# THE UNIT COSTS OF JOINT PRODUCTS IN AN INTEGRATED ENERGY FACILITY*

PETER G. SASSONE
*Associate Professor of Economics*
*College of Industrial Management*
*Georgia Institute of Technology*

### ABSTRACT

The so-called integrated energy facility (IEF) holds some promise as a partial solution to current world energy problems. An IEF combines a conventional electric energy producing plant with a set of appropriately chosen industrial plants which can productively utilize the waste steam, hot water, and flue gases which are by-products of electricity production. In effect, the conventional energy plant becomes a producer of multiple energy forms. One of the key decision points in the analysis of the IEF concept is the unit costs of each of the produced energy forms. *Ceteris paribus,* the IEF must, to be successful, produce these forms at lower unit costs than alternative means.

Because the various energy forms are jointly produced, conventional methods of costing output run into serious difficulties. This paper describes a method for assigning unit costs which is informationally efficient and satisfies three demanding criteria. After developing the costing scheme, we show the approach is consistent with an optimal production level. We then show that the same approach, applied iteratively, can lead production to its optimal level. This approach is an alternative to arbitrary joint cost allocation schemes where the resulting unit costs have little or no significance. Significantly, the results of this new approach are in complete agreement with the results of traditional unit costing approaches when those approaches can be meaningfully applied. Thus, the new approach represents a generalization of unit costing techniques.

# Unit Costing at Given Production Levels

## INTRODUCTION

The integrated energy facility (IEF) is a concept which has recently come under scrutiny as a promising alternative to conventional energy generation techniques. An IEF combines an energy generation facility with an appropriate mix of energy using (industrial) customers so that a maximum amount of the various energy forms produced within the facility is channeled to productive use. For example, a conventional coal fired steam plant may be the energy generation facility within an IEF, but the otherwise waste products of flue gases, various grades of steam and heated water are channelled to customers with demands for those forms of energy. The IEF is an attractive concept because it combines energy conservation with the potential for upgrading the environment. The IEF, by utilizing waste energy, potentially reduces air and thermal pollution.

Assessing the economic viability of an IEF is best accomplished by comparing the unit costs of the energy forms produced by the IEF with the unit costs of those same energy forms if they were produced by conventional means.[1] This raises the question: how does one determine the unit costs of the various energy forms produced by a generating facility within the IEF? The apparent approach would involve identifying all the input elements, assigning total costs to each element, and then allocating the costs on the basis of some criterion, such as the BTU value of the relevant output energy forms. When stated in the abstract, this approach seems entirely reasonable. However, in attempting its actual implementation, it soon becomes evident that so much of the cost allocation depends on arbitrary decisions that the resulting unit costs have little practical meaning. The information content of these unit costs is remarkably uncertain, so their value as the basis of decision-making is highly suspect.

## JOINT COSTS

The practical impossibility of meaningfully dissecting the supply of different energy forms from a single facility is a problem not unique to IEF's. It is an instance of the more general *problem of joint costs*. The classic example of the joint costing problem involves the production of beef and leather from a steer. There is no apparently meaningful method to differentiate the costs of the leather from the costs of the beef. Indeed, economists have failed to provide a general solution to this problem.

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[1] Our emphasis on unit costs stems from the observation that this is the dimension in which most decision-makers, both private and public, appear to prefer to work. Indeed, this paper was stimulated by a Federal research project in which unit costs were specified as the principal desired output, to be used as a decision aid.

The problem of joint costs is said to arise when there does not exist a unique relation between the marginal costs of producing some output and the level of that output. That is, X and Y, two outputs, share the joint costing problem if the cost of producing an extra unit of X (its marginal cost, MC(X)) is not independent of Y. Total Cost of X and Y = $a_0 + a_1 X + a_2 Y^2$ does not involve a joint costing problem since MC(X) = $a_1$ and MC(Y) = $2a_2 Y$. Neither MC function depends on the other output. However, if Total Cost of X and Y = $a_0 + a_1 X + a_2 Y^2 + a_3 X \cdot Y$, then MC(X) = $a_1 + a_3 Y$ and MC(Y) = $2a_2 Y + a_3 X$, and the problem of joint costing appears. The marginal cost of an output is not uniquely related to the level of that output.

It happens that there is one key difference between the general joint costing problem (for which there appears to be no adequate solution) and the joint costing problem at hand. Specifically, the general problem includes situations wherein the outputs are produced in fixed proportions. Beef and leather from cattle, and (theoretically) hydrogen and oxygen from water are examples of joint outputs produced in fixed proportions. An energy generation facility is not limited to producing different energy forms in fixed proportions, however. By varying the engineering parameters of the system, a wide range of different output combinations can be achieved. This observation allows the construction of a meaningful, operational, and relatively straightforward solution to the problem of costing joint energy outputs.

The approach we adopt is a radical departure from traditional costing schemes. Rather than tracing inputs through the production process, and attempting to link each unit of each input to an eventual output, thus developing unit output costs based on unit input costs; our approach is to *assign* a set of unit costs based on considerations of the purposes the unit costs are intended to serve. These purposes are internal production control, external pricing to meet revenue requirements, and (from the social prospective) external pricing consistent with optimal resource allocation. We show not only that our assigned unit costs satisfy these requirements, but that traditionally calculated unit costs do not. In addition, there is good reason to believe the proposed approach is informationally more efficient: its implementation requires a minimum of information about the internal production process.

## A SIMPLE MODEL

Let us again state the problem which must be addressed. In an integrated energy facility, several forms of useful energy are produced in a single production process. Thus, it is not possible to make a simple determination of, say, the cost of steam production. Nonetheless, for both the purposes of *internal control* of production costs and *external pricing* of the final products

(presumably influenced by a regulatory commission), *meaningful* unit costs must be assigned. We now show that a reasonable solution to this problem may be constructed.

Assume that two products, steam and electricity, are *jointly* produced in a process whose inputs are land, labor, and capital. Let

S = No. of units of steam produced, e.g., in BTU's
E = No. of units of electricity produced, e.g., in BUT's
L = No. of units of land used
N = No. of units of labor used
K = No. of units of capital used.

The production function can be represented by the implicit function

$$F (S, E, L, N, K) = 0 \tag{1}$$

In words, the specification of L, N, and K (the inputs) allows certain (maximum) joint outputs of S and E. Graphically, the function can be represented by a set of product transformation curves (PTC), as in Figure 1. $(\overline{L}, \overline{N}, \overline{K})$ represents a specific level of each input, and the lower PTC represents all the possible combinations of S and E which that input combination can achieve. For example $(\overline{L}, \overline{N}, \overline{K})$ can be used to produce either $(S', E')$ or $(S'', E'')$. $(\hat{L}, \hat{L}, \hat{K})$ represents a higher level of inputs, thus the corresponding PTC is northeast of the first.

Note that different proportions of electricity to steam output can be represented by rays eminating from the origin, as represented in Figure 2. For example, say $R_1$ represents an output proportion of electricity to steam of one to one, or 1.0. $R_2$ represents .5 and $R_3$ represents .25.

## CRITERIA FOR "GOOD" UNIT COSTS

There are many ways in which unit costs may be assigned. However, our insistance that the assigned costs be *meaningful* for control and pricing leads to three criteria which the costing approach must satisfy.

1. Assuming the unit costs assigned will provide the basis for pricing the output, the costs (and hence the prices) should induce an optimal consumption mix of the outputs, as gauged by traditional measures of social welfare.
2. Assuming the unit costs will be used in production control decisions, the costs should induce the most efficient use of resources.
3. Assuming the selling prices are based on the assigned unit costs, and assuming a regulatory mandate that the facility earn only a normal return on investment, the assigned unit costs should cover the total costs of production, with no excess profits. That is, if the outputs are sold at
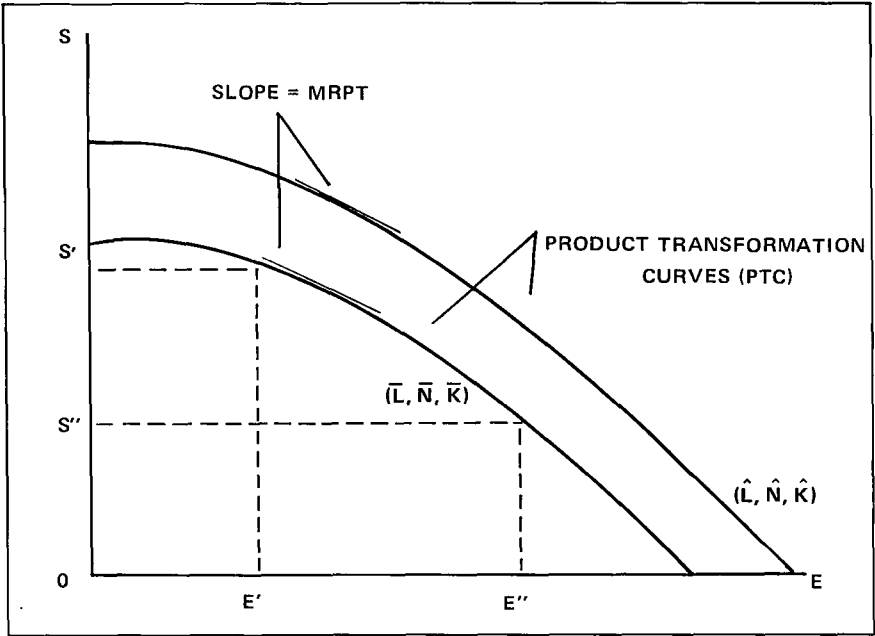
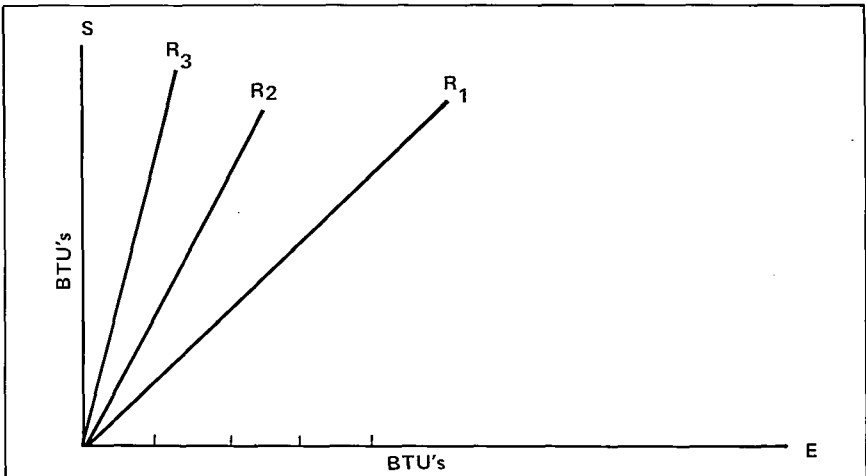Figure 1. Production transformation curves for an IEF.



Figure 2. Representing different output proportions.

unit cost, the resulting revenue should cover total production costs, and allow only a normal rate of return.

## DERIVING UNIT COSTS

We now investigate the implications of each of these criteria for the unit costing approach. We initially assume the optimal production decision has already been made. Our only goal is to assign unit costs. Later, this restrictive assumption is dropped. The first criterion demands that unit costs (and by implication, prices) be consistent with optimizing social welfare. Social welfare is optimized when the value of the consumed goods most exceeds the cost of production. Thus

MAXIMIZE $\int P(S)dS$ + $\int P(E)dE$ − $C(L, N, K)$

SUBJECT TO $F(S, E, L, N, K) = 0$

where $P(S)$ and $P(E)$ are the ordinary demand curves for S and E, $\int P(S)dS + \int P(E)dE$ measures the gross benefits of consumption (area under the respective demand curves), $C(L, N, K)$ is the cost of inputs, and F the production function relating outputs to inputs. Using the Langrangian approach to characterize an optimum, we have

$L = \int P(S)dS + \int P(E)dE − C(L, N, K) + \lambda[F(S, E, L, N, K)]$

whose first order conditions imply that

$$\frac{P_S}{P_E} = \frac{\partial F/\partial S}{\partial F/\partial E} \tag{2}$$

and, by assumption, the price of S will be its unit cost; and the price of E will be its unit cost. Letting $C_S$ and $C_E$ be the unit costs, we have

$$P_S = C_S \tag{3}$$

and

$$P_E = C_E \tag{4}$$

The second criterion demands that the assigned unit costs induce efficiency. That is, for any given quantities of S and E which are produced, the minimum cost of that production must be ahcieved. Thus, we need to

MINIMIZE $C_S \cdot S + C_E \cdot E$

SUBJECT TO $F(S, E, L, N, K) = 0$

The Lagrangian first order conditions imply

$$\frac{C_S}{C_E} = \frac{\partial F/\partial S}{\partial F/\partial E} \tag{5}$$

Finally, the third criterion demands that the energy facility just break even, i.e.,

$$P_S \cdot S + P_E \cdot E = C(L, N, K) \tag{6}$$

or, a total revenue must equal total costs. Conditions (2) through (6) summarize the requirements that a *good measure* of unit costs should satisfy. Fortunately, there is some redundancy in the equations. In particular, using (3) and (4), it is seen that (2) and (5) are equivalent; and that (6) may be expressed as

$$C_S \cdot S + C_E \cdot E = C(L, N, K) \tag{7}$$

Thus, the issue boils down to finding (or assigning) unit costs so that both (5) and (7) are simultaneously satisfied. Remember, our purpose here is to determine unit costs *once the production decision has been made*. Thus, if it is determined that $S^*$ and $E^*$ are to be produced, and the lowest cost $C^*(L, N, K)$ is found, (5) and (7) become two independent equations in two variables $C_S$ and $C_E$. Thus, a unique solution always exists. To see this, let

$$W = \frac{\partial F/\partial S}{\partial F/\partial E}, \text{ and note that } \frac{\partial F/\partial S}{\partial F/\partial E}$$

evaluated at $S^*$, $E^*$ is a constant. Thus (5) may be rewritten as

$$\frac{C_S}{C_E} = W, \text{ or } C_S = W C_E, \text{ or } C_S - W C_E = 0.$$

Likewise $C(L, N, K)$ at $S^*$, $E^*$ is a constant TC. The system comprised of (5) and (7) may be written as

$$\begin{bmatrix} 1 & -W \\ S^* & E^* \end{bmatrix} \begin{bmatrix} C_S \\ C_E \end{bmatrix} = \begin{bmatrix} 0 \\ TC \end{bmatrix}$$

The system has a unique solution as long as the 2 x 2 matrix has an inverse, and an inverse exists as long as the matrix is non-singular. Its determinent is $D = E^* + W S^*$. Since all variables are positive, it follows that D is necessarily positive, hence non-zero. Thus, the 2 x 2 matrix is non-singular, its inverse

exists, and unique $C_S$ and $C_E$ exist. For latter reference, the solution to the system is

$$C_S = \frac{W \cdot TC}{E + WS}, C_E = \frac{TC}{E - WS}.$$

In order to make the approach operational, we need to show that the right hand side (RHS) of (5) is a meaningful expression. In fact, it is easy to show that the RHS of (5) corresponds to the slope of the PTC in Figure 1. That is, it is the rate at which steam and electricity must be traded off when inputs are kept constant. To see this, start with the production function, (1), $F(S, E, L, N, K) = 0$. Take the total differential

$$\frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial E} dE + \frac{\partial F}{\partial L} dL + \frac{\partial F}{\partial N} dN + \frac{\partial F}{\partial K} dK = 0.$$

Note that along a PTC, L, N, K are constant, so dL, dN, dK each equal 0. Using that result, and rearranging, we find

$$\frac{dE}{dS} = - \frac{\partial F/\partial S}{\partial F/\partial E}.$$

That is, the slope of the PTC (the marginal rate of product transformation) is the negative of the RHS of (5).[2]

A simple example will show how the above approach, summarized by (5) and (7), can determine meaningful unit costs.

Suppose current production is $E = 200$ and $S = 400$, and the minimum total cost is 100. Suppose the marginal rate of production transformation is 1.2, i.e., to produce one more unit of electricity while keeping all inputs constant, 1.2 less units of steam must be produced. From (5) we know that

$$\frac{C_S}{C_E} = 1.2$$

and from (7),

$$C_S \cdot 400 + C_E \cdot 200 = 100$$

The solution is that the unit cost of steam, $C_S$, is .176, and the unit cost of electricity, $C_E$, is .147. It is important to note, first, that these unit costs can be assigned *without* dissecting the production process nor tracing the inputs

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[2] Note that in the case of joint production with outputs in *fixed proportions*, $\frac{dF/dS}{dF/dE}$ is not defined since output substitution is not possible. Hence, as mentioned above, this approach will not solve the problem of costing joint fixed proportion outputs.

through that process and, second, there is absolutely no reason to believe that these unit costs would be arrived at by traditional cost allocation schemes. And to reiterate, these unit costs:

1. induce an optimal consumption pattern,
2. induce efficiency in intrafirm resource allocation, and
3. provide a fair return to the energy facility.

Our approach to costing joint energy outputs leads to the somewhat surprising conclusion that it is not necessary to dissect the physical-engineering energy generation facility in order to arrive at the unit costs of the various energy output forms. This is not to suggest, however, that the approach is completely divorced from the engineering parameters of the system. Rather, the minimum requisite engineering information is identified and circumscribed. That minimum is the trade-off rate between or among the various energy output forms in the neighborhood of the desired output as inputs are held constant. While this trade-off rate is rarely readily apparent, it can usually be estimated by a combination of design and operating data. Given our criteria for judging the "goodness" of estimated unit costs, the unit costs derived in the manner we describe must be judged superior to unit costs which are based on a thorough dissection of the energy generating process, even though far more engineering information would presumably be used in calculating the latter. The merits of our approach, then, include economy of information as well as improved unit costs.

## Unit Costs and Achieving Optimal Production

### THE DYNAMIC PROBLEM

So far our concern has been with the purely static problem of assigning unit costs in a joint product situation *after* the production decision has been made. In particular, if the socially optimum production level has somehow been determined, then the unit costs defined by (5) and (7) have the three especially appealing properties.

Now suppose the optimum production level is to be determined. Can prices based on our unit costs aid in this determination? Are conditions (5) and (7) consistent with the socially optimum production level? The latter question is important because we have not yet established that the *quantities demanded* of the various energy forms at the unit prices defined by (5) and (7) will be consistent with the *quantities supplied* which gave rise to those prices. Our plan for this part of the paper to define the concept of optimum production level, examine the relation between production levels and unit costs, suggest an iterative approach which will drive production to its optimal level, and finally demonstrate that the solution iteratively achieved is consistent with the static costing conditions, (5) and (7).

## THE OPTIMUM LEVEL OF PRODUCTION

From society's viewpoint, the best output level for any good is that level at which its marginal revenue product (MRP) equals its marginal cost (MC). A good's MRP is the value added by the last unit produced of the subject good in the production of some other good. MC is, of course, the cost of producing the last unit of the subject good. To see that MRP = MC is the condition for optimality, assume it is violated. Specifically, assume MRP > MC. This means that, say, the last unit of steam used in the production of widgets adds $20 to the value of widgets produced, while the cost of the last unit of steam is $5. Clearly, the production of one more unit of steam yields a net benefit of $15 ($20 in gross benefits less $5 in cost). Equally clear is that the initial production level of steam could not have been optimal since a change in that level improves social welfare. Only if MRP = MC is no improvement possible. The output level must then be optimal.

Figure 3 illustrates the argument. At low levels of S, MRP exceeds MC and benefits are derived by increasing S. At high levels, the converse is true. The MC curve for S can be thought of as being constructed in terms of E. The MC of a unit of S is, indeed, the units of E sacrificed. Thus, the MC curve reflects the values of the slope along a product transformation curve of Figure 1. Since more and more E must be sacrificed to successively increment steam output, $MC_S$ necessarily rises to the right. $MRP_S$ slopes down to right due to the law of diminishing marginal productivity. Extra units of steam add less and less to the value of widget production, *ceteris paribus.*

## UNIT COSTS, MRP, AND MC

The approach detailed under "Unit Costing at Given Production Levels" is based on the premise that output levels are used to determine unit costs. These costs, or prices, are then used by production management to efficiently channel resource use and by customers as the basis for their purchase decisions. It is well known from economic theory that customers will purchase S and E in amounts such that

$$\frac{MRP_S}{MRP_E} = \frac{P_S}{P_E} \tag{8}$$

and that efficient production managers will produce S and E so that

$$\frac{MC_S}{MC_E} = \frac{P_S}{P_E} \tag{9}$$

(8) and (9) are first order optimizing conditions for customers and managers, respectively.
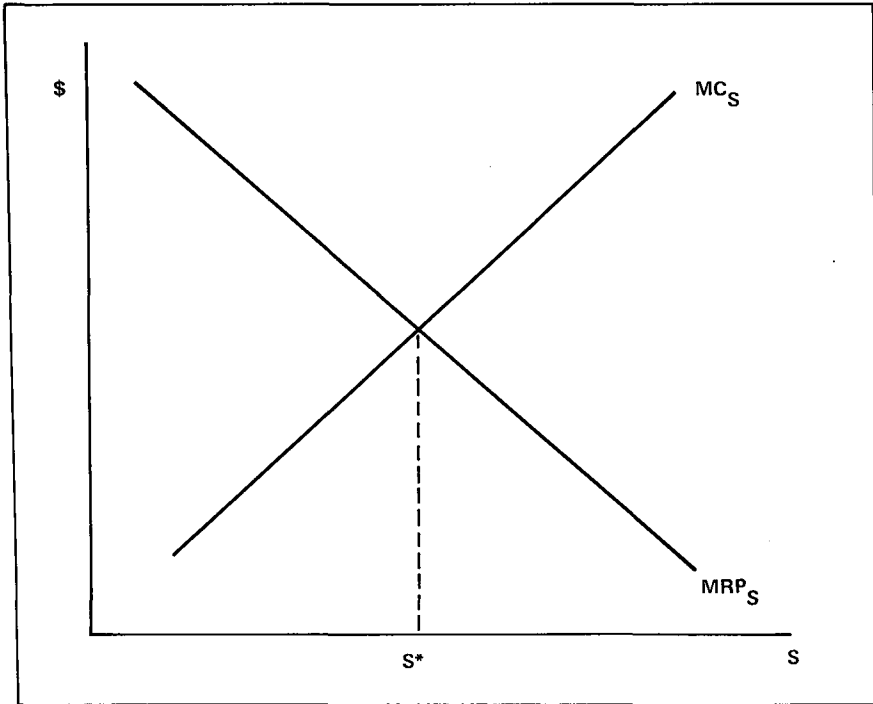
Figure 3. The optimal level of steam production.

Before proceeding, an additional assumption must be added to our model. We assume that total cost function for S and E is linear homogeneous. That is, if

$$TC = g(S, E), \text{ then } \quad a \cdot TC = g(aS, aE)$$

$$\text{for } a > 0 \tag{10}$$

It must be stated that (1) is merely a simplifying assumption. It will be discarded later. Nonetheless, there is a good deal of evidence that cost relations do tend to be linear homogeneous. This assumption enables us to temporarily avoid second-best problems.[3]

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[3] The so-called theory of the second-best addresses decision problems in which the satisfaction of the usual first order conditions for optimality is precluded by constraint. In this case, the optimality condition (to be derived) that the price equal the marginal cost would be precluded by the revenue equals cost constraint if the total cost function were not linear homogeneous. See Joint Costs on page 40.

Suppose the current level of steam output is $S^S$ (steam supplied), and the current level of electricity output is $E^S$. At these outputs there is defined a Marginal Rate of Product Transformation (MRPT). The MRPT determines the ratio of the unit costs $C_S$ to $C_E$ from (5). (7) determines their absolute level. The solution to the set of simultaneous equations, (5) and (7), yields unit costs

$$C_S = \frac{-R \cdot TC}{E - KS} \tag{11}$$

where $R \equiv \dfrac{\partial F/\partial S}{\partial F/\partial E} = -$ MRPT between S and E

From (3), (4), and (5), we see that

$$R = - P_S/P_E \tag{12}$$

and from (9) and (12) we have

$$R = - MC_S/MC_E \tag{13}$$

Substituting (13) into (11) yields

$$C_S = \frac{MC_S/MC_E \cdot TC}{E + \dfrac{MC_S}{MC_E} S}$$

Multiplying the RHS by $\dfrac{MC_E}{MC_E}$ results in

$$C_S = \frac{M_{CS} \cdot TC}{MC_E \cdot E + MC_S \cdot S} \tag{14}$$

Now, by assumption, the total cost function, TC = g(S, E) is linear homogeneous. By Euler's theorem[4] we have

$$TC = \frac{\partial TC}{\partial S} \cdot S + \frac{\partial TC}{\partial E} \cdot E \equiv MC_S \cdot S + MC_E \cdot E \tag{15}$$

Substituting (15) into (14) and simplifying yields

$$C_S = MC_S \tag{16}$$

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[4] Euler's theorem states that if $Y = F(X_1, X_2)$ is homogeneous of degree m, then

$$mY = \frac{\partial F}{\partial X_1} \cdot X_1 + \frac{\partial F}{\partial X_2} \cdot X_2$$

and it's easily shown that

$$C_E = MC_E \tag{17}$$

Where the total cost function is linear homogeneous, the unit costing approach defined by (5) and (7) is nothing more than assigning marginal costs as unit costs! Using these unit costs as prices is simply marginal cost pricing!

## A DYNAMIC ITERATIVE PROCEDURE

Having shown that our unit costs are, in fact, marginal costs, an algorithm to find the optimal output level is easily constructed. As might be expected, the algorithm simulates a competitive market. At the current (assumed non-optimal) output levels, $S^S$ and $E^S$, the unit costs defined by (5) and (7), $C_S$ and $C_E$, are simply $MC_S$ and $MC_E$. Let us concentrate on $MC_S$. The situation is depicted in Figure 4. Since $S^S$ is less than $S^*$, the unit cost (price) defined by $S^S$ (found on the $MC_S$ curve) elicits a quantity demanded in excess of $S^*$. An excess demand for $S$ is management's signal to increase $S^S$, which defines a new $C_S$, a new $S^D$, and so on. At each step, excess demand (or excess supply) signals the direction of changes needed in $S^S$. Convergence toward $S^*$ is assured.[5] A similar series of steps is, of course, carried out in the E market. It is worth noting that the MC curves are not fixed. Rather, because S and E are joint products, $MC_S$ shifts with changes in E and $MC_E$ shifts with changes in S.

Note also that each step in the dynamic process of moving toward the optimum is consistent with (5) and (7). Our unit costing approach is appealing not only at the optimum, but at sub-optima as well. For, applied in an iterative fashion, the same unit costing rules which support an optimum can lead production decisions toward that optimum.

## GENERALIZATION TO DECREASING MARGINAL COSTS

Finally, let us briefly indicate how our approach to determining unit costs in a joint production situation may be extended to the case of decreasing marginal costs. It is well known that marginal cost pricing in the presence of decreasing marginal cost causes negative profits to be earned. Clearly then, marginal cost pricing cannot be adopted by an IEF. Constrained to violate the social production optimality condition, we enter the realm of the second-best. The problem is to choose unit costs which satisfy (7) and yet yield the greatest social welfare. Operationally, the produced quantities of S and E will be less than their socially optimal ("first-best") levels. How should the production costs be distributed between S and E?

[5] We have not defined a rigorous convergence process since the literature abounds with examples. The precise convergence process is only a tangential issue here. The basic idea is simply the simulation of a competitive market.
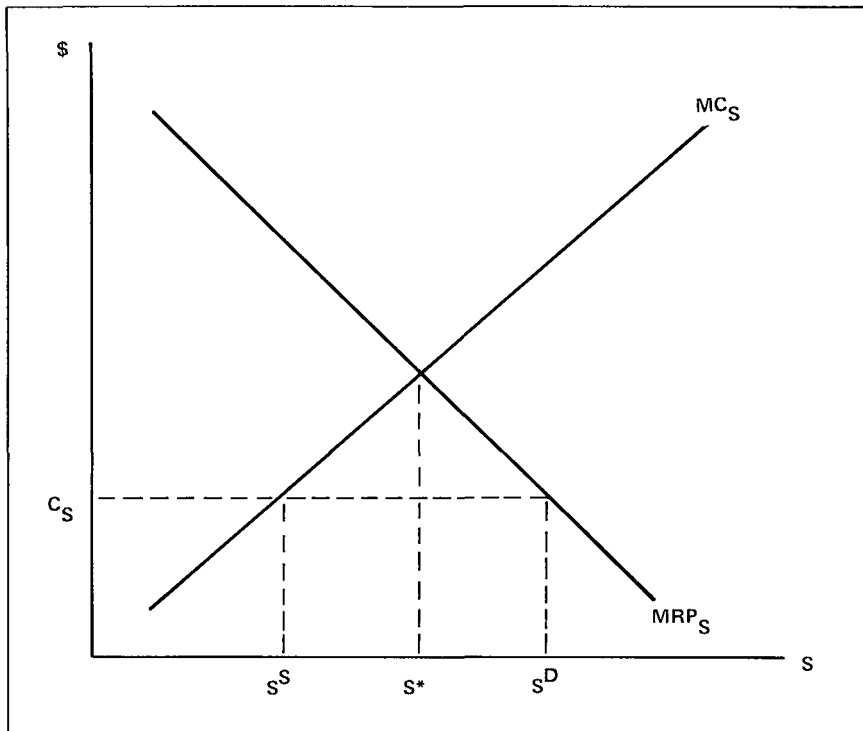
Figure 4. Convergence in the steam market.

The problem can be formulated as follows: maximize the net benefits from the consumption of S and E while maintaining total revenues equal to total costs.

Net benefits = gross benefits − gross costs

$$= \int_0^S P(S)dS + \int_0^E P(E)dE - g(S, E) \qquad (18)$$

where $MRP_S = P(S)$ and $MRP_E = P(E)$. These, of course, are the demand curves for S and E, respectively. g(S, E) is the total cost function for S and E.

$$\text{Total Revenues} = C_S \cdot S + C_E \cdot E \qquad (19)$$

where $C_S$ and $C_E$ are the unit costs to be determined. The appropriate Langrangian expression is, using (18) and (19),

$$\mathcal{L} = \int_0^S P(S)dS + \int_0^E P(E)dE - g(S, E) + \lambda(C_S \cdot S + C_E \cdot E - g(S, E)) \tag{20}$$

By observing that if $C_S$ is the unit price of S, and $\hat{S}$ units are demanded at $C_S$, then the $MRP_S$ at S is $C_S$, we have

$$\frac{\partial \mathcal{L}}{\partial S} = C_S - MC_S + \lambda(C_S + S \frac{\partial C_S}{\partial S} - MC_S) = 0 \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial E} = C_E - MC_E + \lambda(C_E + E \frac{\partial C_E}{\partial E} - MC_E) = 0 \tag{22}$$

Now recall that elasticity of demand (for S) can be expressed as

$$\xi_S = - \frac{C_S}{S} \frac{\partial S}{\partial C_S} \tag{23}$$

Using (23) and the analogous expression for $\xi_E$, (21) and (22) may be rewritten as

$$C_S - MC_S + \lambda (1 - \frac{1}{\xi_S} - \frac{MC_S}{C_S}) = 0 \tag{24}$$

$$C_E = MC_E + \lambda (1 - \frac{1}{\xi_E} - \frac{MC_E}{C_E}) = 0 \tag{25}$$

Solving each of (24) and (25) for $\lambda$, equating, and rearranging terms results in

$$\frac{\dfrac{C_S - MC_S}{C_S}}{\dfrac{C_E - MC_E}{C_E}} = \frac{\xi_E}{\xi_S} \tag{26}$$

The solution to this second-best problem is that each unit cost may be set so that its percentage deviation from marginal cost is inversely proportional to that good's elasticity of MRP.[6] Thus, while retaining condition (7), (5) must be replaced by (26). The simultaneous solution of (7) and (26) define second-best unit costs under joint production. An iterative procedure may also be constructed on (7) and (26) to drive production toward its second-best level.

[6] Baumol and Bradford derived this result in [1]. We believe our derivation is somewhat more straightforward, however.

## Summary

Unit costs of production form the basis for crucial resource allocation decisions. Especially in the case of joint products, unit costs tend to be computed from crude conceptual dissections of the physical-engineering production system and arbitrary rules of thumb for allocating joint costs. Using the example of an IEF, we have shown that even in the "jointest" cases of production, *meaningful* unit costs may be constructed with a minimum of information. Our approach was to characterize "good" unit costs and derive the costing rules from these characterizations. We showed that when total cost is a linear homogeneous function, our unit costs are simply marginal costs. The unit costs assigned by (5) and (7) not only support an optimum production level, but are the foundation for a sequential process of driving output toward the optimum. Finally, in second-best cases, the unit costing approach must be modified by substituting (18) for (5).

Almost paradoxically, while our intent was to circumvent the traditional costing of output which operates via the analysis of input costs and the production process, our conclusion—marginal cost pricing (at least for the linear homogeneous case)—is completely consistent with the traditional approach, *if that traditional approach could be implemented.* As was discussed, the attempt to implement the traditional costing approach runs into an impasse when it encounters joint costs. By focusing on the output side of the production process, the marginal cost of a unit of joint output is defined, not in terms of the inputs, but in terms of the amount of the *other* (joint) output which must be sacrificed to produce that unit. The true opportunity costs of production are thereby revealed. Thus, our overtly radical approach to unit cost estimating results in finding the same unit costs sought, but unattainable, by the traditional methods.

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Direct reprint requests to:

Peter G. Sassone
Associate Professor of Economics
College of Industrial Management
Georgia Institute of Technology
Atlanta, Georgia    30332