Multi-Objective Optimization Methods Applied for Manoeuvre Load Control on Combat Aircraft wing

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Abstract
Manoeuvre Load Control (MLC) is a subset of Active Control Technology (ACT), deals with optimal rotation of control surfaces to effectively redistribute the forces and moments on airframe, resulting in structural benefits and performance improvements. The present formulation consists of flight control parameters as design variables; bending and twisting moment at the wing root as conflicting independent multiple objective functions, together with aircraft stability/trim equations as equality constraints and actuator hinge moments as inequality constraints. This work attempts to bridge the gap observed with computational approaches and contemporary flight test programs, by treating MLC problem as multi-objective functions. Several multi-objective optimization methods such as Goal programming method, Multi-objective Evolutionary Algorithm (MOEA) and its hybrid form, which is a combination of the above two methods are explored for their capabilities when applied to this real world problem. Another significant development is to optimize the combined load cases such as roll pull up and roll push over manoeuvres which define the structural limit for the wing, rather than conventional treatment with unitary manoeuvres. Results from these multi-objective optimization methods indicates optimal load envelope by at least 10% from the baseline moment values can be realized resulting in improved aircraft performance.

NOMENCLATURE

$[A]$	aerodynamic influence coefficient matrix
c	mean chord length (mm)
CS	control surface
$[D]$	resultant aeroelastic mass (kg)
E_j	j^{th} equality constraints
$\{g\}$	acceleration due to gravity (mm / sec^2)
$\{g_r\}$	rigid body acceleration (mm / sec^2)
HM	hinge moment (N - mm)

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H_k	k^{th} inequality constraints
I_{xx}	moment of inertia of the aircraft about roll axis (kg - mm^2)
I_{zx}	product of inertia about yaw and roll axis (kg - mm^2)
$[K]$	stiffness matrix
l	wing semi - span (mm)
L	moment about roll axis (N - mm)
$[m]$	mass of aircraft (Kg)
M	moment about pitch axis (N - mm)
N	moment about yaw axis (N - mm)
n_z	vertical acceleration interms of load factor $[Z / w]$
p	roll rate (radians / sec)
$\dot{p}$	roll acceleration (radians / sec^2)
$[P]$	matrix of rigid aerodynamic force coefficients
$\left(\frac{pl}{v}\right)$	roll damping (radians)
q	dynamic pressure (N / mm^2)
$\{R\}$	resultant aeroelastic trim force (N)
R_j	penalty factors for j^{th} equality constraints
r_k	penalty factors for k^{th} inequality constraints
RBM	root bending moment (N - mm)
RTM	root twisting moment (N - mm)
s	wing surface area (mm^2)
$\{u\}$	structural nodal displacements (mm)
w	weight of the aircraft (N)
w_i	weight coefficients
Y	side force (N)
Z	lift (N)

1. INTRODUCTION

Manoeuvre loads are the forces the airframe experiences in the event of aircraft manoeuvring to maintain its intended flight path. The ability to tailor the distribution of manoeuvre loads over the airframe is known as Manoeuvre Load Control (MLC). Active Control Technology (ACT) deals with optimal actuation of control surfaces to alleviate manoeuvre loads on airframe through redistribution of wing loads, which reduces the stresses on structures and enhances the flight performance. With a slight modification in the flight control system parameters for operating the control surfaces optimally, this method proves to be promising in recent years without significantly affecting the structural weight. The present study is on a typical tailless delta wing Combat Aircraft having symmetrically placed ELEVON (ELEVator and AilerON) on inboard and outboard as primary control surface for longitudinal and lateral control, with RUDDER attached to a vertical fin for directional control. The ELEVONS which are operated in tandem are explored to operate independently to effectively redistribute the wing loading.

Scott Zink et al. [1] have attempted the MLC problem for simple unitary manoeuvre load cases such as unit load factor and unit roll, by optimizing for wing root bending moment as single objective function. Henry [2] used a variation of the ε (epsilon) - constraint method [3] by considering minimization of wing root bending moment and included the ratio of bending to twisting moment as equality constraints. A conventional calculus base optimization method is used to identify the optimal solution. In a similar work, Indira et al. [4] attempted to solve the same formulation using single objective Genetic Algorithm (GA) solver for finding the feasible space, further to which a gradient base optimization is used to estimate the optimal. Recent MLC study on Unmanned Aerial Vehicle (UAV) sensorcraft by Hunten et al. [5] also considers single objective function of wing root bending moment minimization. The objective is to reduce the UAV weight by performing trim optimization for simple unitary manoeuvre load cases.

In the single objective formulation the knowledge of variation in bending moment or the twisting moment is unavailable when either one of the function is optimized. The methods specified above of converting a multi-objective problem to single objective problem by imposing a linear relation between bending and twisting moment are inadequate to estimate the true optimal value. Limitations of these studies are backed up by comprehensive literature survey of several flight test programs performed on contemporary aircrafts. The outcome from the flight test programs signifies the importance of variation in bending and twisting moment values. To mention, Manoeuvre Load Distribution Control System (MLDCS) done on C-5A aircraft [6] and Rolling Manoeuvre Load Alleviation studies (RMLA) [7] done by NASA on Active Flexible Wing (AFW) have captured the variation in bending and twisting moment values. Figure 1 shows the variation of bending and twisting moment values from RMLA study in comparison with baseline values. Having understood the importance of problem formulation this paper attempts to bridge the gap between computational and experimental studies by formulating the problem as conflicting independent multiple objective functions of minimizing wing root bending and twisting moment.

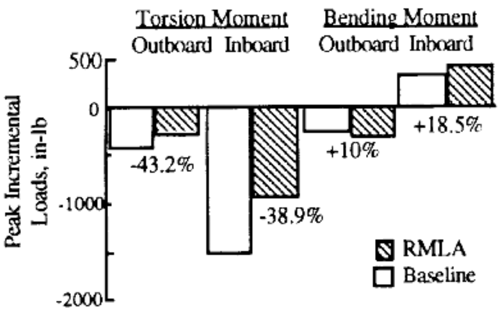

Figure 1. Rolling Manoeuvre Load Alleviation studies on Active Flexible Wing adapted from Woods and Potozky [7]

The other objective of the work is to identify relevant multi-objective optimization methods applicable for solving this MLC problem. Initially, the lacuna with methods of single objective formulation is brought out using a single objective GA solver. Then the familiar Goal programming method [8] that works on sequential quadratic programming to estimate the feasible solution , followed by a line search algorithm to locate the optimal is used for optimization. Later the population based MOEA methods that are ideally suited to solve multi-objective problems by bringing out the best non-dominated solutions on Pareto front is explored. For the MOEA method, Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) [9] with a contemporary penalty parameter less approach to handle constraints is adapted in Matlab© environment. The potency of NSGA-II is combined with Goal programming to form a Hybrid method and results are compared with their original implementations. The performance and capabilities of these methods when applied to this MLC problem is discussed.

To a large extent, conventional treatment to this trim analysis problem of aircraft will only consider simple unitary manoeuvre load cases such as unit load factor, unit roll rate and unit roll acceleration. This works brings closer insight to the reality, by optimizing the load envelope for combined manoeuvres. Critical combined manoeuvres involving symmetric and anti-symmetric ELEVON rotations such as roll pull up and roll push over are taken up for optimization. Industry standard MSC Nastran© module is used for extracting the elastic stability derivatives, hinge moment derivatives, bending and twisting moment derivatives for the flexible aircraft.

2. PROBLEM FORMULATION FOR COMBINED MANOEUVRES

2.1 Configuration

Figure 2 shows a typical tailless delta wing aircraft which consist of four ELEVONS (symmetrically placed at Inboard and Outboard) at trailing edge as the primary control surface for lateral (Roll) and longitudinal (Pitch) control, with a RUDDER hinged to single vertical tail for directional (Yaw) control.

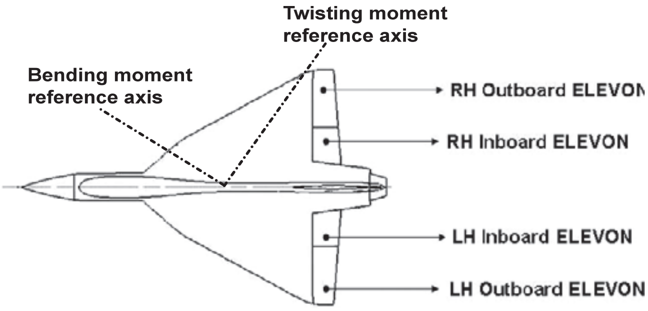

Figure 2. A typical tailless delta wing aircraft

Independent actuators are used to swivel each ELEVON. The baseline model consists of inboard and outboard ELEVONS actuated by identical but individual signals making it to operate as a single control surface. The possibility of independent ELEVON rotations is the motivation for MLC, by which the net resultant of wing loading can be moved towards wing root. In simple terms MLC means unloading of outer wing through independent optimal actuation of ELEVONS.

2.2 Mathematical model

When the ELEVONS are actuated symmetrically (δ_e) on port (LH) and starboard (RH) side it would result in pitch up/down motion whereas the anti-symmetric (δ_a) actuation causes roll motion on the aircraft. The baseline model termed as 'Linked ELEVON' consist of inboard and outboard ELEVONS actuated by identical signal making it to operate as single control surface. For the optimal model termed as 'Split ELEVON' the inboard and outboard ELEVONS are separated, however the starboard and port side inboard ELEVONS are linked. The understanding is that the inboard ELEVONS take the same direction and magnitude for symmetric manoeuvres and opposite direction with the same magnitude for anti-symmetric manoeuvres. The same strategy is adopted for outboard ELEVONS too. Precisely, apart from the other design variables, the baseline model requires input of single ELEVON swivel angle and for the optimal model the optimizer will determine the swivel angles of inboard and outboard ELEVONS to alleviate manoeuvre loads. Hence the design variables consist of swivel angles of inboard and outboard ELEVONS for symmetric (δ_e) and anti-symmetric (δ_a) manoeuvres, rudder (δ_r), side slip angle (β) and angle of attack (α) for symmetric structural configuration. The symmetric structure implies the aircraft weight distribution is even on the starboard and port side.

The problem is formulated for conflicting independent multiple objectives with wing root bending and twisting moment as fitness functions. Industry standard MSC Nastran© is used for extracting the aircraft elastic stability derivatives, actuator hinge moment derivatives and bending/twisting moment derivatives which are applied in equality , inequality constraints and objective functions for the optimal model. To identify the maximum bending and twisting moment location, the values are monitored all along the cross section at several locations of bending moment and twisting moment reference axis as indicated in Figure 2. Maximum values are identified at wing root section which forms the region for optimization. The baseline model is solved to get a closed form solution in MSC Nastran©. For the optimized model, equality constraints are imposed through stability equations as represented in the mathematical formulation. Inequality constraints are imposed through limitation on actuator loads in the form of hinge moments with side constraints on the range of swivel angles for the design variables. All the equality and inequality constraints are normalized suitably to apply in optimization algorithm. The convention followed is, clockwise (cw) rotation is taken to be positive (when control surfaces rotates down with respect to hinge line) and vice versa. In the equations, to generalize, four ELEVONS are represented as (δ_i) with $1 \ldots n_{cs}$, where n_{cs} are the Inboard Right (*IR*), Outboard Right (*OR*), Inboard Left (*IL*) and Outboard Left (*OL*) ELEVONS respectively.

The rationale behind the multi-objective formulation is explained as follows. Referring to Figure 2 for the delta wing configuration, when the inboard ELEVON is deflected, the twisting moment generated by the ELEVON is more influential than the bending moment. This is due to the larger moment arm of inboard ELEVON resultant from the twisting moment reference axis. For the outboard ELEVON, the bending moment generated by the ELEVON is more influential than twisting moment, due to the large moment arm of outboard ELEVON resultant from bending moment reference axis.

There lies an optimal trade-off configuration of inboard and outboard ELEVONS rotation for each of the manoeuvres, wherein the conflicting objectives of bending and twisting moment minimization is satisfied. It is often straightforward to draw conclusion of wing load changes to stress/fatigue implications. With this background, the mathematical model for combined manoeuvre load cases involving vertical load factor and roll is defined as follows

Minimize the independent multi-objective functions,

$$RBM = RBM_0 + \left(\frac{\partial(RBM)}{\partial\alpha}\right)_{flex}\alpha + \sum_{i=1}^{n_{cs}}\left(\frac{\partial(RBM)}{\partial\delta_i}\right)_{flex}\delta_i + \left(\frac{\partial(RBM)}{\partial\beta}\right)_{flex}\beta + \left(\frac{\partial(RBM)}{\partial\delta_r}\right)_{flex}\delta_r \quad (1)$$

$$RTM = RTM_0 + \left(\frac{\partial(RTM)}{\partial\alpha}\right)_{flex}\alpha + \sum_{i=1}^{n_{cs}}\left(\frac{\partial(RTM)}{\partial\delta_i}\right)_{flex}\delta_i + \left(\frac{\partial(RTM)}{\partial\beta}\right)_{flex}\beta + \left(\frac{\partial(RTM)}{\partial\delta_r}\right)_{flex}\delta_r \quad (2)$$

RBM_0 and RTM_0 are the moment experienced at the wing root independent of control surface rotation. From the small disturbance theory [10] the aerodynamic forces and moments perturbations are small and only first order terms of the flexible derivatives such as $\left(\frac{\partial(RBM)}{\partial\alpha}\right)_{flex}, \left(\frac{\partial(RBM)}{\partial\delta_i}\right)_{flex}$ are retained. This assumption holds good for trim analysis problems as it is considered to be of quasi-steady state. The influence of each control surface perturbation on wing root moment is summed up to represent the net bending and twisting moment as multi-objective functions.

The equality constraints are represented in the form of trim equations ('eqns (3-4)' represents the force balance with respect to side force and lift and 'eqns(5-7)' represents balance of pitch , roll and yaw moments),

$$\left(y_0 + \left(\frac{\partial Y}{\partial\alpha}\right)_{flex}\alpha + \sum_{i=1}^{n_{cs}}\left(\frac{\partial Y}{\partial\delta_i}\right)_{flex}\delta_i + \left(\frac{\partial Y}{\partial\beta}\right)_{flex}\beta + \left(\frac{\partial Y}{\partial\delta_r}\right)_{flex}\delta_r\right) * q * s = 0 \quad (3)$$

$$\left(Z_0 + \left(\frac{\partial Z}{\partial\alpha}\right)_{flex}\alpha + \sum_{i=1}^{n_{cs}}\left(\frac{\partial Z}{\partial\delta_i}\right)_{flex}\delta_i + \left(\frac{\partial Z}{\partial\beta}\right)_{flex}\beta + \left(\frac{\partial Z}{\partial\delta_r}\right)_{flex}\delta_r\right) * q * s = m * n_z * g \quad (4)$$

$$\left(L_0 + \left(\frac{\partial L}{\partial\alpha}\right)_{flex}\alpha + \sum_{i=1}^{n_{cs}}\left(\frac{\partial L}{\partial\delta_i}\right)_{flex}\delta_i + \left(\frac{\partial L}{\partial\beta}\right)_{flex}\beta + \left(\frac{\partial L}{\partial\delta_r}\right)_{flex}\delta_r\right) * q * s * l = I_{xx} * \dot{p} + \left(\frac{\partial L}{\partial_{(pl/v)}}\right)_{flex}\delta_{(pl/v)} \quad (5)$$

$$\left(M_0 + \left(\frac{\partial M}{\partial\alpha}\right)_{flex}\alpha + \sum_{i=1}^{n_{cs}}\left(\frac{\partial M}{\partial\delta_i}\right)_{flex}\delta_i + \left(\frac{\partial M}{\partial\beta}\right)_{flex}\beta + \left(\frac{\partial M}{\partial\delta_r}\right)_{flex}\delta_r\right) * q * s * c = 0 \quad (6)$$

$$\left(N_0 + \left(\frac{\partial N}{\partial\alpha}\right)_{flex}\alpha + \sum_{i=1}^{n_{cs}}\left(\frac{\partial N}{\partial\delta_i}\right)_{flex}\delta_i + \left(\frac{\partial N}{\partial\beta}\right)_{flex}\beta + \left(\frac{\partial N}{\partial\delta_r}\right)_{flex}\delta_r\right) * q * s * l = -I_{zx} * \dot{p} \quad (7)$$

for symmetric manoeuvres (Vertical load factor)

$$\delta_{IR} = \delta_{IL} \quad (8)$$

$$\delta_{OR} = \delta_{OL} \quad (9)$$

for anti-symmetric manoeuvres (Roll)

$$\delta_{IR} = -\delta_{IL} \quad (10)$$

$$\delta_{OR} = -\delta_{OL} \quad (11)$$

Inequality constraints in the form of actuator hinge moments

$$HM_{ccw} \le HM_{IR} \le HM_{cw} \quad (12)$$

$$HM_{ccw} \le HM_{OR} \le HM_{cw} \quad (13)$$

$$HM_{ccw} \le HM_{IL} \le HM_{cw} \tag{14}$$

$$HM_{ccw} \le HM_{OL} \le HM_{cw} \tag{15}$$

Where the hinge moment for each control surface is given by

$$HM_{cs} = HM_{cs0} + \left(\frac{\partial(HM_{cs})}{\partial\alpha}\right)_{flex}\alpha + \sum_{i=1}^{n_{cs}}\left(\frac{\partial(HM_{cs})}{\partial\delta_i}\right)_{flex}\delta_i + \left(\frac{\partial(HM_{cs})}{\partial\beta}\right)_{flex}\beta + \left(\frac{\partial(HM_{cs})}{\partial\delta_r}\right)_{flex}\delta_r$$

with side constraints on the design variables

$$\alpha_{ccw},\beta_{ccw} \le \alpha,\beta \le \alpha_{cw},\beta_{cw}$$
$$\delta_{ccw} \le \delta_{IR},\delta_{OR},\delta_{IL},\delta_{OL},\delta_r \le \delta_{cw}$$

the subscript '*flex*' indicates elastic derivatives. Y_0, Z_0, L_0, M_0 and N_0 refers to the forces and moment values that are not dependent on control surfaces rotations. HM_{cs0} is the actuator hinge moment during steady flight. The term (*pl/v*) is the roll damping effect which is purely an aerodynamic balance, n_z is the desired vertical load factor (acceleration expressed in terms of *g's*), p and $\dot{p}$ are the desired roll rate and roll acceleration respectively. 'Eqns (8-11)' introduce symmetric and antisymmetric condition of starboard and port side ELEVON rotations respectively. Owing to mass symmetry, equations can be decoupled and hence for the manoeuvres involving vertical load factor, 'eqns (4, 6 and 8-9)' needs to be satisfied and for the roll manoeuvres 'eqns (3, 5, 7 and 10-11)' needs to be satisfied. The specifications adopted are applicable for any typical tailless delta wing combat aircraft available in standard aircraft specification database such as Janes [11].

3. METHODOLOGY FOR MANOEUVRE LOAD COMPUTATION

3.1 Load Envelope

Typical aircraft design process begins with the requirements from end user of aircraft to the designers. The expected performance is specified as flight envelopes which defines the operating range with respect to Mach-Altitude regime, for which the aircraft is designed. Limits are determined by attainable load factors, rates and accelerations during roll, pitch and yaw manoeuvres. Manoeuvre loads are identified from the corner points of flight envelopes. The combination of roll and vertical load factor termed as roll pull up /roll push over manoeuvres are the most common that occurs during flight. At the instant of roll initiation, the roll acceleration tends to be at maximum with the roll rate being the minimum and progressively the roll rate cumulate to the maximum with the roll acceleration reaching minimum. The combined case examines the manoeuvre loads experienced by the aircraft in the initiation, intermediate (when roll rate and roll acceleration are considerable) and termination phases of each manoeuvre. For the wing structure configuration, the bending and twisting moment values for all manoeuvre load cases are computed at wing root and plotted. The values at the boundary are joined together to define the structural limit termed as 'critical load envelope' for which the aircraft is designed for, as shown in Figure 3. The shaded circles represent the induced bending and twisting moment for several manoeuvre load cases. Shaded circles lying inside the critical load envelope are intermediate manoeuvre load cases that are non-critical. On the

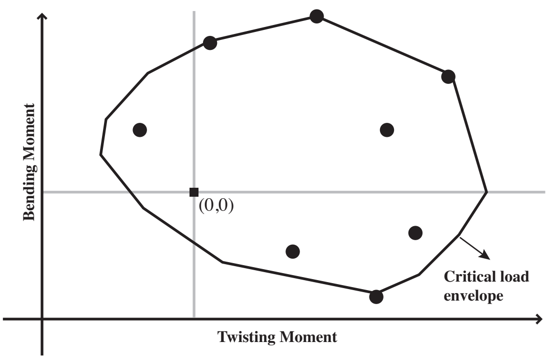

Figure 3. Plot of typical Load Envelope

basis of net resultant location, the direction of bending and twisting moment monitored at wing root gets altered. Hence the clockwise (positive) and counter-clockwise (negative) bending and twisting moment values are located from the origin (shown as shaded square) of load envelope.

For the wing configuration, thirteen of the combined manoeuvre load cases that exist on the boundary are identified as critical and hence the optimization caters to determine optimal values for all these load cases, such that the load envelope can be shrunk to alleviate the wing loads.

3.2 Static Aeroelastic Trim Analysis

The static aeroelastic load computation is performed with MSC Nastran©. The finite element model consists of full aircraft with one hundred thousand degrees of freedom. The aerodynamic grid consists of panels modeled with double-lattice method on the basis of linearized aerodynamic potential theory. Structural nodes and aerodynamic grids are connected by interpolation. Structural nodes are chosen to have independent degree of freedom and the dependent ones, are the aerodynamic degrees of freedom. The interpolation method termed as splining transforms the structural deflections to the aerodynamic grid and the relationship between aerodynamic forces to the structural equivalent forces in the form of aerodynamic influence coefficient.

The basic equation for static aeroelastic analysis by the finite element method is:

$$[m]\{g\}+[[K]-q[A]]\{u\}=[P]\{\varphi\} \tag{16}$$

$\{\varphi\}$ is the non-acceleration trim parameter values (e.g., ELEVON and Rudder deflection). For static aeroelastic trim, 'eqn (16)' is decomposed as represented below,

$$[D]\{g_r\}=[R]\{\varphi\} \tag{17}$$

this in essence, is the simple balance of aeroelastic and inertial forces. The aeroelastic forces and moments can be expressed as a function of the variables by expanding in Taylor series. The inertial forces and moments are obtained from equations of motion considering rigid body accelerations with respect to inertial reference frame [12]. The equality constraints presented earlier in 'eqns (3-7)' are derived from 'eqn (17)'. When the number of free trim parameters is equal to the number of supported degrees of freedom (DOF), 'eqn (17)' has a closed form solution. If only one control surface is used, then the calculation of control surface rotation to trim the aircraft to the user given input is straightforward. However, if multiple control surfaces (i.e., redundant surfaces) are desired to trim the aircraft then the system of equations can be formulated as an optimization problem to find out the best combination of control surface rotations. The method by which the closed form solution is obtained for a given input can be reviewed from theoretical manual of Rodden and Johnson [13].

4. METHODS OF MULTIOBJECTIVE OPTIMIZATION

The stringent demand for performance and closer margin of safety has evoked the aircraft design process to involve synergistic participation of several disciplines. As described by Osyczka [14] the Multi-objective Optimization Problem (MOP) can be defined as the problem of finding a vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions. These functions form a mathematical description of performance criteria which are usually in conflict with each other. Hence, the term "optimize" means finding such a solution which would give the values of all the objective functions acceptable to the decision maker. The development of multiobjective optimization started with identifying various methods of converting multiple objectives into a single objective function. By and large this single objective function is solved iteratively using calculus based methods by updating the direction vector and step size. In the later development the targeted vector approaches such as Goal programming method had gained popularity which minimizes the differences between current solution and target goals. The difficulty in handling trade-off optimal solutions and non-convex space had provided the insight to develop Pareto based solution techniques. Evolutionary principle such as Genetic Algorithm is a well adapted technique for multi-objective optimization which handles the problem with population of solutions to steer toward multiple non-dominated solutions. The set of all non-dominated solutions is formally called as Pareto Front [15]. Survey on MOEA from the application to complex engineering problems [16] and ease of handling [17] perspective revealed a promising potential for applying NSGA-II algorithm to this sort of MLC problem. NSGA-II in Matlab© environment is coded with a contemporary penalty parameter less approach for constraint handling which stands as the frame work for this paper.

4.1 Methods of single objective function

In the weighted sum method, a composite single objective function is formulated by combining the fitness functions of bending and twisting moment with appropriate weight coefficients. Different sets of optimal values are achieved by modifying the weight coefficients, from which the best solution can be selected. The objective function is to

$$\text{Minimize} \quad F(x) = w_1 * RBM + w_2 * RTM \text{ , where } \sum_{i=1}^{2} w_i = 1 \tag{18}$$

Another approach of using a single objective function is the variation of ε – constraint method [3] wherein the minimization of bending moment is considered as the objective and the other fitness function of twisting moment is treated as equality constraint by a linear relationship between bending and twisting moment as presented by Henry [2] and Indira et. al [4]. The ratio of bending to twisting moment is termed as μ which is of the range 1 to 1.2 based on existing data available for the baseline model.

$$\text{Minimize } F(x) = RBM \text{ , such that } \frac{RBM}{RTM} = \mu \text{ with all other equality and inequality constraints} \tag{19}$$

To bring out the lacuna with single objective formulation, a simple load case of 2g vertical load factor is solved with the methods described above. Single objective Genetic Algorithm solver is used for this purpose. A comparative study is made for multi-objective problem formulation by solving the same load case with Goal programming and MOEA methods. For the purpose of clarity, only the results from multi-objective optimization methods are presented here. The procedures are detailed in subsequent sections.

The comparative study results are presented in Table 1. When equal weight coefficients are assigned for bending and twisting moment in weighted sum method, the twisting moment gets reduced by 7%. However the bending moment increases by 10%. To limit bending moment within the baseline value, the weight coefficients are altered in steps of 10%. The reduction in root bending moment is in the range of 4% to 6% from baseline value, for the successive increase in importance of bending moment weight coefficient. However the root twisting moment increases in the range of 9% to 20% from the baseline value for the corresponding decrease in importance of twisting moment weight coefficient. For the ε-constraint method the reduction of bending moment is 3% with the twisting moment being more by 40% when μ is set to 1.2. The inconsistent trend observed for these conflicting objectives when solved using methods of single objective function is evident. The choice of weight coefficients and μ values invariably affects the optimal solutions from these methods. For comparison purpose, the results obtained from multi-objective optimization methods such as Goal programming and MOEA method provides lower optimum of root bending and twisting moment. From this outcome of comparative study these multi-objective optimization methods are taken up for optimizing combined manoeuvres load cases.

Table 1. Comparison of optimal values from methods of single objective function with several multi-objective optimization methods

Si. no	optimization methods	Ratio = Optimal (Split ELEVON) / Base line (Linked ELEVON)	
		RTM	**RBM**
1)	**Methods of Single objective function**		
	a. weighted sum method		
	i. Equal Weights for RTM and RBM	0.931	1.105
	ii. 40% Weight for RTM and 60% for RBM	1.094	0.964
	iii. 30% Weight for RTM and 70% for RBM	1.208	0.943
	b. variation of ε – constraint method RBM as objective function and RBM=1.2*RTM as equality constraint	1.396	0.978
2)	Goal Programming Method	0.837	0.885
3)	MOEA with NSGA – II algorithm	0.863	0.820

4.2 Goal Programming Method

Goal programming method introduced by Gembiki [8] is implemented in Matlab© environment that makes use of sequential quadratic programming and line search algorithm to optimize the given objective with constraints. The method works with a set of design goals, $F(x^*) = \{F_{RBM}^*, F_{RTM}^*\}$, associated with a set of design objectives, $F(x) = \{F_{RBM}, F_{RTM}\}$. The problem formulation allows the objectives to be under or overachieved, enabling the designer to be relatively imprecise about the initial design goals. The relative degree of under or overachievement of the goals is controlled by a vector of weighting coefficients, $w = \{w_{RBM}, w_{RTM}\}$ and is expressed as a standard optimization problem using the formulation,

$$\begin{aligned} &\text{Minimize } \gamma, \\ &\text{such that } F_i(x) - w_i\gamma \leq F_i(x)^* \quad \text{where } i = RBM, RTM \end{aligned} \tag{20}$$

With all other equality and inequality constraints in place the deviation measured in terms of attainment factor γ is minimized. The term $w_i\ \gamma$ gives flexibility to the problem, which otherwise imposes the goals to be rigidly met. For instance, making weights equal to the initial goals indicates the same percentage of under or over achievements of the goals. Setting weights equal to zero incorporates hard constraints. In this way this method can provide an intuitive interpretation of the design problem. For the combined manoeuvre load cases considered the goals are fixed not to be lesser than 10% reduction from the base line values. Initial goals are assigned for the weight coefficients to allow the same percentage of under-or overachievement. Results using this method for all combined manoeuvre cases are presented in the results and discussion section.

4.3 Multi-objective Evolutionary Algorithm Method

The primary objective for any multi-objective optimization algorithm is to determine diversified trade-off solutions with superior spread on Pareto front. Over the past years vigorous developments around the globe to key out the solution methods, had rendered some popular Multi-objective Genetic Algorithms such as VEGA [18], SPEA [19], NPGA [20], NSGA-II [9] and PAES [21]. NSGA – II is one of the popular algorithm which uses Pareto based fitness assignment and ranking as presented by Goldberg [22]. Elimination of providing interior and exterior niching parameters apart from the standard evolutionary algorithm parameters such as population size and generations, render a good popularity among researchers to solve complex engineering problems [23, 24]. The algorithm uses an elite preserving operator and explicit diversity preserving mechanism. The NSGA-II procedure is well described by Deb [25] in his book on MOEA.

4.3.1 Constraint Handling

The involvement of large number of constraints to be handled for this MLC problem has resulted in investing significant effort to identify and assess the best method to handle constraints. Interior and exterior penalty methods are simple and most popular approach to handle constraints in multi-objective optimization. In this work the constraints are handled using contemporary penalty parameter less approach coded in Matlab© environment. Applications of this penalty parameter less approach with NSGA-II for complex practical engineering problem are least reported so far and hence attempted. Since the penalty parameter less approach is a variation of the exterior penalty approach, the procedure is briefed out.

In the exterior penalty approach, the penalty factors are selected for equality and inequality constraint and added to the violation [26]

$$F(x,R,r) = f(x) + \sum_{j=1}^{J} R_j \left\langle E_j(x) \right\rangle + \sum_{k=1}^{K} r_k \left[h_k(x) \right] \tag{21}$$

Where $\vec{x}$ are the design variables, $f(\vec{x})$ fitness value for the infeasible solution, $E_j(\vec{x})$ and $h_k(\vec{x})$ are the constraint violation values for j^{th} equality constraints and k^{th} inequality constraints with R_j and r_k as penalty factors to scale up the infeasible solution. It is elusive to exactly determine the penalty factor prior to the solution. Researchers such as Homaifar et al. [27] used a multi-level penalty function depending on the constraint violation. Joines and Hauck [28] tried with dynamic penalty factor that are varied on each generations. All of this research suggests that the choice and technique of applying penalty factor are problem specific to drive towards optimum.

The contemporary penalty parameter less approach will eliminate the difficulty of selecting right penalty factor by making use of a constrained tournament selection operator and fast non-dominated sorting as suggested by Deb and Agarwal [29]. The constrained tournament selection operator compares two solutions and chooses the best, based on the following criterion:

a) Any feasible solution is preferred over an infeasible solution.

b) Among two feasible solutions, the one having better fitness is preferred.

c) Among two infeasible solutions, the one with lesser violation is preferred.

$$F(x) = \begin{cases} f(x), & \text{if } x \text{ is feasible} \\ f_{max} + \sum_{j=1}^{J} \langle g_j(x) \rangle + \sum_{k=1}^{k} |h_k(x)|, & \text{Otherwise} \end{cases} \tag{22}$$

The fundamental difference from the penalty based approach is that the objective values for infeasible solutions are not computed. Instead, for the infeasible solutions, maximum fitness (f_{max}) among the feasible space is added to the violated constraint values as indicated in ‘eqn (22)’. Constrained tournament selection operator functions on relative difference in fitness values with preference for feasible solution over infeasible solution. Fast non-dominated sorting and constrained tournament selection has enabled the penalty parameter less method to complement well with NSGA-II thereby making it inapplicable for conventional calculus based technique. To maintain diversity among the ranked population, crowding distance operator estimates the distance between the populations. Population with larger crowding distance are retained to promote diversity. The procedures explained above are applied to some of the constrained test problems presented by Elaoud et.al [30]. These test problems were compared for penalty and penalty parameter less approaches with a population size of 100 and for 50 generations. The deviation from the true Pareto front for several combinations of penalty factors are studied and compared with penalty parameter less approach on these test problems. By gaining confidence with the behaviour of NSGA-II algorithm with penalty parameter less approach, this method is implemented to handle combined manoeuvre cases. The results from NSGA-II method for combined manoeuvre load cases are presented in results and discussion section.

4.4 Hybrid Method by combining MOEA with Goal Programming

To promote faster convergence and to have superior spread the NSGA-II algorithm is combined with Goal programming method to bring out a Hybrid form. The strength of MOEA is to reach the region near an optimal Pareto front relatively faster, yet the bottleneck is on convergence to the optimum. By updating the search direction and step size the conventional calculus base algorithm used for Goal programming performs faster in terms of convergence provided the initial starting point is feasible. Hence for certain number of generations the MOEA method is used to get to the near optimum front and from thereon the Goal programming method is used for local search. For the Goal programming method the pseudo weights are assigned based on the location of solution on the Pareto front and the ideal objective vector is taken as goal. The ideal objective vector indicates the minimum of all the objectives on Pareto front. Thus each point on the Pareto front is optimized by minimizing the deviation measured in the form of attainment factor γ and the Pareto front is updated. The schematic representation of the method is shown in Figure 4. With this implementation in place the combined manoeuvre load cases are solved and the results are compared with their original form of multi-objective optimization routine which will be discussed in the subsequent section.

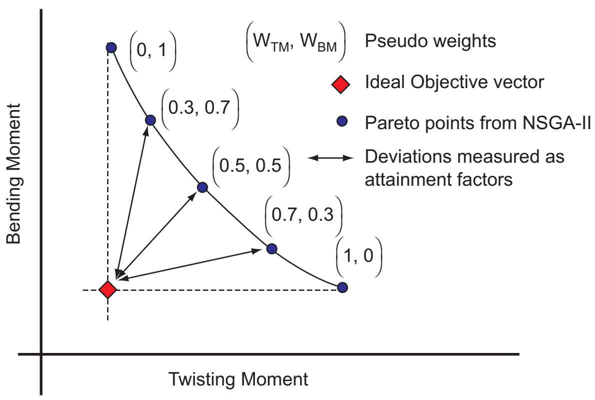

Figure 4. Schematic representation of Hybrid Method for two objective problems

5. RESULTS AND DISCUSSION

The combined manoeuvre load cases are presented in Table 2 which consists of combination of vertical load factor, roll rate and roll acceleration normalized with respect to their maximum values $(n_{z\max}, p_{\max}, \dot{p}_{\max})$ to assimilate the roll pull up and roll push over manoeuvres. For these load cases the conflicting independent multi-objective problem is formulated and solved to minimize wing root bending and twisting moment, with angle of attack (α), side slip angle (β), rudder swivel angle (δ_r), symmetric (δ_e) and anti-symmetric (δ_a) ELEVON swivel angles as design variables. Methods such as Goal programming, MOEA with NSGA-II and its Hybrid form are implemented as solution methods. The peculiarity with this MLC problem lies in delicate way to treat the clockwise (positive) and counter-clockwise (negative) values of design variables and fitness functions, involvement of extensive static aeroelastic computation and fine tuning of algorithms to handle large number of constraints. Table 3 presents the values of design variable for the baseline model (Linked ELEVON) obtained from closed form solution. The vertical load factor and the roll manoeuvres are computed separately and summed up to represent the combined load cases. As detailed earlier, for the baseline model the inboard and outboard ELEVON swivels by the same magnitude.

Table 2. Combined Manoeuvre Load cases

Load case Ref No.	Vertical Load factor $n_z / n_{z\max}$	Roll Rate $p / p_{\max}$	Roll Acceleration $\dot{p} / \dot{p}_{\max}$
10	1	-0.370	0
20	-0.125	1	0
30	-0.437	0.370	0
40	1	0	0.424
50	1	0.261	0.299
60	0.5	0.558	1
70	0.5	-0.558	-1
80	0.25	0.707	1
90	0.25	-0.707	-1
100	-0.125	0.707	1
110	-0.125	-0.707	-1
120	-0.25	0.528	1
130	-0.25	-0.528	-1

Table 3. Design variables values for the Baseline model (Linked ELEVON)

Load case Ref No.	Angle of attack (α)	Symmetric ELEVON swivel angle (δ_e)	Anti-symmetric ELEVON swivel angle (δ_a)	Angle of Side slip (β)	Rudder swivel angle (δ_r)
10	0.246	-0.126	-0.024	-0.007	-0.005
20	-0.031	0.016	0.064	0.020	0.014
30	-0.108	0.055	0.024	0.007	0.005
40	0.246	-0.126	0.024	-0.026	0.084
50	0.246	-0.126	0.033	-0.013	0.063
60	0.123	-0.063	0.091	-0.051	0.207
70	0.123	-0.063	-0.091	0.051	-0.207
80	0.061	-0.031	0.101	-0.048	0.209
90	0.062	-0.031	-0.101	0.048	-0.209
100	-0.031	0.016	0.101	-0.048	0.209
110	-0.031	0.016	-0.101	0.048	-0.209
120	-0.062	0.031	0.089	-0.051	0.206
130	-0.062	0.031	-0.089	0.051	-0.206

All the design variable values are in radian

Table 4 shows the optimized values obtained from Goal programming method. With the targets taken to be not less than 10% reduction from the baseline model the optimizer is able to satisfy the goals without violating any of the constraints imposed. The under or over achievement of the targets are measured in the form of attainment factor γ. A under achievement threshold limit of maximum 3% is set for the optimum values to deviate from the target goals. For all the load cases the goals are either overachieved or exactly achieved leaving away a negligible attainment factor γ. From Table 4 , the overall reduction of twisting moment considering all the load cases is of the range 7% minimum (load case reference 110) to 14% maximum (load case reference 80)and the bending moment reduction being in the range of 8% minimum (load case reference 100) to 29% maximum (load case reference 90)with no constraint violation. Table 4 also infer that the bending moment reduction is on an average 5% higher than the twisting moment and the outboard ELEVON swivel angle is comparatively greater than inboard ELEVON swivel angle. These aspects signal an optimistic sign of bringing in the resultant of wing load towards wing root. Sensitivity analysis of objective functions with respect to design variables shows that the optimum values for manoeuvres involving load factor are sensitive for the change in angle of attack (α) and symmetric ELEVON swivel angle (δ_e).For the roll manoeuvre the optimum values are sensitive for the change in anti-symmetric ELEVON swivel angles (δ_a), followed by slide slip angle (β) and Rudder swivel angle (δ_r).

The result from the MOEA method with NSGA-II and penalty parameter less constraint handling approach is presented in Table 5. By carrying out convergence trial the population size of 200 and number of generation as 100 are finalized to apply for this MLC problem. The probability values for Simulated Binary Crossover (SBX) and polynomial mutation are selected based on the literature made available by Deb [25]. One complete run with hardware specification of workstation machine with core 2 Duo processor and 2GB memory RAM in Matlab© environment handles the problem within 300 seconds on an average with computational complexity of the order MN^2 , where M is the number of fitness function and N is the number of design variables [29]. The results are comparable with Goal programming method for the overall load cases, as reduction in twisting moment being in the range of 4% (load case reference 130) to 18%(load case reference 90) and bending moment from 6%(load case reference 100) to 36% (load case reference 120) with no constraint violation as presented in Table 5. The same trend of outboard ELEVON swivel angle being greater than inboard is also observed in this method. Population base exploration within the constrained space and the split surface area of the ELEVONS can be attributed for larger swivel angles of ELEVONS when compared to the baseline model.

With the intention to maintain better spread and speed up the convergence, combined manoeuvre load cases are solved using Hybrid method, a combination of NSGA-II with Goal programming. The optimization is carried out in dual phases. In the first phase for initial 50 generations the problem is solved with NSGA-II method and in the second phase the Goal programming method is used for the remaining 50 generations. The result from the first Pareto ranked solution at 50th generation is captured, to assign ideal objective vector as goal and the pseudo weights are assigned according to the position of population on the Pareto front appropriately. With this set up in place the Hybrid method is able to capture better spread of population with faster convergence, in agreement to the results from the other methods. From the tabulated results of Table 6 the overall range of optimum for all the load cases is from 7% (load case reference 40) to 16% (load case reference 100) for twisting moment and 10% (load case reference 100) to 36% (load case reference 120) for bending moment with no constraint violation.

A sample load case reference no.50 is taken to compare the Pareto fronts from NSGA-II and Hybrid method as presented Figure 5. *Zone A* of Figure 5 depicts the better spread of optimal solution from the Hybrid method compared with NSGA-II. Spread measures the change of Pareto between consecutive generations. The larger change of spread observed in Hybrid method is beneficial from convergence perspective even at the expense of nominal loss in diversity. *Zone B* of Figure 5 indicates the nominal loss of diversified population the reason being non-existence of diversity preserving mechanism in the local search algorithm. The non-dominated sorting and tournament selection of MOEA preserves the diversified population which is absent in local search algorithms. The result from Goal programming method is also superposed on the Pareto front for comparison.

For the same load case reference no.50, the crowding distance metric for four sample population on the first Pareto ranked front from NSGA-II and Hybrid method is compared in Table 7. The crowding distance metric which is an indicator to promote diversity, estimates the Euclidean distance between solutions on the same Pareto front. Large crowding distance observed in NSGA-II is a measure of better

Table 4. Design variables and optimal values from Goal Programming Method

load case Ref No.	Optimal design variables							Ratio= Optimal / Baseline	
		Inboard		Outboard					
	α	δ_e	δ_a	δ_e	δ_a	β	δ_r	RTM	RBM
10	0.234	0.089	0.019	-0.261	-0.029	0.069	-0.175	0.883	0.859
20	-0.028	-0.038	0.006	0.050	0.105	0.076	-0.164	0.893	0.865
30	-0.098	-0.134	0.000	0.177	0.038	0.020	-0.041	0.888	0.876
40	0.234	0.089	0.057	-0.261	0.019	0.064	-0.091	0.879	0.890
50	0.232	0.089	0.000	-0.261	0.049	-0.013	0.049	0.880	0.885
60	0.111	0.153	0.003	-0.202	0.133	-0.047	0.162	0.877	0.887
70	0.114	0.153	-0.003	-0.202	-0.133	0.047	-0.162	0.885	0.867
80	0.054	0.077	0.001	-0.101	0.147	-0.046	0.163	0.862	0.890
90	0.058	0.077	-0.001	-0.101	-0.147	0.046	-0.163	0.888	0.719
100	-0.030	-0.038	0.001	0.050	0.147	-0.046	0.163	0.893	0.928
110	-0.026	-0.038	-0.001	0.050	-0.147	0.046	-0.163	0.925	0.891
120	-0.058	-0.077	0.003	0.101	0.130	-0.047	0.162	0.891	0.733
130	-0.054	-0.077	-0.003	0.101	-0.130	0.047	-0.162	0.875	0.887

Table 5. Design variables and optimal values from Multi-objective Genetic Algorithm (NSGA-II) Method

load case Ref No.	Optimal design variables							Ratio= Optimal / Baseline	
		Inboard		Outboard					
	α	δ_e	δ_a	δ_e	δ_a	β	δ_r	RTM	RBM
10	0.234	0.188	0.072	-0.346	0.085	-0.087	-0.161	0.882	0.889
20	-0.032	-0.066	0.126	0.074	0.172	0.139	-0.253	0.912	0.688
30	-0.099	-0.290	0.072	0.306	0.085	0.087	-0.161	0.874	0.728
40	0.234	0.188	0.000	-0.346	0.039	0.012	0.000	0.938	0.885
50	0.233	0.188	0.110	-0.346	0.001	-0.014	0.080	0.918	0.888
60	0.113	0.196	-0.056	-0.243	0.158	-0.044	0.140	0.931	0.841
70	0.115	0.196	0.056	-0.243	-0.158	0.044	-0.140	0.873	0.820
80	0.053	0.171	-0.056	-0.165	0.172	-0.043	0.141	0.879	0.806
90	0.056	0.171	0.056	-0.165	-0.172	0.043	-0.141	0.825	0.900
100	-0.030	-0.066	-0.056	0.074	0.172	-0.043	0.141	0.844	0.944
110	-0.027	-0.066	0.056	0.074	-0.172	0.043	-0.141	0.871	0.824
120	-0.058	-0.154	-0.095	0.165	0.172	-0.050	0.141	0.850	0.647
130	-0.055	-0.154	0.095	0.165	-0.172	0.050	-0.141	0.964	0.758

Table 6. Design variables and optimal values from Hybrid Method

load case Ref No.	Optimal design variables							Ratio= Optimal / Baseline	
		Inboard		Outboard					
	α	δ_e	δ_a	δ_e	δ_a	β	δ_r	RTM	RBM
10	0.210	0.200	0.081	-0.327	0.083	-0.076	-0.182	0.827	0.836
20	-0.057	-0.072	0.139	0.073	0.183	0.157	-0.286	0.877	0.742
30	-0.086	-0.258	0.081	0.319	0.083	-0.076	-0.182	0.824	0.714
40	0.269	0.212	0.009	-0.328	0.043	0.013	0.000	0.930	0.868
50	0.269	0.212	0.109	-0.328	0.006	-0.014	0.080	0.906	0.892
60	0.128	0.198	-0.067	-0.267	0.143	-0.526	0.139	0.813	0.845
70	0.132	0.198	0.067	-0.267	-0.143	0.526	-0.139	0.829	0.788
80	0.064	0.163	-0.032	-0.152	0.187	-0.053	0.162	0.831	0.827
90	0.067	0.163	0.032	-0.152	-0.187	0.053	-0.162	0.816	0.827
100	-0.014	-0.072	-0.032	0.073	0.187	-0.053	0.162	0.746	0.908
110	-0.010	-0.072	0.032	0.073	-0.187	0.053	-0.162	0.808	0.760
120	-0.048	-0.149	-0.099	0.137	0.151	-0.328	0.162	0.840	0.648
130	-0.044	-0.149	0.099	0.137	-0.151	0.328	-0.162	0.921	0.765

Table 7. Sample Pareto solutions with crowding distance and Pareto rank for Load case reference no: 50

Method	Ratio= Optimal / Baseline		Pareto rank	Crowding distance metric
	RTM	RBM		
MOEA with NSGA-II	1.000	0.700	1	Infinity (extremum)
	0.950	0.802	1	0.0211
	0.918	0.888	1	0.0740
	0.852	1.010	1	Infinity (extremum)
Hybrid Method	1.051	0.898	1	Infinity (extremum)
	0.924	0.834	1	0.0183
	0.906	0.892	1	0.0663
	0.881	1.000	1	Infinity (extremum)

The extremum of solutions from the same Pareto front is indicated as infinity in crowding distance metric.

diversity within the population in the same Pareto front. Figure 6 shows the comparison of optimal and baseline load envelope involving all manoeuvre cases. It is indicative that all of these multi-objective optimization methods are capable to effectively optimize the load envelope by at least 10% from the base line values.

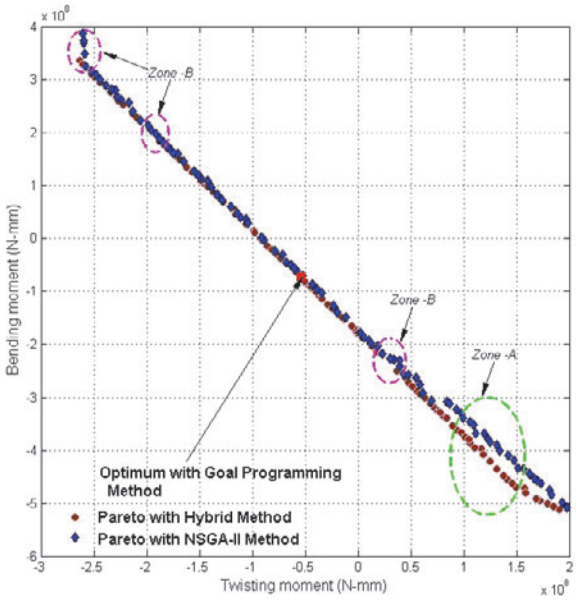

Figure 5. Pareto plot from several methods for Load case reference number: 50

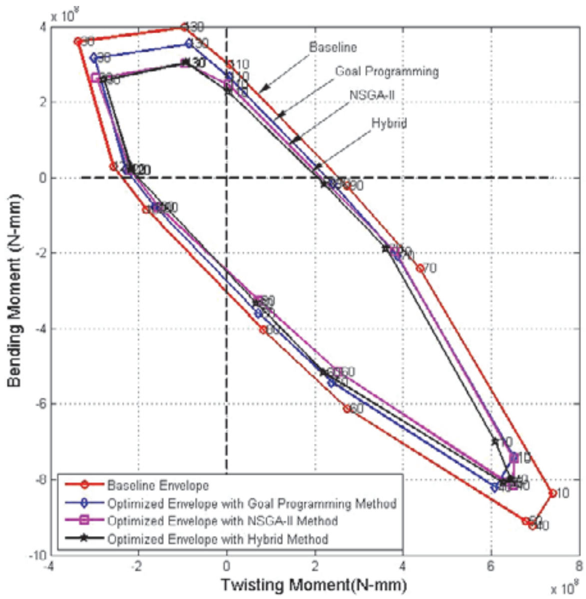

Figure 6. Comparison of baseline and optimum Load Envelope from several methods

Finally, the parameters of average, best and worst fitness solutions with the computational complexity, merits and demerits of all these methods are discussed in Table 8. It's understandable that, the Goal programming method being the traditional of all these methods can ensure guaranteed optimal for the constraint problems in convex space. However the non-Pareto approach and intricacy to handle non-convex problems pose a limitation to this approach. Even though computationally expensive, the Multi-objective Evolutionary Algorithm (MOEA) are best suited to handle these sort of conflicting objectives by brining out the spread of non-dominated solutions on Pareto front. For a faster convergence and better spread the Hybrid method appears to be promising by blending up the best of

Table 8. Parameter comparison of several methods

Parameters	**Goal Programming**		**MOEA with NSGA-II**		**Hybrid Method**	
Load case Ref No: 50	**RTM**	**RBM**	**RTM**	**RBM**	**RTM**	**RBM**
Average fitness	0.880	0.885	0.900	0.899	0.883	0.905
Best fitness	0.880	0.885	0.918	0.888	0.906	0.892
Worst fitness	0.880	0.885	1.108	0.732	1.023	0.853
Convergence	Best		Fair		Better	
Fitness evaluation	$O(N)$		$O(MN^2)$		$O(MN^2)$	
Average Computational time	90 sec		300 sec		280 sec	
Merits	Faster convergence.		Pareto front in single run			
	Guaranteed optimal.		Works at its best to handle conflicting objectives.		Pareto front with better convergence and spread.	
Demerits	Non-pareto / unique solution.		Slower convergence.		Problem specific customization.	
	Intricacy to handle non-convex space.		Computationally intensive.		Chance of loosing diversity.	
MLC applicable for	Aircrafts in service Where the load envelope is known apriori.		Prototype and aircrafts in service.		Prototype and aircrafts in service.	

O - Order of computation, M – number of objectives, N – population size

Pareto approach and line search algorithm. However the assignment of pseudo weights and the ideal vector as goals needs hand crafted approach according to the problems.

6. CONCLUSION

The efforts to formulate the problem as multiple conflicting objectives and systematic application of several multi-objective optimization methods, had conceived a through insight on nature of this MLC problem. The performance measures of multi-objective optimization methods such as Goal programming, MOEA and its hybrid form are compared on real world MLC problem. The outcome is realized with an optimized load envelope by at least 10% from base line values for structural load alleviation. The other way to comprehend is to have enhanced performance in terms of manoeuvrability and agility for the same baseline envelope. The method of Goal programming being the most traditional can optimize the overall envelope by minimum of 7% to maximum of 14% for root twisting moment and 8% to 29% for root bending moment. Limitations to accommodate significant changes in hardware configuration for the aircrafts in service make this method more practical to work with achievable target goals wherein the existing envelope is well defined and known a prior. MOEA and its Hybrid form can optimize the envelope by a minimum of 4% to maximum 18% for root twisting moment and 6% to 36% for root bending moment. For the aircrafts in design phase that have wider scope for design flexibilities, the MOEA method and its Hybrid form can be applied with the MLC implementation in place resulting in optimal structural design and enhanced performance. Completeness can be achieved when asymmetric structural configuration are included that can cover total flight envelope and bring additional complexity with coupled product of inertia terms in trim equations.

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