Numerical simulations of explosive volcanic eruption: Blast waves and pyroclastic flows

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ABSTRACT

An explosive type volcanic eruption causes devastating disaster due to extremely high energy release rate.

Numerical predictions of hazardous phenomena accompanied by explosive type volcano eruptions are carried out in order to establish a practical means of providing quantitative database for establishing better safety measures for specific volcanoes. Conservation laws of mass, momentum and energy, together with the perfect-gas equation of state are solved. In order to solve problems for specific volcanoes, three dimensional (3D) computations with corresponding ground geometry of the interested area are carried out. The 3D simulations with a computational domain large enough for collecting meaningful data are categorized as large scale computations even for today's large computer systems. Parallel computations with MPI and OpenMP together with vector processing techniques are employed in this study and some details are given. Also considered in this paper are the eruption models that describe how the energy is released. The method is applied to an imaginary eruption of Mt. Usu located in Hokkaido Japan to demonstrate how numerical simulations can be utilized for investigating volcanic eruptions.

1. INTRODUCTION

Shock wave research, in its earlier development, is aimed mainly at solving problems in high-speed gasdynamics in relation to aerospace technological developments. However, shock wave is a comprehensive representation of a nonlinear wave motion [1] and it exists in nature and the artificial world [2, 3]. Accordingly, shock wave research has been expanded to other fields of science and technology as its interdisciplinary applications to medicine, geophysics and industries etc. [4]. For example, extracorporeal shock wave lithotripsy (ESWL) [5, 6] is one of the most successful examples and further developments of shock wave therapy are now being expanding to different fields of medicine [7]. Numerical works on wave and flow phenomena induced by explosive type volcanic eruptions have been done as an interdisciplinary application of shock wave research to geophysics. This paper describes the numerical methods that have been developed during the research activities in that field.

Extremely large amount of energy is abruptly released in the event of explosive type volcanic eruptions and different kinds of devastating damages are induced in the surrounding environment. Those include blast waves, pyroclastic flows, volcanic blocks, volcanic gas and ashes, etc. When a finite amount of energy is suddenly released to an open area, blast waves consisting of expanding shock waves and rarefaction waves are generated. Since the strength of blast waves decays rather quickly and the waves finally become sound waves, the area suffered from severe blast-wave damage is relatively small in cases of small to middle size eruptions. However, for an extremely large volcano eruption such as the case of Mt. St. Helens' in 1980, the damages caused by the blast waves were tremendous [8].

Pyroclastic flow is another kind of hazardous ground phenomena occasionally observed in volcanic eruptions. It is extremely dangerous and destructive due to large momentum of the flowing medium and the high temperature. Numerical method introduced here can easily be extended and applied to simulate such flows under certain practical assumptions.

Numerical predictions of hazardous phenomena accompanied by explosive type volcano eruptions are carried out in order to establish a practical means of providing quantitative database for establishing better safety measures. Conservation laws of mass, momentum and energy, together with a perfect gas equation of state are solved. Several works have been done with the TVD scheme of Harten and Yee [9, 10] and with the Weighed Averaged Flux method of Toro [11] in the past [12-14].

In order to solve problems for a specific volcano, computations are carried out in the three dimensional (3D) space coordinates including the corresponding ground geometry. The 3D numerical simulations with a computational domain wide enough for collecting meaningful data require huge amount of computational resources. A combination of parallel techniques using MPI and OpenMP together with vector processing is employed in the current study and details of the parallel techniques relevant to this study are described.

The initial and boundary conditions for numerical simulations of volcano eruptions (eruption models) are introduced and discussed. In this paper imaginary eruptions of Mt. Usu are simulated to demonstrate how numerical simulations can be utilized for studying the gasdynamic phenomena induced by volcanic eruptions. Mt. Usu is located in Hokkaido Japan and is known to erupt periodically with intervals of about 20 to 30 years. Its recent eruptions occurred in 1976 and 2000.

2. NUMERICAL METHOD

Basic equations for conservation laws, boundary and initial conditions and numerical schemes that are adopted in this study are fairly standard and well established in the field of supersonic gasdaynamic problems. They are described in this section for the sake of completeness.

2.1. Basic equations and numerical scheme

For the current study, the assumption of an inviscid, non-heat-conducting gas is appropriate since the space resolution is not high enough to accurately resolve the boundary layer on the ground. Therefore conservation laws of mass, momentum and energy for inviscid non-heat-conducting gases (Euler equations) are numerically integrated together with the equation of state for perfect gases. As to the numerical scheme, the weighed average flux method (WAF) introduced by Toro [11, 15] is used in the current study. The method is a Godunov-type second-order scheme both in space and time. It forms Riemann problems at cell interfaces with piecewise constant data in the same way as the first-order Godunov scheme. The second-order accuracy of the scheme is achieved by space averaging of the whole Riemann solutions at the middle point of numerical time step. The high-order Godunov type scheme WAF has been successfully applied to many gasdynamic problems.

2.2. Parallel computations

Three dimensional numerical simulations such as the present study require extensively large memory area and take much CPU times even for today’s advanced computer systems.

High performance computations today are based on various kinds of parallel techniques and a numerical code must be optimized for a specific computer. Figure 1 shows a typical configuration of recent computer systems. It connects multiple computer nodes that consist of several Processing Elements (PEs) and a common memory area (M).

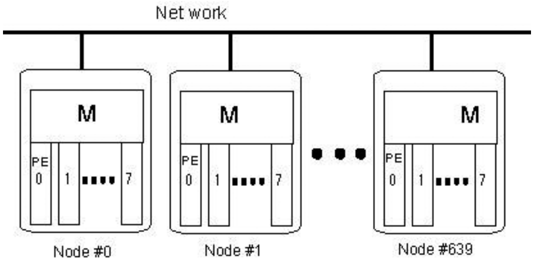

Figure 1. Schematic diagram of typical memory-PE (CPU) configuration.

There are several possible ways of utilizing such computer systems. For example, it is possible to write a code using HPF. If the compiler is mature and reliable, it is perhaps the easiest. In case if the program size is small enough to fit within a computational node, shared memory programming with OpenMP is a reasonable choice.

In the current study, we employed a hybrid method of combining distributed and shared memory strategies as shown in Figure 2. The portability of the code from a computer system to another is relatively flexible with this method. More specifically, the whole three-dimensional computational region is divided evenly by the number of computer nodes and each sub-region is distributed to a corresponding node. This part of parallelization together with inter-node communications is done by using MPI. The domain decomposition is done either in a single direction or multiple directions. In this study, the computational domain is divided only in the z-direction (vertical direction) just because of simplicity. Within each node, calculations of the sub-region are executed in parallel by using OpenMP since the memory is common to all PEs in the node.

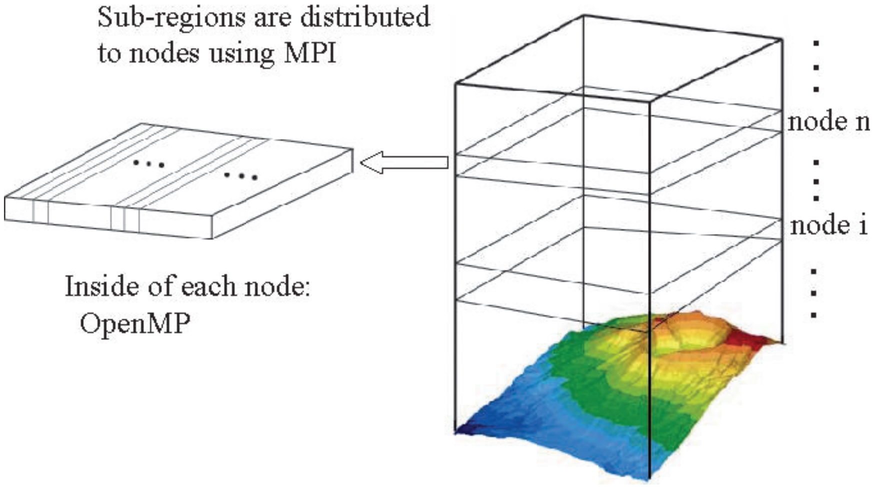

Figure 2. Hybrid parallel strategy employed in this study.

Although we have not spent too much effort in optimizing the code yet, speed up of 160 is obtained by utilizing 320 PEs of the Earth Simulator located at the Earth Simulator Center in Yokohama Japan. The processing element of the Earth Simulator consists of vector pipelines and numerical codes are vectorized when they are run on the Earth Simulator. Computations are also done on Cray XD1 computer system that has similar memory-PE configuration of the Earth Simulator but without vector pipelines.

3. ERUPTION MODELS

Knowledge about how and how much of the energy is released during an eruption is required for constructing initial and boundary conditions of the numerical simulations. However the energy release processes of volcanic eruptions are quite complex and, in many cases, not enough data are available. Therefore modeling of eruptions based on physically reasonable assumptions is necessary. Three different eruption models have been tried in the past. They are:

1. **High pressure reservoir model:** Eruption is modeled as an explosion of a pressurized reservoir placed at the eruption cite. The reservoir is filled with a high-pressure gas whose thermodynamic conditions are determined from the expected amount of released energy. This model is simple and easier to implement and commonly used in many blast wave simulations. However, even for this simple model, there are many parameters to be determined. From the amount of assumed total release energy, the pressure, density, temperature and the velocity (this is usually zero) of the initial filling gas are computed with assumptions of the size and shape of the reservoir based on available data from observations.

2. **Jet model:** Eruption is modeled as a gas jetting from a crater. It is also simple and implemented as a part of boundary conditions on the ground. It requires the velocity and the thermodynamic conditions of jetting volcanic gas at the crater. Parameters such as the duration time and the size of jetting area etc. are also necessary. The jet pressure at the crater is usually assumed to be atmospheric and the density is calculated by the equation of state from the values of pressure and temperature.

3. **Shock tube model:** In this model, a volcanic conduit is included in the computational region. High-pressure gas is placed at the bottom of the conduit just like a driver section of a shock tube and the other tube end is open at the crater. Eruption is modeled by the shock wave and the flow behind the shock that come out from the crater. Modeling the volcanic conduit, this may be considered as the most realistic one among the three. However, very few information about the structure of volcanic conduit is obtained for almost all volcanoes. Consequently, there are so many uncertainties in the model parameters such as the geometries and initial thermodynamic conditions inside the conduit. It is expected that the behavior of blast waves and the successive flows above the crater depend so much on what takes place in the conduit. Therefore, this model is practical only when enough information on the shape and initial thermodynamic states of the conduit is well known.

4. NUMERICAL RESULTS

Typical results of numerical simulations of blast waves and pyroclastic flows induced by volcanic eruptions are shown. Numerical conditions for computations shown in this chapter, Figs.3 to 5are summarized in Table 1. These computations were carried out on Cray XD1 computer system. Typical computation time was about 4 days by using four processing elements. Due to limited computer resources, numerically simulated area of these cases is relatively small for obtaining practically useful information for hazard prediction. However, these results demonstrate capability and usefulness of such numerical simulations for establishing practical safety measure against explosive-type volcano eruptions and for studying the mechanisms of volcano eruptions.

Table 1. Numerical conditions for simulations of volcano eruptions

	Figure 3	Figure 4	Figure 5
Jet velocity [m/s]	200	200	150
Crater radius [m]	30	30	460
Mass discharge rate [kg/s]	2.48×10^6	2.48×10^6	4.38×10^8
Time from eruption started	2s	30s	60s
Cell numbers (x, y, z-direction)		200 × 200 × 400	
Initial density of eruption cloud [kg/m^3]		4.39	
Specific heat ratio of eruption cloud		1.0145	
Specific heat ratio of ambient air		1.4	
Temperature of erupting cloud [K]		1000	
Temperature of ambient air [K]		273	
Pressure of erupting cloud [Pa]		101300	
Pressure of ambient air [Pa]		101300	

4.1. Blast waves induced by volcano eruptions

Figure 3 shows pressure distributions obtained for an imaginary eruption of volcano Usu Mt. in Hokkaido Japan. Jet model is used and the jet velocity of 200 m/s is assumed. The figures are the three-dimensional representation (left) of the primary blast wave and the pressure distribution on a two-dimensional vertical cut plane (right). The digital elevation map with space resolution of 10m supplied by the Geographical Survey Institute, Ministry of Construction Japan is used to construct the numerical grids of the terrain. The diameter of the crater is 30 m and the figures represent results at 2s after the eruption started. Although the primary shock wave, at a glance, seems quite smooth and spherical, details of pressure contours on the vertical plane exhibits complex structure due to local wave reflections and diffractions on the complex terrain. The pressure at the expanding blast wave front rapidly decays to a sound wave. Therefore, although annoying, the blast wave itself usually is not a serious threat unless it is such a large scale eruption of Mt. St. Helena or Mt. Pinatubo.

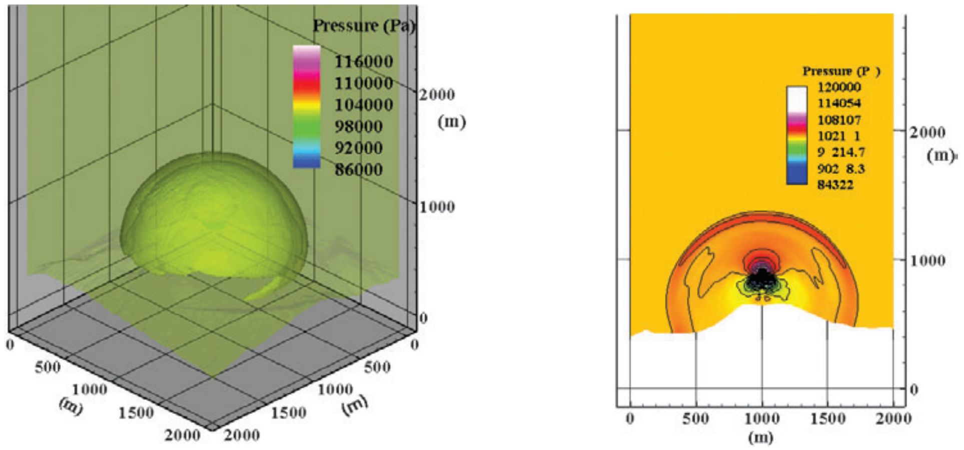

Figure 3. Blast wave induced by jet eruption model: 2s after the eruption, (Left) Iso-surface of pressure showing blast-wave shock front, (Right) Pressure distribution on the vertical cut-plane.

4.2. Pyroclastic flows induced by volcano eruptions

Pyroclastic flows are the high speed high temperature flows of volcanic gas mixed with solid particles sliding down the mountain side due to gravitational force. Since the flow velocity typically is several hundreds of m/s and the temperature could be several hundreds to even a thousand K, pyroclastic flow is one of the most devastating phenomena induced by volcano eruptions. Therefore, database for the pyroclastic flows obtained from numerical simulations can be valuable in view of hazard predictions.

Pyroclastic flows consist of two phases of gas and solid. In this study, however, the solid phase of particles is assumed to be continuum and treated like a gas. This assumption is not so unrealistic since most particles are considered to be small enough to follow the movement of the erupting gas. Therefore, the problem becomes simulations of two different gases, i.e. air and the volcanic gas. The system of basic equations for conservation laws of the gas mixture is expressed in the integral form as follows,

$$\int_V U\,dV\Big|_{t_0}^{t} + \int_{t_0}^{t} dt \int_\sigma \left(F_x n_x\,ds + F_y n_y\,ds + F_z n_z\,ds\right) = 0\ , \tag{1}$$

where V is a gas volume bounded by the closed surface σ with the outward normal unit vector $\vec{n} = (n_x, n_y, n_z)$. The conserved variable vector U and the flux components in x, y and z directions and F_x, F_y and F_z are written as,

$$U = \begin{pmatrix} \rho_1 \\ \rho \\ \rho u_x \\ \rho u_y \\ \rho u_z \\ E \end{pmatrix},\quad F_x = \begin{pmatrix} \rho_1 u_x \\ \rho u_x \\ \rho u_x^2 + p \\ \rho u_x u_y \\ \rho u_x u_z \\ (E+p)u_x \end{pmatrix},\quad F_y = \begin{pmatrix} \rho_1 u_y \\ \rho u_y \\ \rho u_y u_x \\ \rho u_y^2 + p \\ \rho u_y u_z \\ (E+p)u_y \end{pmatrix},\quad F_z = \begin{pmatrix} \rho_1 u_z \\ \rho u_z \\ \rho u_z u_x \\ \rho u_z u_y \\ \rho u_z^2 + p \\ (E+p)u_z \end{pmatrix},$$

where p is the pressure and the symbols u_x, u_y, u_z are the velocity components in x, y, z directions, respectively. The symbol ρ_1 is the density of the air and the one for the volcanic gas ρ_2 is obtained by $\rho_2 = \rho - \rho_1$ with ρ being the density of the gas mixture. E is the total energy per unit volume, $E = e\rho + \frac{\rho}{2}(u_x^2 + u_y^2 + u_z^3)$, and e is the specific internal energy. In order to close the system of equations, the caloric equation of state, $e = p/(\gamma - 1)\rho$, is used.

The specific heat ratio γ of the mixture must be specified as a gasdynamic parameter. It is determined from those of air γ_1 and the volcanic gas γ_2 by the following formula at each numerical cell,

$$\gamma = x\gamma_1 + (1-x)\gamma_2 \, , \tag{2}$$

where x is the mass fraction of the air in the mixture. Since gravity plays the main role in the pyroclastic flows, it is included in the basic equations. In the computation, the gravity effect on the flow is taken into account by the operator splitting technique [16] by integrating the following set of equations at the end of each time step,

$$\frac{dU}{dt} = I(U), \quad I = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \rho g \\ \rho g u_z \end{pmatrix} \tag{3}$$

where g is gravitational acceleration. The integration is carried out with either second of forth-order Runge-Kutta method.

Figure 4 shows typical numerical results of distributions of the mass 30s after the eruption started. The jet velocity at the crater is 200 m/s and the crater radius is 30m. The blast wave generated at the beginning of the eruption has already gone outside the computational region. It is shown clearly that the volcanic-gas jet changes from laminar to turbulent.

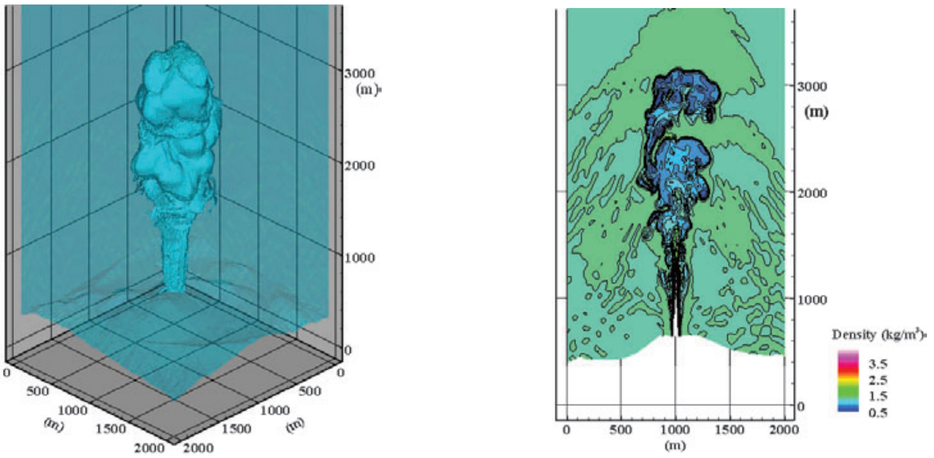

Figure 4. Volcanic ash cloud induced by jet eruption model: 30s after eruption started, (Left) Iso-surface of density showing ash cloud, (Right) Density distribution on the vertical cut-plane.

Figure 5 is the numerical results at 60s after the eruption. Vortices of different sizes are generated and developed. This figure is quite similar to some of actual cases. In this case, the jet velocity is 150 m/s and the crater radius is 460m. It is seen that the volcanic cloud starts to come down as pyroclastic flow at its root near the ground. It is also noticed that the cloud becomes unstable and it is observed that the erupting gas falls downward due to gravity. Extensive studies on behavior of volcanic ashes are reported by Suzuki et al. [17]. It is found that current numerical results compare well with their results.

5. CONCLUSIONS

Numerical simulations of hazardous phenomena accompanied by explosive type volcano eruptions are carried out in order to demonstrate capability and usefulness of such numerical tool as a practical means of providing quantitative database for establishing better safety measures. In order to solve problems for a specific volcano, 3D computations including actual terrain of the area is carried out. Different eruption models as the initial and boundary conditions for computations are introduced and discussed.

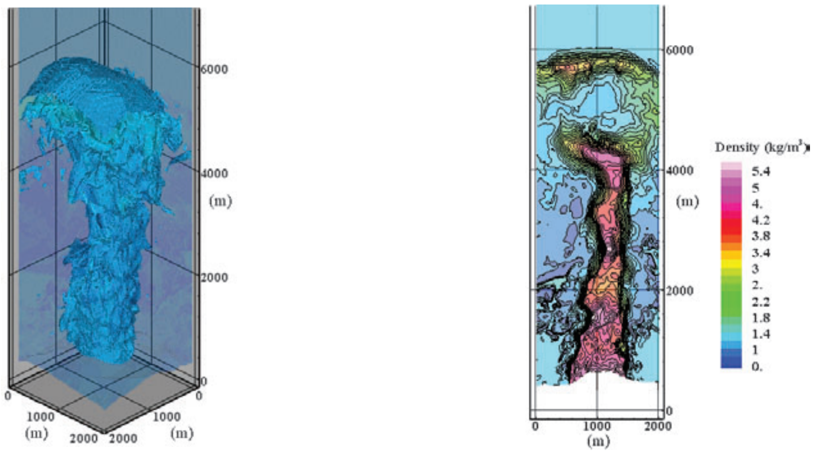

Figure 5. Volcanic ash cloud induced by jet eruption model: 60 s after eruption started, Jet velocity 150 m/s, (Left) Iso-surface of density showing ash cloud, (Right) Density distribution on the vertical cut-plane.

Parallel computational method with combination of MPI and OpenMP is employed and reasonable performances are obtained. The numerical method with jet eruption model is applied to an imaginary eruption of Mt. Usu to demonstrate how the blast wave is created and propagated. Simulations of volcanic gas flows are also shown as a part of investigations of pyroclastic flows.

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