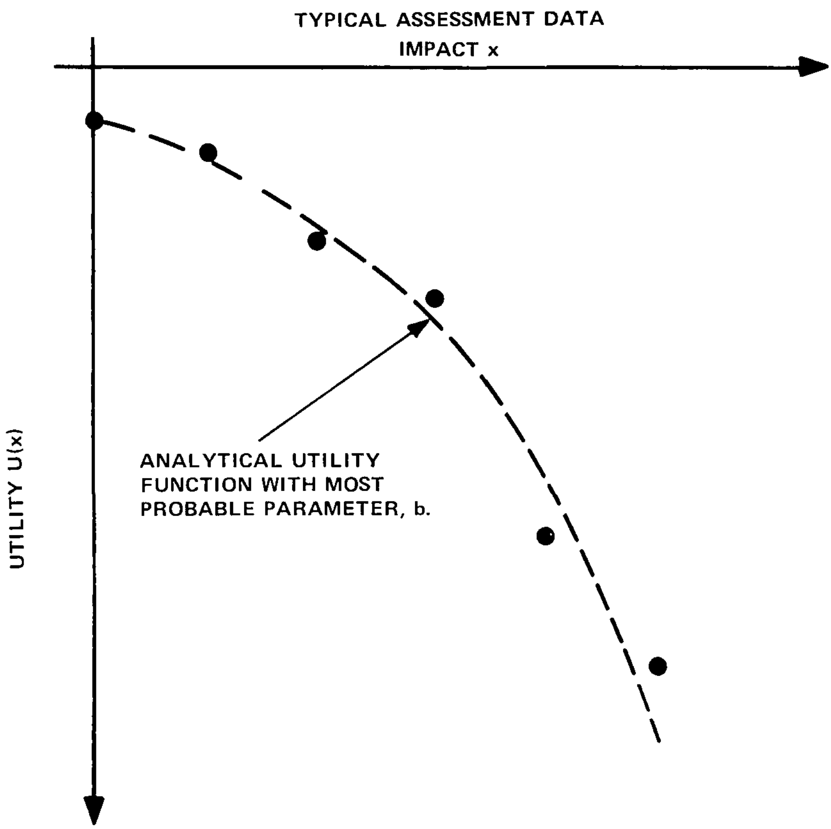

Figure 5.

$$\propto f^{o}(a) \prod_{i} \int_{b_i} N(b_i|a)\, f'(b_i|\text{data})\, db_i, \qquad (23)$$

which can then be included directly in equation (8) for expected group utility. As this equation becomes rapidly intractable, numerical solutions would probably need to be resorted to for solution.

Prior Information

A strength of the present approach to group utility sampling is that prior information from economic sources, opinion surveys, past assessments, and informed political opinion can be analytically included and balanced off against sample data in drawing final conclusions. This data enters the analysis through probability distributions on a, the parameters of the population distribution of the utility parameters b. This allows an intermeshing of more than one type of information and may contribute to a lessening of

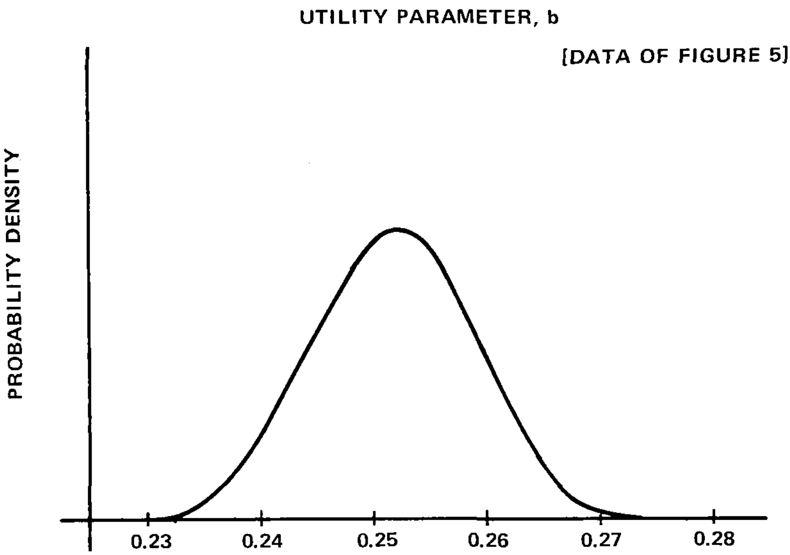

Figure 6.

apparent conflict between those workers who prefer purely market data and those who prefer direct approaches.

Conclusions

We have attempted to structure a rigorous approach to the problem of assessing group utility functions for inclusion in decision-analytic approaches to plan evaluation. The advantages of the present approach are that it offers more realistic estimates than most surrogates for group utility functions, and allows information of other types, like market data, to be analytically included. While the mathematical formulations become complicated, numerical techniques can be easily used for actual evaluation.

This analysis has only considered single attributed utility functions, although the precise functional form of the utility function in no way changes the analysis. A clear next step would be to expand the analysis to multi-attributed functions, which are of more relevance in actual plan evaluations.

ACKNOWLEDGMENT

The author would like to acknowledge the support of the Rockefeller Foundation under its Conflict in International Relations Program Fellowship, RF 74025, allocation 21, during the tenure of which the present report was written.

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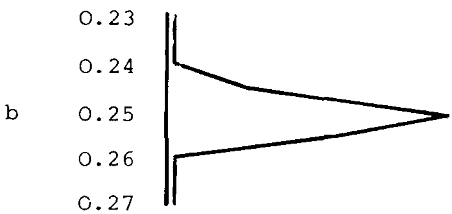
APPENDIX A

Probability Density Functions of Utility Parameters
Inferred From Subjects' Responses
(error of the second kind)

SUBJECT 1



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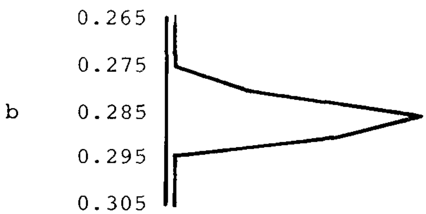

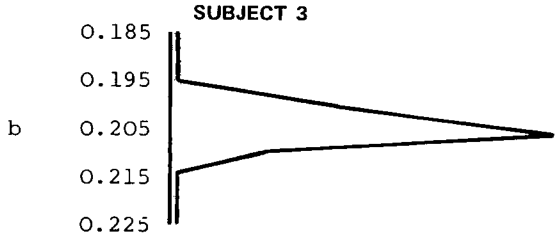
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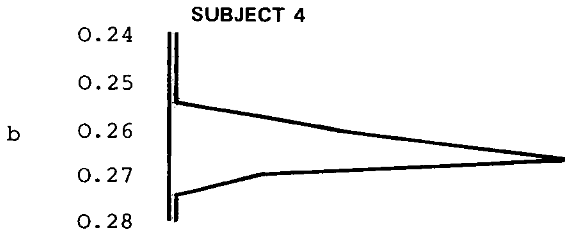
0.195
b
0.205
0.215
0.225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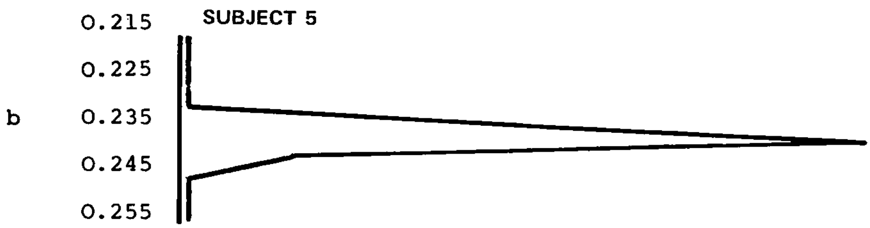
b
0.26
0.27
0.28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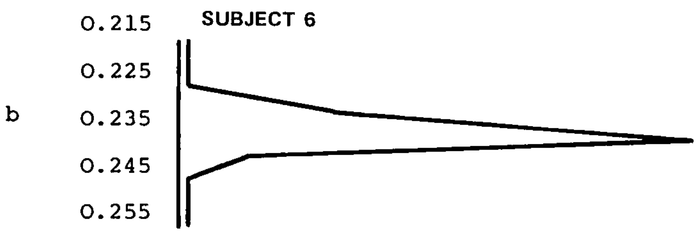
0.225
b
0.235
0.245
0.255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b
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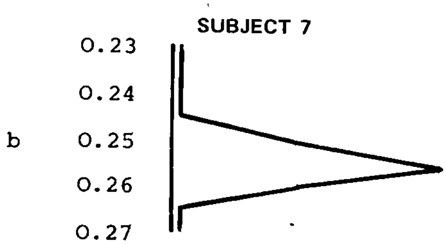
0.245
0.255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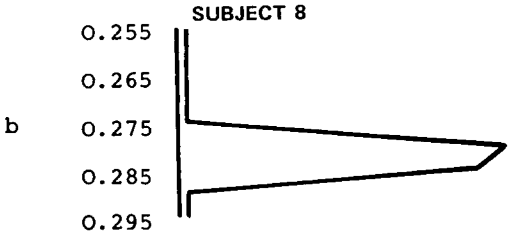
0.24
b
0.25
0.26
0.27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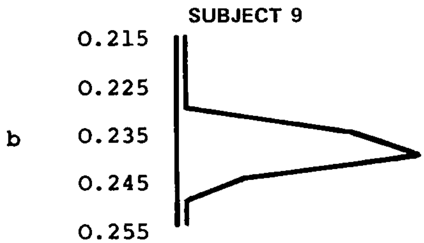
0.285
0.295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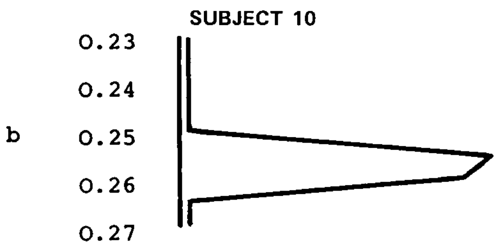
0.255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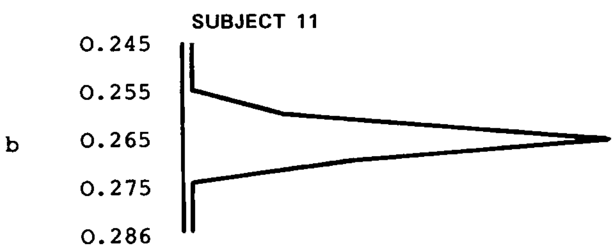
0.25
0.26
0.27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0.265
0.275
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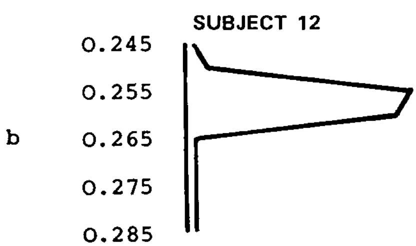
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b
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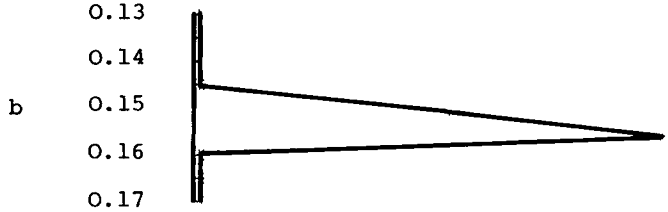
0.275
0.285

SUBJECT 13



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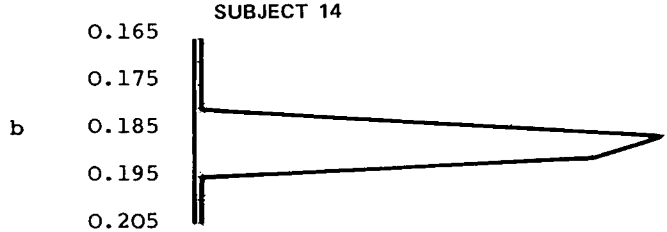

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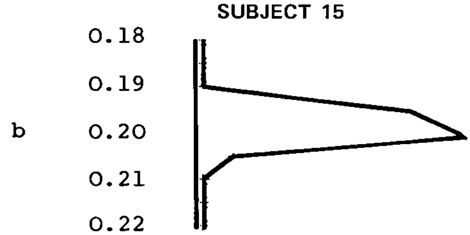

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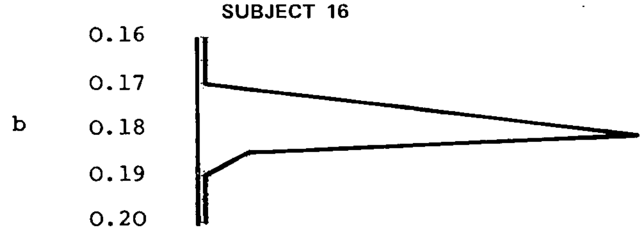

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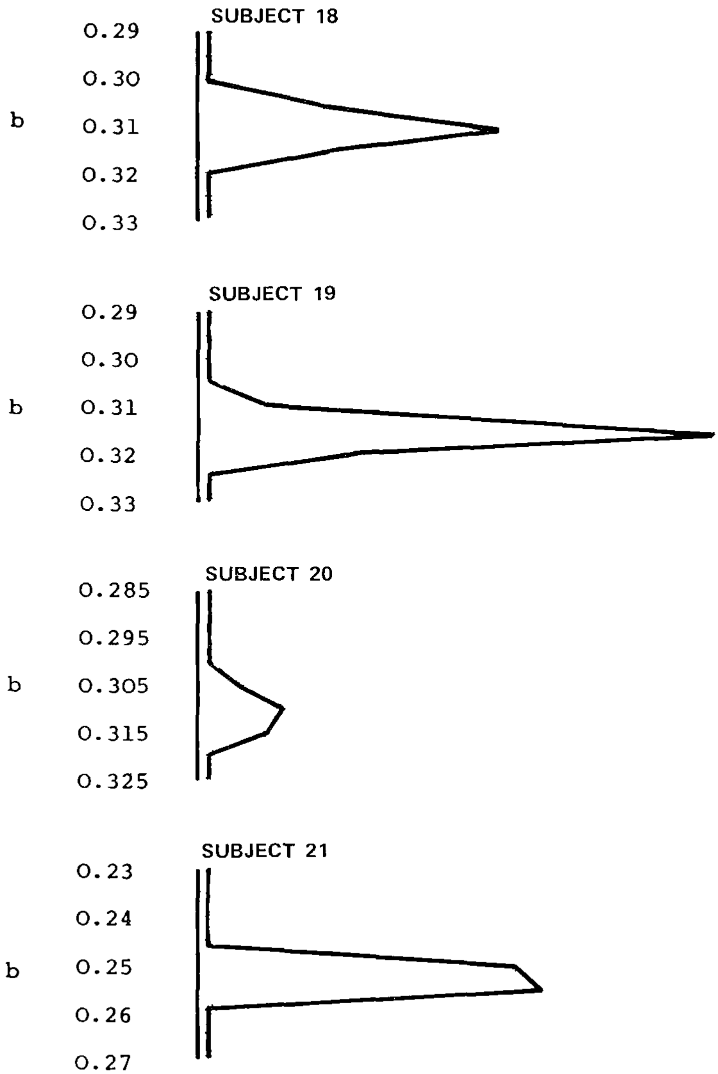

Direct reprint requests to:

Gregory B. Baecher
Dept. of Civil Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139